

CICERO Working Paper 2003:04

# Option Values and the Timing of Climate Policy<sup>1</sup>

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September 2003

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<sup>1</sup> Paper originally prepared for the 12<sup>th</sup> Annual Conference of Eaere, Bilbao, Spain, June 2003

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**Tittel:** Option Values and the Timing of Climate Policy

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CICERO Working Paper 2003:04  
33 sider

**Finansieringskilde:** Norges Forskningsråd

**Prosjekt:** Deciding Who does What and When: Four Essays on the Economics of Global Climate Change (doktorgradsavhandling)

**Prosjektleder:** Camilla Bretteville  
**Kvalitetsansvarlig:** Jon Vislie

**Nøkkelord:** klimapolitikk, beslutningskriterier, global oppvarming, irreversibilitet, læring, opsjonsverdi, valg av tidspunkt, usikkerhet

**Sammendrag:** Denne artikkelen ser på betydningen av irreversibiliteter i klimapolitikken. Modellen som blir presentert her fanger både opp irreversible endringer av klimaforandringer som følge av klimagassutslipp og irreversibiliteter knyttet til utslippsreducerende investeringer, som begge påvirkes av usikkerhet som gradvis er i endring pga. læring. Klimairreversibilitet knytter en opsjonsverdi til 'tidlig handling' strategien så lenge det er en positiv sannsynlighet for at irreversibilitetsskranken er bindende. Investeringsirreversibilitet, derimot, skalerer ned fremtidige klimaeffekter på samme måte som en økning i diskonteringsraten ville gjort. Effekten opsjonsverdien av tidlig handling har på politikkutformingen reduseres jo mer irreversible investeringene er, men denne effekten er mindre jo lengre tidsperiodene vi ser på er. Hvilken politikk som foretrekkes avhenger av den relative størrelsen på disse opsjonsverdiene og på de mulige justeringskostnadene. Hvis summen av klimaopsjonsverdien og justeringskostnaden forårsaket av for beskjedene utslippsreducerende tiltak overgår summen av investeringsopsjonsverdien og justeringskostnaden forårsaket av for store utslippsreducerende tiltak, er nettoopsjonsverdien positiv, klimairreversibiliteten dominerer, og 'tidlig handling' bør velges framfor 'vent og se' strategien.

**Språk:** Engelsk

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**Title:** Option Values and the Timing of Climate Policy

**Author(s):** Camilla Bretteville and H. Asbjørn Aaheim

CICERO Working Paper 2003:04  
33 pages

**Financed by:** The Research Council of Norway

**Project:** Deciding Who does What and When: Four Essays on the Economics of Global Climate Change (dissertation)

**Project manager:** Camilla Bretteville  
**Quality manager:** Jon Vislie

**Keywords:** climate change policy, decision criteria, global warming, irreversibility, learning, option value, timing, uncertainty

**Abstract:** This paper reconsiders the importance of irreversibilities in climate change policy. The model presented here captures the irreversibility of both climate change due to greenhouse gas emissions and abatement technology investments, both of which are subject to uncertainty that will necessarily change gradually over time as a result of learning. The climate irreversibility adds an option value to the 'early action' strategy as long as there is a positive probability of encountering the climate irreversibility constraint. In contrast, irreversibility in abatement investments scales down the future climate effects in the same way as an increase in the discount factor would. The effect the option value of early abatement has on policy making is reduced the more irreversible investments are, but this effect decreases with the length of the time periods. The preferred policy option depends on the relative size of these option values and the possible adjustment costs. If the sum of the climate option value plus the adjustment costs resulting from too low initial abatement exceeds the sum of the investment option value plus the adjustment costs resulting from too high initial abatement, the net option value is positive, the climate irreversibility effect dominates, and 'early action' should be preferred over the 'wait-and-see' policy.

**Language of report:** English

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Or be downloaded from:  
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# Acknowledgements

We are grateful to Espen Froyn, Fredric C. Menz, Jon Vislie, participants at the 12<sup>TH</sup> Annual Conference of EAERE in Bilbao, Spain, June 2003, in particular Ronan Congar, and our colleagues at CICERO for helpful discussions on this material and comments on earlier drafts. We also thank Espen Froyn for excellent Excel macro programming, Lynn Nygaard for editorial assistance, Tone Veiby for drawing figure 1, and the Research Council of Norway for financial support. Responsibility for errors and opinions are our own.

“All things are so very uncertain,  
and that's exactly what makes me feel reassured.”  
Too-Ticky in *Moominland Midwinter*  
by Tove Jansson

## 1 Introduction

Intuitively it seems very plausible that environmental irreversibilities would play an important role in global climate change. In the economics literature, however, this is widely disputed. Due to the long time horizon and the uncertainty aspects of the problem, climate change policy decisions made today may influence welfare hundreds of years into the future. It is also likely that many of the changes caused by global warming will be irreversible. One example is the possible change in the path of the Gulf Stream; it can hardly be assumed that it could be moved back to its original basin once change has occurred. Another example is that changes in the climate might destroy unique ecosystems or drive certain species to extinction if their habitats are destroyed. Also, since greenhouse gases accumulate in the atmosphere, the emissions themselves are irreversible.<sup>4</sup>

Irreversibilities and learning in environmental economics were first formalized by Arrow and Fisher (1974) and Henry (1974), and the implication for climate change policy was to introduce mitigation strategies now to keep future options open ('early action'). However, later contributions (Kolstad 1996a,b; Ulph and Ulph 1995) downplay the importance of the irreversibility of climate change due to greenhouse gas emissions (henceforth referred to as *climate irreversibility*) in favor of the irreversibility in abatement technology investments (henceforth referred to as *investment irreversibility*), and thus advocate a 'wait and see' strategy. The purpose of this paper is to isolate policy effects of option values linked to the two irreversibilities and to explore the question: Are irreversibilities in climate change important?

This question is also addressed in Pindyck (2002), analyzed as an optimal stopping problem. He looks at when (if ever) climate policy should be adopted. So adopting a policy today competes not only with never adopting the policy, but also with adopting it next year, in two years, and so on. He focuses largely on a one-time adoption of an emission reducing policy, whereas this paper compares adjustable policies of doing a little, doing a lot, or doing something in between.

We use a stylized model to highlight three important uncertainty effects on climate change policy. The simplicity of the model is chosen to enable an explicit illustration of these effects. When uncertainty is high, the likelihood of making a perfect guess is low. Under a process of learning, no matter what strategy is chosen, costs must thus be expected because of adjustments desired due to new information. This is the first effect we study. In addition, there are two opposing option values of policies that maintain flexibilities: one related to climate irreversibility and the other related to investment irreversibility. These conflicting irreversibilities were first treated in Kolstad (1996a). He finds that, if the learning process is sufficiently slow, compared with the rates of pollution decay, and capital depreciation, learning does not influence the decision. However, if learning is significant, the two irreversibilities can affect the desired first-period level of emissions in opposite directions. The dominate effect is determined by the relative sizes of the rates of pollution decay, and capital depreciation, as well as the expectations about damages. We show that the climate irreversibility imposes an option value to the 'early action' strategy if, and only if, there is a positive probability of encountering the climate irreversibility constraint, and similarly that the investment irreversibility imposes an option value to the 'wait and see' strategy if, and only if, there is a positive probability of encountering the investment irreversibility constraint.

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<sup>4</sup> Every gas has a specific atmospheric lifetime – that is, the time it takes before  $1/e$  of an emitted quantity of the gas is left in the atmosphere, e.g. 114 years for  $N_2O$ , 12 years for  $CH_4$ , and up to 200 years for  $CO_2$ . No single lifetime can be defined for  $CO_2$  because of the different rates of uptake by different removal processes (IPPC 2001).

We also show that irreversibility in abatement investments scale down the future climate effects in the same way as an increase in the discount factor. Furthermore, the effect the option value of early abatement has on current policy choice is reduced the more irreversible the investments, but this effect decreases as the time horizon increases.

Our framework allows a clear distinction between costs caused by desired adjustments due to new information and the two opposing option values. The preferred policy option depends on the relative size of these option values and of the ex ante adjustment costs. We conclude that if the sum of the climate option value plus the expected adjustment cost resulting from too low initial abatement exceeds the sum of the investment option value plus the expected adjustment cost resulting from too high initial abatement, the net option value is positive, the climate irreversibility effect dominates, and ‘early action’ should be preferred over a ‘wait and see’ policy.

## **2 Background**

Although insights into the basic science of climate change have improved substantially over the past decades, there is still uncertainty around almost every aspect – including future levels of emissions, temperature changes resulting from greenhouse gas concentrations, impacts from changes in global mean temperature, technological developments, and costs of abatement and adaptation. One might think that the degree of uncertainty would significantly influence the political processes and decisions, and that the economic analyses thus would treat uncertainty as a central feature. Nevertheless, the main perceptions about economic issues related to climate change and climate policy, for example as assessed and reported by IPCC (2001), are based on deterministic studies. This does not necessarily mean that the uncertainties have been ignored, but rather that this part of the economics literature is inconclusive.

One possible explanation is that most of the climate change studies are based on numerical models. The uncertainties per se are therefore difficult to trace. Results derived from numerical models depend on assumptions about economic relationships such as damage and abatement cost functions, which are far from well known. It is therefore difficult to tell, for example, how better ability to predict temperature change in the future will affect current policy choices, unless we assume that we know all there is to know about possible climate change impacts and the costs of mitigation. This is somewhat of a paradox, because the impact of learning is one of the main issues that have been studied in analyses of climate policy choice under uncertainty.

The issue of irreversibility and learning in environmental economics was introduced in the seminal papers by Arrow and Fisher (1974) and Henry (1974). Focusing on a one-time development decision, their basic idea was that making an irreversible decision induces additional costs because the current decision restricts future decision possibilities. This implies that an extra value, an option value, properly attaches to the reversible alternatives. This is the value of retaining the option to choose any of the alternatives in the light of new information – an option that is lost if an irreversible alternative was chosen in the first place. Thus, if there is a chance of learning, they argue, it becomes more important to keep future options open. With regard to climate change policy, the implication is that the current level of greenhouse gas emissions should be lower if there is a possibility of learning more about irreversible damages in the future. Arrow and Fisher (1974) refer to this as the ‘irreversibility effect.’

Kolstad (1996a,b) and Ulph and Ulph (1995) pointed out that there are two kinds of irreversibilities in the context of climate change. In addition to the climate irreversibility, there is also the investment irreversibility due to investment in sunk capital. Once the

abatement investment has been undertaken, the resulting capital is usually not easily converted back to consumption or other forms of capital. These papers downplay the importance of climate irreversibility relative to investment irreversibility, and thus conclude that the climate irreversibility effect need not dominate for greenhouse gas emissions (see Fisher 2001; or Heal and Kriström 2002 for more details). Brekke and Lystad (2000), however, observe that Kolstad is not depreciating the capital: his capital lasts forever. They show that with a small depreciation rate on capital, Kolstad's conclusion is turned around. Fisher and Narain (2003) observe that in Kolstad (1996b) the obvious reason why there are no findings of impacts from climate irreversibility is that, in the parameterization of the model, the non-negativity restriction on emissions, used in the model to define emissions irreversibility, is never binding. Fisher and Narain (2003) introduce endogenous risk to the problem and find that, like emissions irreversibility, this has a positive effect on abatement investments. Aaheim (2003), using a stochastic model with Brownian motion, shows that both investment irreversibility and climate irreversibility may affect optimal abatement significantly, but if the policy is updated frequently in accordance with new information, the two effects tend to weigh each other out.

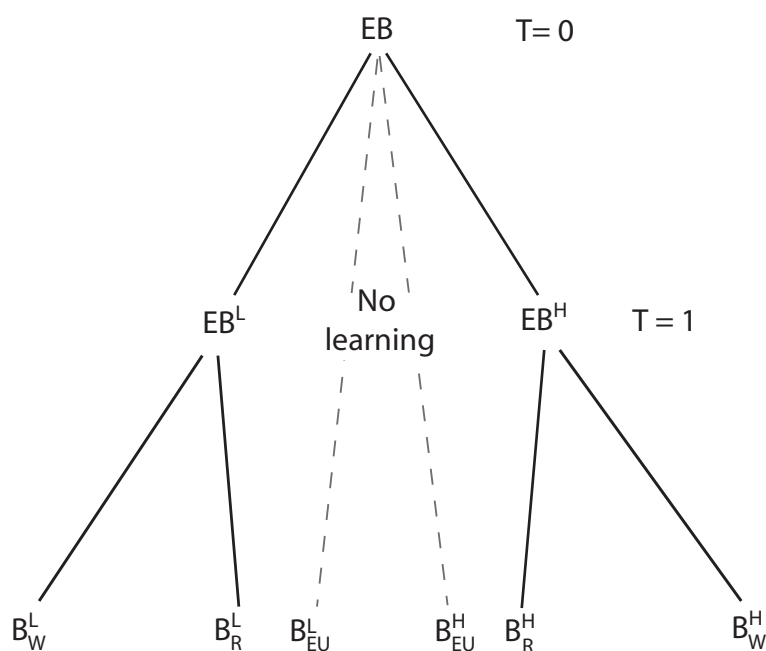
All in all, the rather sparse economic literature on the implications for the timing of climate policy of uncertainty and irreversibility seems to favor the 'wait and see' policy over 'early action.' Fisher (2001) argues that the reason for this result, which may appear counterintuitive to most, is that, given the assumptions or parameter values built into the models, there is relatively little cost in deferring abatement while waiting to learn more about the benefit. The climate irreversibility, therefore, plays less of a role in driving current policy recommendation on controlling emissions.

Contrary to the standard assumptions of economic climate change models, Fisher and Narain (2003) assume that the probability of a catastrophic impact at some point in the future (the next period), although very small, may be positively related to the level of greenhouse gas concentration in the atmosphere; the probability is endogenous. Their model also allows for the possibility of regrettable first period investments. Fisher and Narain find the effect of the investment irreversibility to dominate the effect of the climate irreversibility essentially because the concentration of greenhouse gases in the atmosphere does not change much over a 10 year period. However, Torvanger (1997), using a three-period stochastic dynamic programming model, shows that in the case of endogenous probability of irreversible climate change the climate irreversibility effect dominates.

The difference between a static and a dynamic analysis under uncertainty is that the decisions are taken sequentially in dynamic analyses. While a static problem gives a best solution once and for all, a dynamic problem aims at finding the best strategy under a given set of information. This is why learning becomes relevant: If uncertainty affects the decision, future amendments to information will change decisions in the future. Whether decisions of today are affected by future learning therefore depends on whether future decisions depend on present decisions, or, in the words of the option value literature, whether present decisions are irreversible.

The alternative strategies can be illustrated schematically by Figure 1. This is a two-period decision tree with alternative strategy paths. The dotted paths in the middle represent static expected utility maximization without learning. According to this strategy, at time  $t = 0$  (the first decision node) we choose abatement levels based on a weighted sum of the probabilities of states (our expectations). The probabilities are formed in accordance with our beliefs. If damages turn out to be high, we get the net benefit of  $B_{EU}^H$ ; and if they turn out to be low, we get  $B_{EU}^L$ . The outer branches represent sequential decision paths: At time  $t = 0$  we have to decide which state we perceive as the most likely, and choose our first period abatement level accordingly. Either way we risk making the wrong guess. The actual state is, however, now

revealed at time  $t = 1$ . This means that if we initially made the wrong guess we can now adjust our abatement effort according to the new available information. If we guess the state of damages to be low at  $t = 0$  (the left hand side in figure 1), the net benefit is  $B_R^L$  if we are correct and  $B_W^L$  if we are wrong. If we guess the state of damages to be high, the net benefit is  $B_R^H$  if we are right and  $B_W^H$  if we are wrong. Regardless of our choice of path, we



**Figure 1: Alternative decision strategy paths**

get a lower benefit if we make the wrong guess because the level of first period abatement is not chosen optimally.<sup>5</sup>

Such a sequential decision-making process aims to identify short-term strategies in the face of long-term uncertainties. The next several decades will offer many opportunities for learning and mid-course corrections. The relevant question is not “What is the best course of action for the next 100 years?”, but rather “What is the best course for the near-term given the long-term objective?” (IPCC 2001).

### 3 What can we expect to learn about climate change?

There is a vast amount of ongoing research aimed at improving our understanding of the implications of anthropogenic emissions of greenhouse gases. We will therefore learn more about the problem in the future, and most likely the scientific background for climate policy

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<sup>5</sup> The benefits in Figure 1 are in arbitrary order. Generally we can only say that  $B_R^H > B_W^H$  and  $B_R^L > B_W^L$ .



will improve. To the extent that learning has an influence on decision making, it should therefore be taken into account also in current climate policy choice.

However, a better understanding of climate change issues does not necessarily mean less uncertainty. The connection between uncertainty and the state of knowledge is illustrated by comparing reports from the UN Intergovernmental Panel of Climate Change (IPCC). The Second Assessment Report (IPCC 1995) estimates that a doubling of CO<sub>2</sub> concentrations in the atmosphere (2 x CO<sub>2</sub>) will lead to a global mean temperature increase between 1.8 °C and 4.2 °C. In the Third Assessment Report (IPCC 2001), the possible range of the future increase in global mean temperature increased to between 1.4 °C and 5.6 °C. The reason for the increase in uncertainty is improved knowledge in the underlying science linking greenhouse gas emissions to atmospheric accumulation and global temperature change. A better and more precise description of the relationships between policy, economic and human development and emissions, on the one hand, and atmospheric processes, on the other, improved the consistency between the scenarios in the Third Assessment Report (TAR), and resulted in a wider range of possible outcomes.

The Third Assessment Report was the first to also indicate the degree of uncertainty associated with its predictions by stating the degree of confidence the leading authorities in the field of climate change report with their forecasts, ranging from 'very high' (≥95%) to 'very low' (≤5%). Most of its findings are assigned a probability ranking (see the TAR or e.g. Heal and Kriström (2002) for more details). Even though improved knowledge may not reduce the total uncertainty in the future, the aspect of learning is still important, simply because some of the uncertainty is resolved as time goes by. Just as it is easier today to predict the climate in 2010 than in 2100, it will be easier in 2090 than it is today to predict the climate in 2100.

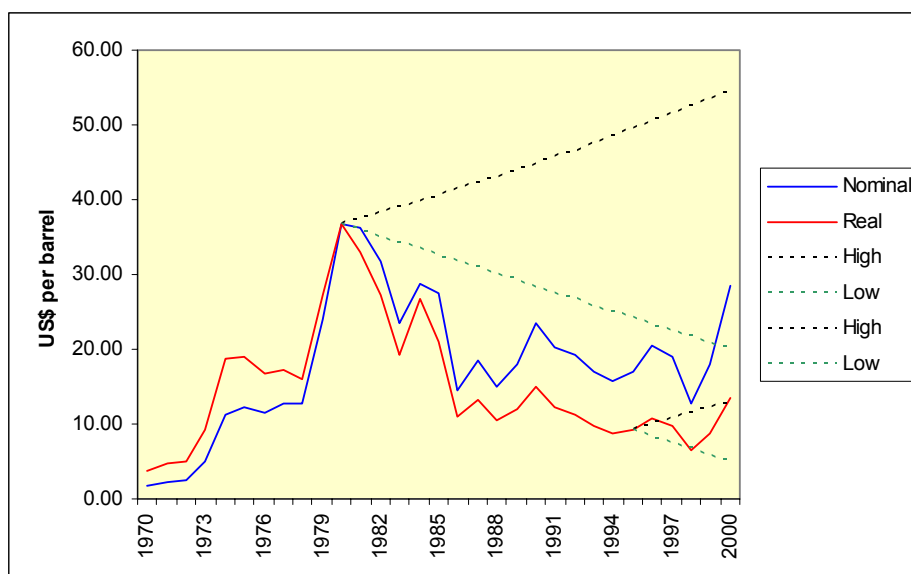
To illustrate this more or less passive way of learning, consider the price of crude oil over the past 30 years. In the wake of oil price hikes in the 1970s, significant effort was put into forecasting the oil price around 1980. Forecasts were based on uncertainty and Hotelling's rule,<sup>6</sup> which predicted an increase in the price of exhaustible resources, such as petroleum.

Figure 2 shows the actual development of oil prices from 1970 to 2000. Most forecasts of the trend made in the early 1980s were found in the upper half of the area between the dotted lines, typically in the range between US\$ 35 and US\$ 55, in real terms.<sup>7</sup> Thus, most pessimistic (low price) forecasts turned out to be far too optimistic. However, one cannot claim that oil price changes are better understood today than they were 20 years ago. It may be granted that the importance of the Hotelling rule was exaggerated, but this is also a result of past observations. Therefore, a forecast for the year 2000 made the year before with the same uncertainty had, of course, a much higher probability of being correct than a forecast made in 1980. The main point is that short-term predictions are more accurate than long-term predictions, since learning occurs along the way, regardless of whether the uncertainty has decreased.

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<sup>6</sup> See e.g. Hanley et al (1997) for a description of Hotelling's rule.

<sup>7</sup> Lorentsen et al. (1985) denotes by US\$ 20 a 'collapse price scenario.'



**Figure 2. Spot prices for Saudi light (1970–1979) and Brent Blend (1980–2000) in nominal prices and real 1990 prices.**

One may expect that gradual resolution of uncertainty will be considered the main source of learning in the climate policy process, at least in the foreseeable future. Some of the uncertainty may be irreducible in principle, and hence decision makers will have to continue to take action under significant degrees of uncertainty. Thus, the problem of climate change evolves as a subject of risk management in which strategies should be reformulated as new knowledge arises (IPPC 2001).

## 4 Irreversibilities and option values

This section analyses the abatement decision taken by an agent subject to uncertainty both in benefits of mitigating climate change (damages) and in abatement technology investment costs. We will concentrate on irreversibility in one of these variables at a time to see how both types of irreversibility affect the optimal solution. We start with a presentation of the general two-period model with uncertainty and learning. Then, we analyze the case of climate irreversibility, which corresponds to the traditional option value discussion. Next, we look at how investment irreversibility may affect the solution.

### 4.1 A two-period model with a binary distribution of outcomes

We consider climate policy over two periods, ‘the present’ and ‘the future.’ We do not discount ‘the present,’ hence this is referred to as period 0, and ‘the future’ is referred to as period 1. Without abatement, emissions of greenhouse gases are fixed in both periods and denoted  $e_0$  and  $e_1$ , in the initial and future time periods, respectively. Abatement may be invoked in either period at costs of  $a_0$  and  $a_1$ , respectively. The abatement cost per unit of emissions cut,  $c_i$ , is assumed to be independent of scale in each period. Hence, the actual abatement is  $a_0/c_0$  in period 0 and  $a_1/c_1$  in period 1.

The emissions of greenhouse gases in period  $t$  can then be written as  $[e_t - a_t/c_t]$ .

These add to the previous level of concentrations of greenhouse gases, denoted by  $y_{t-1}$ . Then, the development in atmospheric concentration<sup>8</sup> can be written  $y_t = y_{t-1} + e_t - a_t/c_t$ . For the two specific periods we have:

$$y_0 = \hat{y} + e_0 - \frac{a_0}{c_0}, \quad (1)$$

$$y_1 = y_0 + e_1 - \frac{a_1}{c_1}, \quad (2)$$

where the historical level of concentrations,  $\hat{y}$ , is assumed known.

Assume, moreover, that the relationship between concentrations and economic damages has a logarithmic form, with an exponent larger than one.<sup>9</sup> Without loss of generality, we choose a quadratic damage function for the sake of simplicity,  $f(y_t) = \alpha y_t^2$ , where  $\alpha$  is a constant. (See e.g. Aslaksen (1990) for a discussion on the choice of exponent.)

Recall that the periodic emissions levels are exogenously given. The benefits of abatement can thus be expressed by the damage avoided. For the initial period, this is

$$b(y_0) = q_0 (f(\hat{y} + e_0) - f(y_0)) = q_0 (\alpha(\hat{y} + e_0)^2 - \alpha(\hat{y} + e_0 - \frac{a_0}{c_0})^2) \quad (3)$$

where  $q_0$  is the price on damages or the willingness to pay for avoiding them in period 0. Similarly, the benefits of abatement in the future can be written,

$$b(y_1) = q_1 (f(\hat{y} + e_0 + e_1) - f(y_1)) = q_1 (\alpha(\hat{y} + e_0 + e_1)^2 - \alpha(\hat{y} + e_0 + e_1 - \frac{a_0}{c_0} - \frac{a_1}{c_1})^2) \quad (4)$$

In the initial period, there is certainty about present costs and benefits but uncertainty about the state in period 1 (the future). Furthermore, assume that learning takes place during the initial period, such that the decisions in the future are adjusted according to this new information. Since we simplify the problem to look at two periods, this is modeled as if the future decisions are made under full certainty. However, when we assume that the uncertainties resolve in the future, we refer only to the information revealed since we made the initial policy decision. In real life there will be substantial uncertainty remaining also in the future.

When deciding upon abatements, we take into account that our initial decisions will influence future benefits. We simplify the uncertainty by considering only two future states of

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<sup>8</sup> Note that concentrations are usually measured in terms of parts per million or billion by volume (ppmv and ppbv respectively), whereas emissions are measured in tons. We therefore convert emissions to concentration units and calculate costs in corresponding terms.

<sup>9</sup> The usual choice in economic studies is somewhere between 2 and 3.

the world: low damages (state A) and high damages (state B). Denote by  $\pi$  the probability of state A.

A standard expected utility maximizing agent determines

$$W = \max_{a_0, a_1} E \left\{ \sum_{t=0}^1 \frac{b(y_t) - a_t}{(1+r)^t} \right\}, \quad (5)$$

where  $E$  is the expectations operator over possible outcomes, and  $r$  is the discount rate. Note that, so far, we have not taken any kind of irreversibility into account. That is, the solution to this problem gives the optimal policy, from the initial-period perspective, when there are no other restrictions.

Inserting for  $b(y_t)$  we can write the welfare function as

$$W = \max_{a_0, a_1} E \left\{ \begin{aligned} & q_0 \left[ f(\bar{y} + e_0) - f\left(\bar{y} + e_0 - \frac{a_0}{c_0}\right) \right] - a_0 \\ & + \frac{1}{(1+r)} \left[ q_1 \left( f(\bar{y} + e_0 + e_1) - f\left(\bar{y} + e_0 + e_1 - \frac{a_0}{c_0} - \frac{a_1}{c_1}\right) \right) - a_1 \right] \end{aligned} \right\} \quad (6)$$

Assume that the damage cost function has a quadratic form  $f(y_t) = \alpha y_t^2$  and that uncertainty in period 1 is characterized by two states, A and B with a vector of outcomes,  $x = (q_1, \alpha_1, c_1)$ . such that

$$x = \left\{ \begin{aligned} & P(x^A) = \pi \\ & P(x^B) = 1 - \pi \end{aligned} \right\}$$

Then, the first order condition for abatement in period 0 can be written as:

$$1 = \frac{2q_0\alpha_0}{c_0} \left( \hat{y} + e_0 - \frac{a_0}{c_0} \right) + \frac{1}{1+r} \left[ \pi \frac{2q_1^A\alpha_1^A}{c_0} \left( \hat{y} + e_0 + e_1 - \frac{a_0}{c_0} - \frac{a_1}{c_1^A} \right) + (1-\pi) \frac{2q_1^B\alpha_1^B}{c_0} \left( \hat{y} + e_0 + e_1 - \frac{a_0}{c_0} - \frac{a_1}{c_1^B} \right) \right] \quad (7)$$

where  $\alpha^i$  is the initial damage per unit concentration, that is, the damage of an increase in concentrations by, for example, 1 ppmv carbon from “today” if the present damage is zero.

The term  $\frac{2q_t \alpha_t}{c_t} y_t$  is the marginal damage per euro of abatement when the concentration level is  $y_t$ . This term may be interpreted as the shadow price of concentrations at level  $y_t$ , and because of the quadratic damage function it is linear in the level of concentrations.

Similarly, the first order condition for abatement in the future period can be written as:

$$1 = (\hat{y} + e_0 + e_1 + \frac{a_0}{c_0}) \left( \pi \frac{2q_1^A \alpha_1^A}{c_1^A} + (1 - \pi) \frac{2q_1^B \alpha_1^B}{c_1^B} \right) - a_1 \left( \pi \frac{2q_1^A \alpha_1^A}{c_1^A} \frac{1}{c_1^A} + (1 - \pi) \frac{2q_1^B \alpha_1^B}{c_1^B} \frac{1}{c_1^B} \right) \quad (8)$$

Equations (7) and (8) imply that the marginal expected value of the concentrations in each period should be equal to 1, which is the marginal cost of abatement in value terms.

In order to simplify the expressions, define the following constants:

$$C_0 = (\hat{y} + e_0) \frac{2q_0 \alpha_0}{c_0} + \frac{\hat{y} + e_0 + e_1}{1 + r} \left[ \pi \frac{2q_1^A \alpha_1^A}{c_0} + (1 - \pi) \frac{2q_1^B \alpha_1^B}{c_0} \right] - 1,$$

$$C_1 = (\hat{y} + e_0 + e_1) \left[ \pi \frac{2q_1^A \alpha_1^A}{c_1^A} + (1 - \pi) \frac{2q_1^B \alpha_1^B}{c_1^B} \right] - 1.$$

$C_0$  and  $C_1$  are both expressions for the maximum of expected marginal damage per euro abatement., They both refer to the concentrations with no abatement, that is ‘business as usual’, but differ in terms of reference to abatement cost and the length of the period for which abatement will work. Thus  $C_0$  refers to abatement in period 0, and  $C_1$  to abatement in period 1, Moreover, define

$$\Phi_0 = \frac{2q_0 \alpha_0}{c_0} \frac{1}{c_0} + \frac{1}{1 + r} \left[ \pi \frac{2q_1^A \alpha_1^A}{c_0} + (1 - \pi) \frac{2q_1^B \alpha_1^B}{c_0} \right] \frac{1}{c_0},$$

$$\Phi_1 = \pi \frac{2q_1^A \alpha_1^A}{c_1^A} \frac{1}{c_1^A} + (1 - \pi) \frac{2q_1^B \alpha_1^B}{c_1^B} \frac{1}{c_1^B},$$

$$\Psi_0 = \frac{1}{1+r} \left[ \pi \frac{2q_1^A \alpha_1^A}{c_0} \frac{1}{c_1^A} + (1-\pi) \frac{2q_1^B \alpha_1^B}{c_0} \frac{1}{c_1^B} \right],$$

$$\Psi_1 = \pi \frac{2q_1^A \alpha_1^A}{c_1^A} \frac{1}{c_0} + (1-\pi) \frac{2q_1^B \alpha_1^B}{c_1^B} \frac{1}{c_0}.$$

Instead of interpreting these constants note that  $\Phi_i a_t$  ( $i = 0,1$ ) is a parallel to the marginal expected benefit of the abatement quantity  $a_t/c_t$  ( $t = 0,1$ ) emissions over the periods that the abatement works.  $\psi_t a_t$  is the marginal expected benefit of the abatement of the quantity  $a_t/c_t$  the period  $i \neq t$

Now the first order conditions can be written as the linear equation system:

$$C_0 = \Phi_0 a_0 + \Psi_0 a_1, \tag{9}$$

$$C_1 = \Psi_1 a_0 + \Phi_1 a_1. \tag{10}$$

Equations (9) and (10) require that the maximum marginal damage of concentrations without abatement in each period equal the marginal benefit of abatement in optimum. Now the level and the distribution of abatement in the two periods can be written as two linear functions in  $a_0$  and  $a_1$ . We rewrite (9) and (10) and get

Period 0: 
$$a_0 = \frac{C_0}{\Phi_0} - \frac{\Psi_0}{\Phi_0} a_1 \tag{11}$$

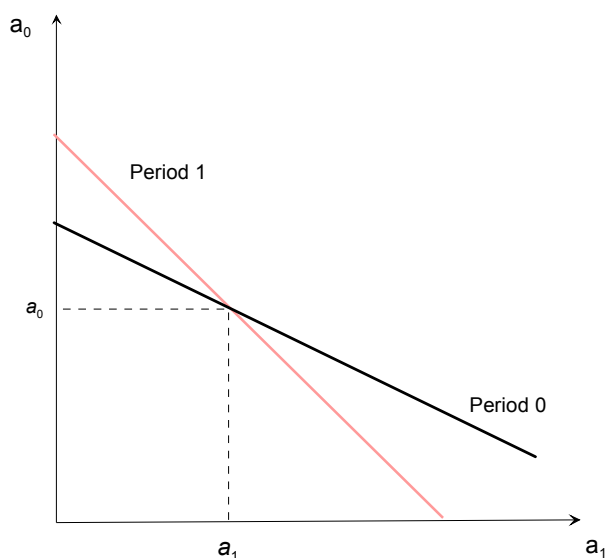
Period 1: 
$$a_0 = \frac{C_1}{\Psi_1} - \frac{\Phi_1}{\Psi_1} a_1 \tag{12}$$

These two linear functions express the trade-off between abatement costs in the two periods. Thus, for period 0, a reduction of 1 € in present abatement cost,  $a_0$ , must be replaced by an increase of  $\psi_0/\Phi_0$  € in period 1 in order to keep marginal costs equal to marginal benefits. For period 1, a 1 € reduction in period 0 must be replaced by increased abatement cost of  $\Phi_1/\psi_1$  € in period 1, if marginal costs are to be equal to marginal benefits.

Figure 3 illustrates the optimal allocation of abatement costs in the two periods. The curves correspond to (11) and (12) and represent all abatement allocations whereby equal marginal costs equal marginal benefits. That is, the period 0 curve shows all combinations of  $a_0$  and  $a_1$  that satisfy the first order condition for period 0, and the period 1 curve shows all combinations that satisfy the first order condition for period 1. Where the curves intersect, both conditions are satisfied simultaneously.

Abatement costs for period 0 start at a lower level and increase at a lower rate compared to the line for period 1. From the definitions of the constants,  $\psi_0/\Phi_0 < \Phi_1/\psi_1$ , unless there is large uncertainty in abatement costs (see appendix). If so, the trade-off curve is steeper in period 1 than in period 0. Then, a necessary, but not sufficient, condition for interior solution is that  $C_1/\psi_1 > C_0/\Phi_0$ . Alternatively, if  $\Phi_1/\psi_1 < \psi_0/\Phi_0$  we must have  $C_1/\psi_1 < C_0/\Phi_0$  to obtain an interior solution. It is seen from the definition of  $C_1$  and  $C_0$  that this depends on the relationship between damage costs and abatement costs in period 0 and period 1. ‘Corner solutions’ with abatement in only one period is possible, but the following discussion will be based on the case of an interior solution. Then a reduction in present abatement may be compensated by more abatement in the future.

The optimal solution is found where the two lines intersect. Consider an allocation with abatement in period 0 only. Then the total benefit of abatement would be  $C_0/\Phi_0$ . This value, however, would increase if some abatement were postponed to period 1. Reallocation from today until the future would, in fact, increase the total benefit of abatement until the two lines intersect. From this point on, further reallocation of abatement between the two periods will give lower total benefit. Intertemporal optimality is therefore attained at the intersection.



**Figure 3. Optimal allocation of abatement between the present and the future**

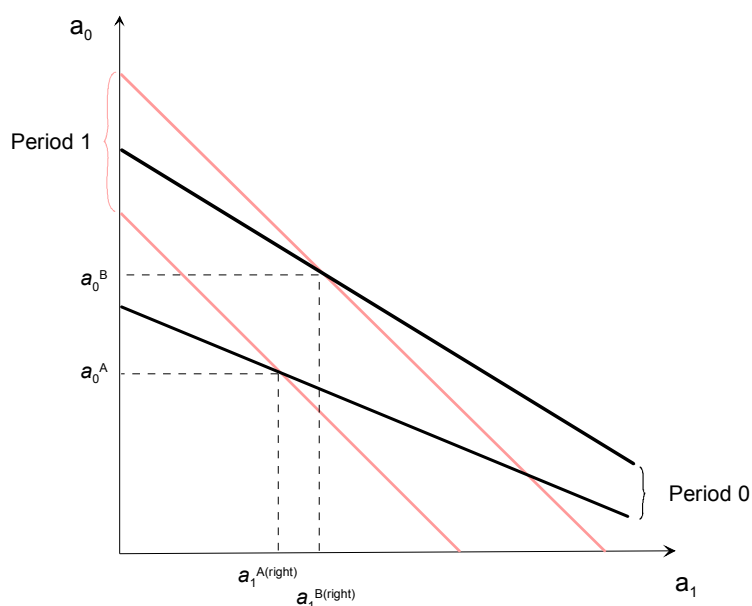
Changes in the parameters will affect total abatement, as well as the allocation between the periods. If  $q_1\alpha_1$  increases, the constants for abatement assigned to both periods increase, but more so in equation (12), which applies for the future (period 1), than in equation (11), which applies for the present (period 0). Also the multiplicative term in equation (11) increases, while the multiplicative term in equation (12) is unchanged. In sum, both total and future abatement increase compared with present abatement. An increase in the expected unit cost of

future abatement does not affect the constant term, but reduces the multiplicative term for period 0. The constant term for period 1 decreases, while the multiplicative term in period 1 increases. All these three shifts thus contribute to an allocation of abatement from the future to the present.

An increase in the discount rate affects only the abatement assigned to period 0 in this model. The effect is that the present value of abatement in period 1 is reduced. This contributes to a reduction of the abatement in period 0. At the same time, abatement in period 1 becomes relatively less expensive. This tends to change the trade-off between abatement in the two periods. Both effects reduce abatement in period 0. Some may be allocated to period 1, but the total amount is likely to be unchanged.<sup>10</sup>

Changes in uncertainty on future beliefs will also affect optimal abatement in the two periods. One exception is the case where there is uncertainty only in future damage costs, while the future abatement costs are certain. Appendix 1 shows that all the above-defined constants remain unaffected if the expected damage remains unchanged. However, if future damage costs are certain, but abatement costs for this period are not, we show in the appendix that the constants will increase. Except for the constant term in equation (11), the terms in equations (11) and (12) will also be affected. The multiplicative term for period 0 increases, while both terms for period 1 decrease. All changes contribute to a reallocation of abatement from the present to the future.

An illustration of the case with uncertain damages is given in Figure 4A. This is simply a more complicated version of Figure 3, where there is uncertainty regarding the position of the trade-off curves. Figure 4A displays two alternative trade-off curves between the abatement in period 0 (the present) and period 1 (the future): one pair representing state A (low future damage costs) where the optimal solution is  $a_0^A$  and  $a_1^{A(right)}$ , and one pair representing state B (high future damage costs), where the optimal choice is  $a_0^B$  and  $a_1^{B(right)}$ . These two solutions are to be considered the boundaries for the abatement choices. The rational decision maker would choose an allocation between the two extremes, based on his subjective probability distribution over states. For example, insertion of the parameters in equations (11) and (12) gives the best ex ante allocation of abatement investments for the expected utility maximizer.

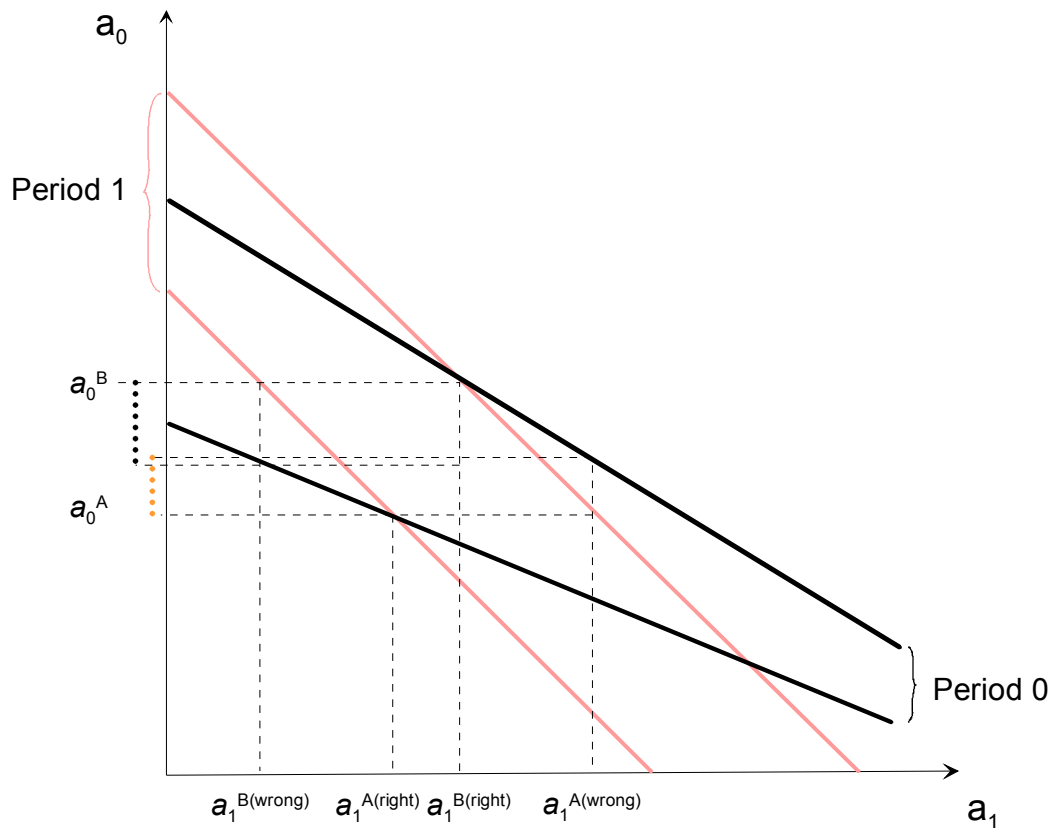


**Figure 4A. Optimal abatement allocation when damage costs are uncertain**

<sup>10</sup> Appendix 2 shows the changes in these parameters for a specific numerical example.



Because of uncertainty, the initial period decision is based on guesses about the future state of the world and the amount of next period abatement. If learning takes place (uncertainty is lessened) before the final decision for the next period, the decision maker may want to make adjustments to her beliefs in period 0. Irreversibilities place a constraint upon these adjustments which will be addressed below, but let us first take a closer look at the costs of a policy adjustment. As a benchmark, assume that the initial period decision is based exclusively on beliefs about either state A (the lower pair of trade-off curves) or B (the upper pair of trade-off curves) in Figure 4A, and not on maximization of expected utility. Furthermore, assume that one of these states is realized in period 1. If the guess is right, future abatement ( $a_1$ ) will be chosen as predicted. If the guess is wrong, however, period 1 abatement should be adjusted according to the new information. Since the choice of initial period abatement ( $a_0$ ) was conditioned upon a guess about  $a_1$ , the benefits will be higher when the guess is right. The situation for when the guess is as wrong as it can get is illustrated in Figure 4B. Here we put all our eggs in one basket, but after learning we wish we had put them in the other. The best abatement solutions after learning that the guess was wrong are marked  $a_1^{A(wrong)}$  and  $a_1^{B(wrong)}$ , respectively (see below for explanations).



**Figure 4B. Cost of guessing wrong under damage cost uncertainty**

Recall that each curve represents abatement allocations that satisfy the period's first order condition for optimal abatement, and that the social loss of guessing wrong can be measured in present value terms. According to the objective function (6), this can be measured by differences in initial (period 0) abatement costs. In Figure 4B these losses are indicated by the two vertical lines to the left of the  $a_0$ -axis. The black dotted line represents the loss incurred when the wrong guess was the high damage cost scenario, and the grey one the loss for when

the wrong guess was the low cost alternative, and in both cases the other extreme was learned to be the real state of the world. The explanation goes as follows:

Start with supposing we based our decision on the high damage cost scenario (early action). Then the initial period choice of abatement cost was  $a_0^B$ . Suppose further that at the start of the next period we discover that damage costs are instead low. Thus, now we wish we had chosen  $a_0^A$  initially, but since we cannot move back in time, this is no longer a possible option. For the optimality condition to hold for period 1, abatement cost must be on the period 1 trade-off curve for state A, and the best move is to choose  $a_1^{B(wrong)}$ . This is a reduction compared with our initial guess for period 1 abatement, and the social loss of guessing wrong can be measured by the difference between the  $a_0$  value where  $a_1^{B(right)}$  crosses the trade-off curve for period 0 in scenario A and the  $a_0$  value that was chosen, namely  $a_0^B$ . This social loss is thus the difference between what we thought our initial abatement was worth and what it turned out to be worth according to the information received. Since we abated too much in period 0, the initial abatement is worth less than we thought. The loss is illustrated by the black dotted line along the  $a_0$ -axis.

Suppose instead that the initial guess is the low damage cost alternative (wait-and-see). The period 0 abatement cost is then  $a_0^A$ . Suppose further that at the start of the period 1, we discover that costs instead are high. Since the optimal allocation for state B is no longer attainable, the best we can do is to satisfy the optimality condition for period 1, given this new information. In this situation we abated too little initially and have to compensate by abating more than we thought we would in period 1. The best choice is  $a_1^{A(wrong)}$ . The social loss of guessing wrong, in this case, can be measured as the difference between  $a_0^A$  and the  $a_0$ -value where  $a_1^{A(wrong)}$  intersects with the trade-off curve for period 0 in state B. The loss is illustrated by the grey dotted line along the  $a_0$ -axis in Figure 4B.

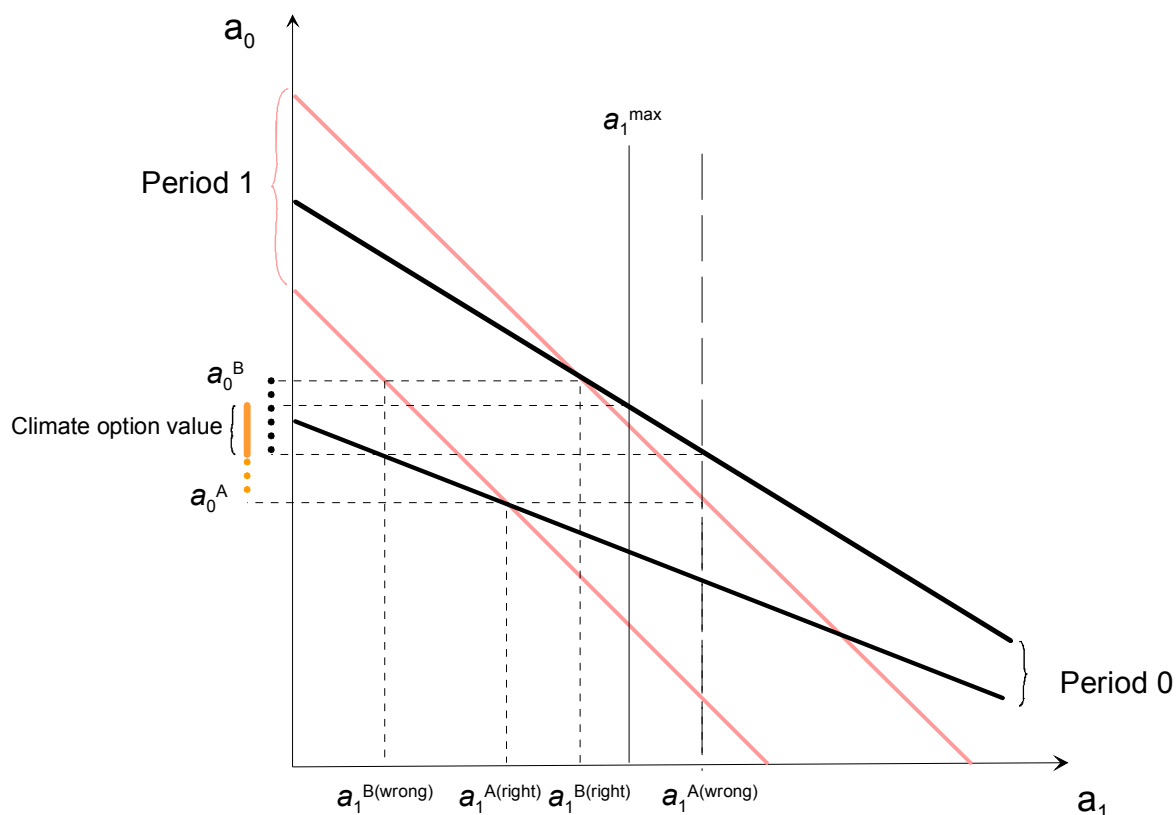
#### **4.2 Irreversible emissions of greenhouse gases.**

So far, we have imposed no restriction on possible abatement in each period. Since emissions of greenhouse gases are considered irreversible, however, abatement in each period cannot exceed the emissions in that period. This gives the constraint

$$a_1^* \leq c_1^* e_1 \tag{13}$$

where  $c_1^*$  is the actual unit costs in period 1 and  $a_1^*$  is the optimal abatement cost in this period. (Note that both  $c_1$  and  $e_1$  are exogenous.) If the decision in the initial period was based on a prediction of higher future abatement than  $c_1^* e_1$ , the optimal solution cannot be achieved. If future emissions are low or uncertainty is large, or both, there will be a positive probability for encountering this irreversibility constraint. The lower we set present abatement, the higher is this probability, and an extra value thus attaches to the choice of high initial abatement. This is the option value of early abatement.

Assume that there is uncertainty in the damage costs only and that these turn out to be higher than expected. Initial abatement was then lower than optimal, when evaluated in hindsight. A relative loss has therefore occurred and adjustments are needed. In this case abatement will have to be adjusted upwards, something that can cause a violation of the maximum abatement constraint (13). If (13) is violated, the optimal level of abatement is not attainable, the climate irreversibility constraint is binding, and the best we can do is to choose the maximum level of abatement.



**Figure 4C. Cost of guessing wrong under climate irreversibility**

The option value of early abatement is illustrated in Figure 4C. The maximum period 1 abatement ( $a_1^{max}$ ) is illustrated by the vertical dotted line. Suppose, as a benchmark, we choose a ‘wait-and-see’ strategy and base our initial period decision on low damages (state A). Suppose further that we learn that damages instead are high. We thus want to choose abatement costs equal to  $a_1^{A(wrong)}$ . This point is, however, to the right of  $a_1^{max}$ . Since we cannot choose an abatement cost level that is higher than the maximum, the climate irreversibility constraint is binding. The second best solution is thus not reachable, and the best we then can do is to choose  $a_1$  equal to  $a_1^{max}$ . The social loss from guessing wrong is represented by the grey line along the  $a_0$ -axis, where the solid segment represents the option value, and the dotted segment represents adjustment costs<sup>11</sup>. Thus, the loss imposed if the climate irreversibility constraint is encountered adds to the costs of the ‘ordinary’ adjustments to resolved uncertainty. The black dotted line to the left of the  $a_0$ -axis is the same as in Figure 4B and represents the loss when the ‘early action’ policy is chosen and we learn that we should have chosen ‘wait-and-see’.

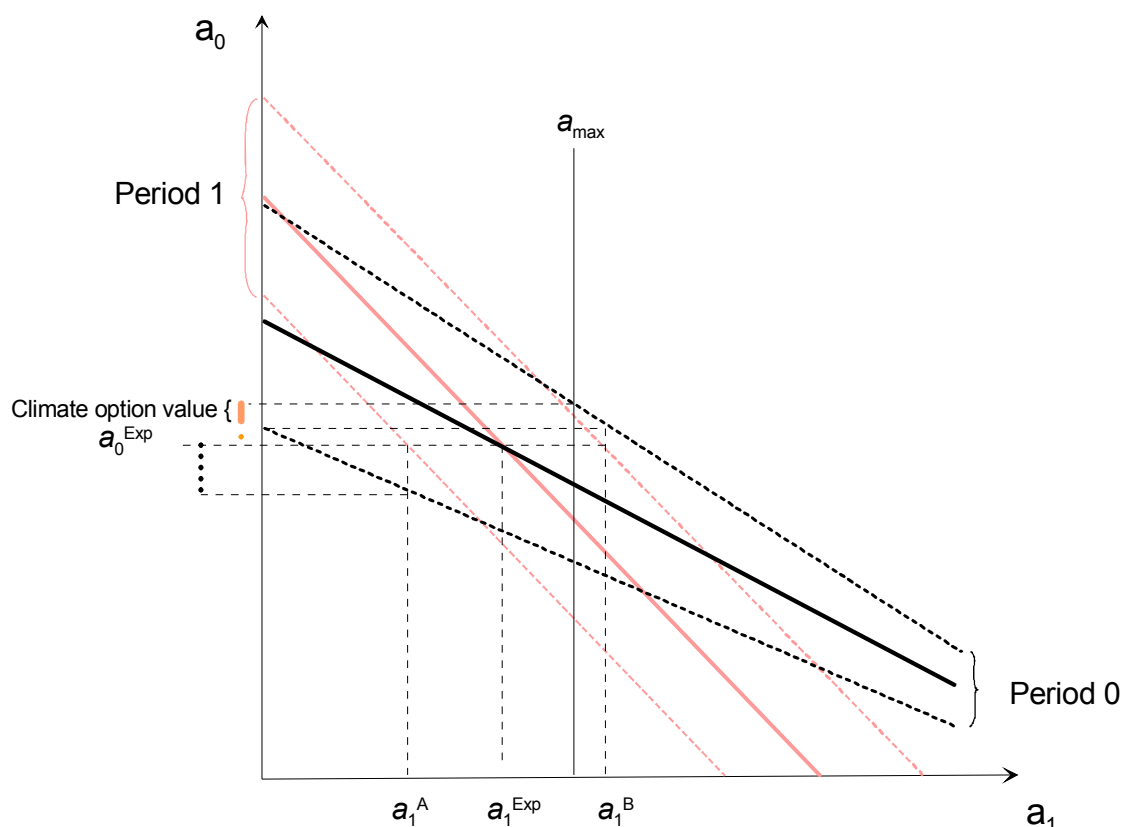
In the special case of the figure, if wrong, the total cost of assuming low damages exceeds the total cost of assuming high damages. Whether this is the case depends on the extent of the option value of early abatement. The option value depends on the probability of encountering the climate irreversibility constraint, which depends on  $a_1^{max}$ , and the maximum possible abatement depends on the periodic emissions, which are exogenously given. In the special

<sup>11</sup> The graphic representations shows the relative, rather than the actual, size of the different costs linked to the three uncertainty effects studied.

case of Figure 4C, the irreversibility constraint will be encountered under high damage costs if the policy in period 0 is based on low damage costs. Note that  $a_1^{A(wrong)}$  represents the maximum possible abatement that the decision maker might want to choose in period 1. Hence, if  $a_1^{max}$  is to the right of  $a_1^{A(wrong)}$ , there is no option value of early abatement. Kolstad (1996b) does not have a binding climate irreversibility constraint, which means that there is no option value of early abatement, and the climate irreversibility therefore, by assumption, has no impact on decisions.

If instead the damage costs turn out to be lower than initially expected, one could think of a situation where it is optimal to emit more than  $e_1$ . This constraint does not depend on the initial period decision, as in this subsection, but the likelihood of encountering it is of course higher with higher initial levels of abatement.

Recall that the rational decision maker will choose an allocation between the two extremes discussed above. Figure 4D illustrates the option value when the decision is based on expected utility maximization. Compared to Figure 4C, the trade-off curves for state A and B are now dotted. The solid lines in between are the ones that apply for expected utility maximization with equal probabilities ( $\pi = 0.5$ ). The optimal ex ante solution is  $a_0^{Exp}$  and  $a_1^{Exp}$ .



**Figure 4D. Cost of guessing wrong under expected utility maximization**

The social losses of guessing wrong are represented by the vertical lines to the left of the  $a_0$ -axis. Again we look at the most serious mistakes. First, take the case where we learn at the end of period 0 that the state of the world is low damage costs (state A). Since we cannot do anything about our period 0 abatement cost choice, we must compensate by lowering the

period 1 abatement cost until the first order condition for this period is satisfied, namely to  $a_1^A$  in Figure 4D. The loss can be measured as the difference between  $a_0^{Exp}$  and the  $a_0$  value at the point where  $a_1^A$  crosses the period 0 trade-off curve for state B. It is represented by the black dotted line to the left of the  $a_0$ -axis.

Second, suppose instead that we learn that the state of the world is high damage costs (state B). We then wish we had done more initially, and this must be compensated for by increasing future abatement. To satisfy the first order condition for period 1, we must choose  $a_1^B$ . This level is, however, not attainable since it is to the right of  $a_1^{max}$ . The best we can do then is to choose period 1 abatement equal to  $a_1^{max}$ . The cost of guessing wrong is represented in the figure by the grey line to the left of the  $a_0$ -axis, where the solid segment is the option value and the dotted segment is the adjustment cost.

To conclude so far, the option value of early abatement relates only to the costs of meeting the climate irreversibility constraint. Under a process of learning, additional costs will occur because of adjustments desired as a result of new information. In the model discussed here, the costs related to these adjustments depend on the slope and the position of the trade-off curves for the two periods. These positions depend on the choice of damage functions and abatement costs, and to which of these that are being subject to uncertainty. For example, Figure 4 is restricted to uncertainty in damage costs. If abatement costs are uncertain, the slope of the trade-off curve for period 0 becomes less steep for high costs without changing the intersection with the  $a_1$ -axis. The trade-off curve for period 1 shifts downwards and becomes steeper. Therefore, whether a ‘wait and see’ type policy should be preferred to ‘early action’, or vice versa, depends on the specification of the model. The only conclusion so far, unless a numerical study is carried out, is that the climate irreversibility imposes an option value to the ‘early action’ strategy as long as there is a positive probability of encountering the climate irreversibility constraint. This is, however, not sufficient for recommending such a strategy, because such a recommendation also requires that the option value exceed the loss resulting from not following the expected utility choice in period 0. On the other hand, the ‘wait-and-see’ option can never be expected to do better than maximization of expected utility when there is climate irreversibility only. The expected cost of encountering the climate irreversibility constraint always is higher in the ‘wait-and-see’ policy option, unless additional restrictions are imposed.

### **4.3 Irreversible abatement technology investments**

The model (1) to (4) assumes that the abatement level is set in each period, independent of previous actions. As Kolstad (1996a,b) and Ulph and Ulph (1995) point out, when focusing on the importance of irreversibility, it would be particularly inappropriate to disregard the fact that capital costs contribute to a large share of the costs of mitigating climate change. These investments are to a large degree sunk, thereby representing an irreversible cost.

Capital irreversibility is easily included in our model. We include the initial investment costs,  $a_0/c_0$ , also in the next period, that is, if the rate of depreciation is zero. Equation (2) is replaced with

$$y_1 = y_0 + e_1 - \frac{a_0}{c_0} - \frac{a_1}{c_1}. \quad (14)$$

The capital irreversibility has two implications for future options. First, the limit for the possible *minimum* level of concentrations in period 1 is lowered, as long as  $a_0 > 0$ , because the initial abatement investments will lower the emissions also in the future period. Consequently, the option value of damages, the value of keeping open the option of further increasing the abatement in the future, will be reduced. Second, the *maximum* level of concentrations in the future is now  $y_0 + e_1 - a_0/c_0$  compared with  $y_0 + e_1$  in the previous sections. This means that now the maximum level of abatement in the future also depends on the investment decision taken initially. In other words, a wrong guess in period 0 may lead to negative optimal abatement in period 1. Since negative abatement is not possible, an extra cost may occur in period 1 because of the positive probability of encountering this capital constraint. This we choose to call the option value of late investments. Inserting for (5) in the benefit function yields

$$b(y_1) = q_1(f(\hat{y} + e_0 + e_1) - f(y_1)) = q_1(\alpha(\hat{y} + e_0 + e_1)^2 - \alpha(\hat{y} + e_0 + e_1 - 2\frac{a_0}{c_0} - \frac{a_1}{c_1})^2) \quad (15)$$

which corresponds to equation (4) above.

The introduction of capital irreversibility thus implies that the initial abatement becomes ‘twice as important’ as before. Apart from this, the model is unchanged. The optimal solution can, in principle, thus be discussed within the same framework as in section 4.1 and 4.2. However, with capital irreversibility, the constants  $C_0$ ,  $\Phi_0$  and  $\Psi_0$  change to

$$C_0^* = (\hat{y} + e_0) \frac{2q_0\alpha_0}{c_0} + \frac{2(\hat{y} + e_0 + e_1)}{1+r} \left[ \pi \frac{2q_1^A\alpha_1^A}{c_0} + (1-\pi) \frac{2q_1^B\alpha_1^B}{c_0} \right] - 1,$$

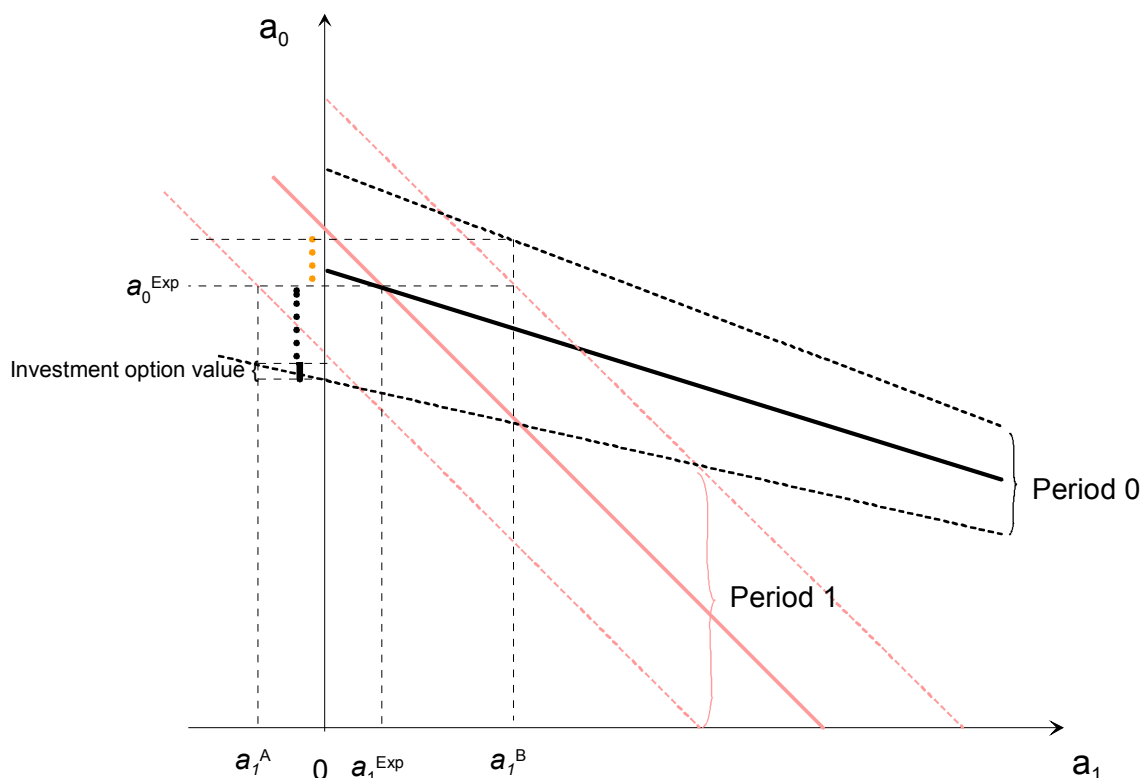
$$\Phi_0^* = \frac{2q_0\alpha_0}{c_0} \frac{1}{c_0} + \frac{2}{1+r} \left[ \pi \frac{2q_1^A\alpha_1^A}{c_0} + (1-\pi) \frac{2q_1^B\alpha_1^B}{c_0} \right] \frac{1}{c_0},$$

$$\Psi_0^* = \frac{2}{1+r} \left[ \pi \frac{2q_1^A\alpha_1^A}{c_0} \frac{1}{c_1^A} + (1-\pi) \frac{2q_1^B\alpha_1^B}{c_0} \frac{1}{c_1^B} \right],$$

while the expressions for  $C_1$ ,  $\Phi_1$  and  $\Psi_1$  are unaltered. Note that in this model, capital irreversibility with no depreciation implies only that the discount term  $1/(1+r)$  is doubled. Ceteris paribus this leads to an allocation of abatement from period 1 to period 0. Total abatement is, however, likely to decrease. (See the discussion in section 4.1.)

Compared to Figure 4, the introduction of capital irreversibility leads to a positive shift in the trade-off curves for period 0, and they also become steeper, since  $\psi_0/\Phi_0 < \psi_0^*/\Phi_0^*$ . The probability of encountering the climate irreversibility constraint is thus lowered. Recall that this constraint is due to the restriction  $a_1 \geq 0$ ; we cannot have negative abatement. The

likelihood of encountering this constraint depends on the exogenous emissions under “business as usual” in the future ( $e_1$ ).



**Figure 5. Cost of guessing wrong under irreversible investments and expected utility maximization**

The option value of late investment is illustrated in Figure 5. If the decision in period 0 is based on expected utility maximization with equal probabilities, we choose  $a_0^{Exp}$ , expecting to choose  $a_1^{Exp}$  in period 1. If the low damage outcome occurs in period 1, it turns out that  $a_0^{Exp}$  was more than enough for both periods. The optimal choice, according to the new information, is to lower  $a_1$  until the low damage cost trade-off curve for period 1 is reached (the state A curve). In Figure 5 this point is to the left of the  $a_0$  axis and is not attainable due to the requirement of non-negative investments. The loss due to this overinvestment can be measured by the difference between the chosen policy,  $a_0^{Exp}$ , and the value of  $a_0$  where  $a_1 = 0$  crosses the low damage cost trade-off curve for period 0. The loss is indicated by the black bar along the  $a_0$  axis in Figure 5, where the option value of late investment represents the solid segment.

Correspondingly, the loss when the high damage cost scenario is realized is the difference between the  $a_0$ -value at the point where  $a_1^B$  crosses the trade-off curve for period 0 in the high damage cost state and  $a_0^{Exp}$ . This is indicated by the grey dotted line to the left of the  $a_0$  axis in Figure 5.

Note that we cannot say whether it is the climate irreversibility constraint or the investment irreversibility constraint that represent the larger ex ante costs. This depends on the choice of parameters, and the ‘business-as-usual’ emissions. It may be noted, however, that a doubling of the discount factor to account for the investment irreversibility represents the maximum possible implication of this irreversibility, namely for the case where the rate of capital depreciation is zero, which is equivalent to the case analyzed by Kolstad (1996b). Introducing depreciation would reduce this term to somewhere between 1 and 2, depending on the rate of

depreciation. If the periods under consideration are 20–25 years or more, the term is probably close to 1, which would be equivalent to case analyzed by Arrow and Fisher and Henry discussed in Section 4.2. Note, however, that climate irreversibility is also subject to ‘depreciation,’ but the time perspective is substantially longer. The concentrations of CO<sub>2</sub>, for example, are reduced by 2/3 over a period of 150–200 years, and 10 percent of emissions will remain in the atmosphere for thousands of years.

Thus, within the framework of this model, the introduction of irreversibility in investments is not likely to have large impact. In principle, the requirement for an option value of early abatement to arise is similar to the case of reversible damage costs. However, the investment irreversibility scales down the future climate effects in the same way as an increase in the discount factor would. This implies that the possible cost of choosing a too low initial level of abatement is smaller. The conclusion is nevertheless similar: whether or not a possible option value of early abatement has an effect on policy making partly depends on the degree of climate irreversibility, but this effect is reduced the more irreversible investments are. This last effect is, however, smaller the longer the time periods we make decisions for. We interpreted this as having two option values that point in different directions, and whether the preferred policy option is ‘early action’ or ‘wait and see’ depends on the relative size of these option values plus the possible adjustment costs. If the sum of the climate option value plus the adjustment costs due to too low initial abatement exceeds the sum of the investment option value plus the adjustment costs due to too high initial abatement, the net option value is positive, the climate irreversibility effect dominates, and ‘early action’ should be preferred. This shows that it is not possible to advocate either ‘early action’ or a ‘wait-and-see’ strategies merely on the basis of the existence of irreversibilities, nor on a direct comparison of option values. The additional adjustment costs resulting from deviation from expected utility maximization, which *ex ante* are always non-negative, must also be considered.

## **5 Decision criteria**

The question of irreversibilities in climate change policy has traditionally been addressed in an expected utility framework. In fact, all the papers cited above use this framework, and this literature indicates that climate irreversibilities should not have too much impact on the design of climate policy. It would be interesting to reconsider this result under alternative decision criteria. The discussion of figures 4A, B, and C was not tied to the principle of expected utility maximization. Hence, the figures could be used to consider implications of the use of alternative decision criteria that are based on the economic principle of equalizing marginal costs and benefits, given beliefs about future states of the world. We will in this subsection indicate briefly how this could be analyzed in the framework of this paper, saving the in-depth analysis for our future work.

The complexity of global warming makes it impossible to completely overlook the consequences of alternative choices. One might therefore ask whether this problem, which exhibits such severe forms of uncertainty, should be analyzed in a framework of ignorance, or at least partial ignorance. Theories of rational behavior under complete ignorance can be found for example in Arrow and Hurwicz (1972). Non-probabilistic criteria build on such a notion of ignorance. Critics of these criteria have put forward that the decision maker must at least have some vague partial information about the true state of nature (Luce and Raiffa 1957). The question remains, however, if this vague partial information is sufficient to assign subjective probabilities to the possible states of the world.

In Bretteville (2003) and Aaheim and Bretteville (2001), we examine the implication of the choice of a number of different decision criteria within a static setting for abatement investments. We found that there might be good reason to base climate policy on other



decision making criteria than maximization of expected utility. However, the conclusions from a static analysis of climate policy may change if the timing of policy, irreversibilities, and the possibility for learning are taken into account.

The most well-known non-probabilistic criterion is perhaps the *maximin* principle (Rawls 1971). This principle implies maximization of the welfare in the worst possible case, and essentially it allows risk aversion to become infinite. It has been claimed that the conservatism of the maximin principle makes good sense in the context of climate irreversibility. Chev  and Congar (2002), for instance, argue that maximin is consistent with the precautionary principle. However, knowing the actual nature of the worst case is problematic. Another problem with the maximin principle is that the worst case might be a catastrophe of such dimensions that deciding between policy options might have no significant impact on the outcome. A third problem is that the conclusion depends on what you define as the worst case.

Applied to the problem of climate change, the maximin criterion can be interpreted as choosing the level of abatement that maximizes the social welfare in the worst possible state of the world. In the context of this paper, the worst state would mean that the decision in period 0 is based on both high damages and high abatement costs. If there is uncertainty attached to policy effectiveness, ‘early action’ cannot be rationalized as the appropriate maximin strategy because the worst case scenario would be to implement a costly remedial policy that fails to avert severe damages. Bouglet and Vergnaud (2000) analyzes the maximin criterion in a context of irreversibility theory and concludes that it does not necessarily lead to more flexible decisions than expected utility maximization.

Minimax regret (/risk/loss), suggested by Savage (1951) as an improvement on the maximin criterion, aims at minimizing the difference between the best that could happen and what actually does happen. The decision-maker tries to minimize possible regrets for not having, in hindsight, made the superior choice. In this global warming example, it can be interpreted as choosing the strategy that minimizes the difference in benefits between guessing right and wrong. This is easily connected to the discussion in Section 4. We found that the effect of a possible option value of early abatement on the maximum regret is less the more irreversible investments are, but this last effect is smaller the longer the time horizon. The possible mistake of choosing a too low level of abatement in the initial period is thus reduced compared to the case with climate irreversibility only. The preferred policy option depends on the relative size of the two option values and the possible adjustment costs. If the sum of the climate option value and the adjustment costs due to too low initial abatement exceeds the sum of the investment option value and the adjustment costs due to too high initial abatement, the net option value is positive, and the maximum regret is minimized by choosing ‘early action’ over the ‘wait-and-see’ policy.<sup>12</sup>

It is obvious from the discussion above that the probability distribution over states will influence the net option value, and thus the preferred policy. Our model supposes two future scenarios. This can be interpreted as picking two of all the possible scenarios – one in the low cost range and one in the high cost range – or it could be interpreted as some sort of a mean or median in the two groups. Our model can thus be used as a framework for analyzing the *Generalized Maximin/Maximax* criterion, also known as *the pessimism-optimism index criterion* of Hurwicz (1951). This criterion states that the level of abatement should be chosen in order to maximize a weighted average of the net benefit in the best and the worst state. The size of the pessimism-optimism index (the weight) should reflect the decision-maker's beliefs about the probabilities of facing different future states of the world. Whether the net option value is positive or negative with this criterion depends on the choice of focus, which states

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<sup>12</sup> Chev  and Congar (2002), however, claims that the minimax regret criterion is not consistent with the precautionary principle.

we choose as the worst and the best, and also the choice of weight. If the states in focus are the same as in the discussion above, the conclusions are, of course, unchanged.

Another criterion mentioned in the literature on decision-making under ignorance is *the principle of insufficient reason*, first formulated by Bernoulli in the 17th century. This criterion states that if there is no evidence leading us to believe that one event is more likely to occur than another, the events should be judged equally probable. In our model we have two possible scenarios, A and B, and they should thus each be assigned a probability of 0.5. In our two-period model with uncertainty and learning, this means that when we decide on the initial abatement we treat the two scenarios as equally likely, and when we decide on future abatement we have acquired new information and can adjust our emissions accordingly. If state B really is a low-probability extreme event, it would be weighted too heavily relative to the weight we would assign to it if we had had enough information to apply expected utility maximization. The occurrence of B is fortunate because then we are more likely able to increase our abatement sufficiently to avoid most of the damage. If the future optimal abatement turns out to be higher than the maximum, the climate irreversibility constraint will be encountered. If B does not occur, which is the most likely outcome, we did too much in the initial period and would thus like to increase emissions after learning the true state of nature. If future optimal abatement is less than zero, the investment irreversibility constraint is encountered.

Combinations of probabilistic and non-probabilistic criteria are also possible candidates for decision-making. The *limited degree of confidence* criterion is one example. It implies that we maximize a weighted sum of the expected utility criterion and the maximin criterion. The weight reflects the degree of confidence in the underlying probability distribution. In the case of full confidence, the weight is equal to 1 and the expected utility criterion is used, whereas under complete uncertainty the weight is equal to 0 and the maximin decision rule is applied. Lange (2003) compares expected utility with this criterion in a two-period model. He finds that more weight on the worst case (less weight on EU) may lead to increased first-period emissions and that the irreversibility effect holds if and only if the value of learning is negative.

## **6 Concluding remarks**

Because of the option value of early abatement, a question arises of whether a more environmentally cautious policy initially (a policy based on beliefs of high future damages), could yield a higher expected benefit than a decision based on expected values. This depends on three factors: first, the size of the climate option value; second, the size of the investment option value; and third, the possible loss from deviating from the principle of maximizing expected utility. The net option value must thus exceed the expected value of this loss if a cautious initial policy is to be preferred.

The model discussion showed that whether the social loss of an ‘early action’ policy exceeds the social loss of choosing ‘wait and see’, in the case of climate irreversibility only, depends on the option value of early abatement as well as the possible abatement adjustments resulting from initial period mistakes. The option value of early abatement depends on the probability of encountering the climate irreversibility constraint, which depends on the size of the maximum abatement in the future.

Including irreversibility in abatement investments scales down the effects of climate irreversibility equivalent to the effect of an increase in the discount factor. *Ceteris paribus* this leads to an allocation of abatement from period 1 to period 0. Total abatement is, however, likely to go down. The possible mistake of choosing a too little abatement in period 0 is reduced compared to the case with irreversibility only in emissions of greenhouse gases. We

found that the effect of a possible option value of early abatement on the policy choice should be less the higher the option value of late investments, but that this effect is smaller the longer the time horizon.

The option values relates only to the costs of encountering the irreversibility constraints. Under a process of learning, additional costs will occur because of adjustments desired as a result of new information. The option value of early abatement is realized only if the desired future abatement, after learning takes place, is higher than what is actually possible. Then the climate irreversibility constraint is binding. Similarly, the option value of late abatement is realized if the desired abatement investment, after learning takes place is negative. Then the irreversibility constraint on investment is binding. The loss imposed when encountering the irreversibility constraints adds to the costs of the uncertainty adjustments resulting from learning.

Changes in the parameters will affect total abatement, as well as allocation between the periods. If  $q_1\alpha_1$  increases, total abatement goes up, and future abatement increases relative to present abatement. An increase in expected future period unit costs contributes to an allocation of abatement from the future to the present. An increase in the discount rate affects, in this model, only the abatement assigned to period 0 which is reduced. Some of the reduced abatement may be allocated to the future, but the total amount will most likely go down. Changes in uncertainty contribute to a reallocation of abatement from the present to the future.

Choice of strategy in climate policy is not only a question of comparing costs, but also a choice of criterion. The choice of criterion will to a large degree influence policy choice and is therefore a political question. The model in this paper is applicable for analyzing and comparing alternatives to expected utility maximization. For example, if the choice between criteria is subject to a comparison between the required adjustment costs under extreme outcomes, the model can be used to attach relative numerical values to alternative strategies. This is a subject for future research.

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## Appendix 1

### Implications of the constants of a mean-preserving spread of risk

The constants on which the optimal solution is based are all subject to uncertainty in damage costs and abatement costs. Each constant will, however, be affected differently depending on which variable is subject to uncertainty. As a consequence, also the value of abatement will change when the policy is adjusted to account for new information, depending on the nature of this information. Below follows a discussion of how a mean-preserving spread of risk will affect the constants defined in Section 4. This affects, in particular, how the abatement is adjusted when new information arrives in period 1.

In general, the constants in chapter 4 can be represented by three categories of expected values:

$$EX = \pi x^A + (1 - \pi)x^B, \quad (A1)$$

which applies for all the constants when there is uncertainty only in the damage costs;

$$EX = \pi \frac{1}{y^A} + (1 - \pi) \frac{1}{y^B}, \quad (A2)$$

which applies for  $C_1$ ,  $\psi_0$  and  $\psi_1$  when there is uncertainty in the damage costs; and

$$EX = \pi \frac{1}{(y^A)^2} + (1 - \pi) \frac{1}{(y^B)^2}, \quad (A3)$$

which applies for  $\Phi_1$  when there is uncertainty in the damage costs.

A mean-preserving spread of risk means that  $dX = \pi dx^A + (1 - \pi) dx^B = 0$ , or  $dx^B = -\pi/(1 - \pi) dx^A$ . Hence, for (A1),  $dEX = 0$  by definition. That is, the expected value is unaffected by a mean-preserving spread of risk in damage costs, if the abatement costs are certain.

For (A2), we have

$$\begin{aligned}
 dEX &= \pi \frac{-1}{(y^A)^2} dy^A + (1-\pi) \frac{-1}{(y^B)^2} dy^B = -\pi \frac{1}{(y^A)^2} dy^A - (1-\pi) \frac{1}{(y^B)^2} \frac{-\pi}{1-\pi} dy^A \\
 &= -\left[ \pi \frac{1}{(y^A)^2} dy^A - (1-\pi) \frac{1}{(y^B)^2} \frac{\pi}{1-\pi} dy^A \right] = -\pi dy^A \left[ \frac{1}{(y^A)^2} - \frac{1}{(y^B)^2} \right]
 \end{aligned}$$

(A4)

Therefore,  $dEX$  is positive or negative depending on whether  $y^A$  is higher or lower than  $y^B$ . If we talk about a spread of risk,  $dy^A$  will be negative if  $y^A$  is lower than  $y^B$ , and vice versa. Therefore,  $dEX$  will increase with ‘more’ uncertainty in the abatement costs.

For (A3), we have

$$\begin{aligned}
 dEX &= \pi \frac{-2y^A}{(y^A)^4} dy^A + (1-\pi) \frac{-2y^B}{(y^B)^4} dy^B = \pi \frac{2y^A}{(y^A)^4} dy^A - (1-\pi) \frac{2y^B}{(y^B)^4} \frac{\pi}{1-\pi} dy^A \\
 &= 2\pi dy^A \left[ \frac{1}{(y^A)^3} - \frac{1}{(y^B)^3} \right]
 \end{aligned}$$

(A5)

Since a spread of risk means that  $dy^A < 0$  if  $y^A$  is lower than  $y^B$  and vice versa,  $dEX > 0$  when the uncertainty in abatement costs increases.

## Appendix 2

### Sensitivity analysis - A numerical example

In this numerical example we have, as far as possible, tried to base the choice of parameters on assumptions that seem to be standard in studies of optimal climate policy. The simple structure of the model in this paper nevertheless implies that the results can only be considered as illustrations.

Each period counts 25 years, making coverage of the period 2000 to 2050. We assume that the ‘business-as-usual’ emissions grow between 1.5 and 2.5 percent per year, proportional to the growth in GDP. What this implies for the exogenous variables  $e_0$  and  $e_1$  is not easy to say. This is because the model expresses emissions in units of concentrations, but it does not include a natural rate of decay in the concentrations. Hence, using the sum of emissions would, first, exaggerate the contribution to concentrations, and second, disregard the fact that emissions contribute less to concentrations the higher the concentrations are. We have therefore replaced emissions with additions to concentrations in each period from the chosen emissions assumption. The natural decay was set to 1/250, which is an approximation of the CO<sub>2</sub> decay rate in the long run.

The unit cost of abatement is based on an assumption that reducing emissions by 10 percent costs 0.75 percent of GDP. This is relatively high compared with other studies. For the damage costs, we assume 5 percent of GDP at a concentrations level of 550 ppmv. Again, this is high. The reason for choosing an upper level for both is that we assume that the policy is to be implemented for the entire world, whereas most cost estimates apply for developed countries, assuming that only these countries abate under the allowance of trading and CDM engagement.

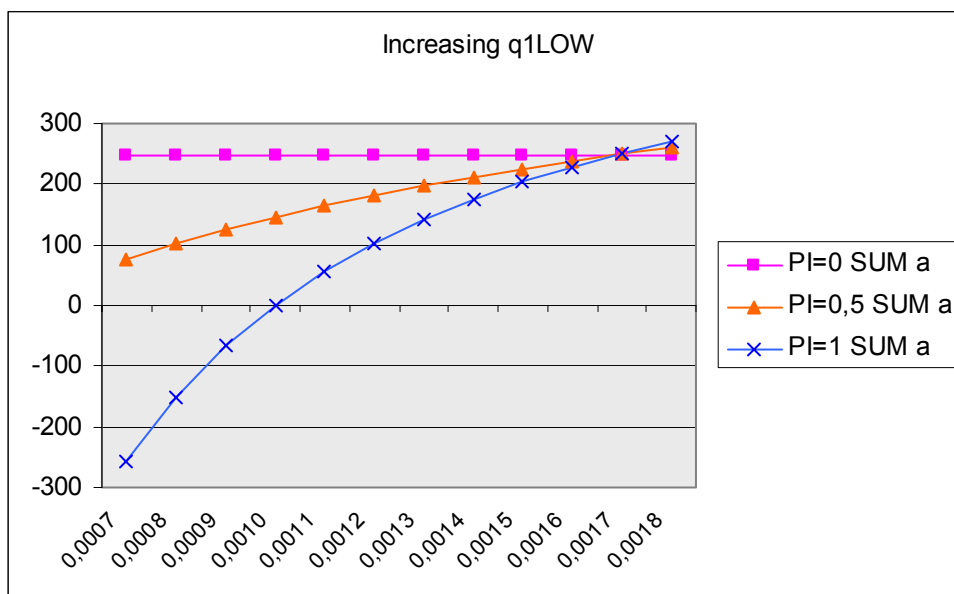
The assumed unit cost of abatement corresponds to a cost lower than 0.5 USD/tC. This is extremely low compared with estimates of marginal costs in other studies (see e.g. IPCC, 2001). The reason is that we assume a constant unit cost of abatement, which means that the cost represents the average, rather than marginal cost, which is usually reported. The damage resulting cost per unit of concentrations is approximately 2 USD/ppmv. Based on the assumption for the damage costs, the constant of the damage cost function was set to 0.0015.

The base case for our numerical example is thus:

y_hat	e0	e1	c0	c1A	c1B
368	63	116	1.15	1.1	1.3

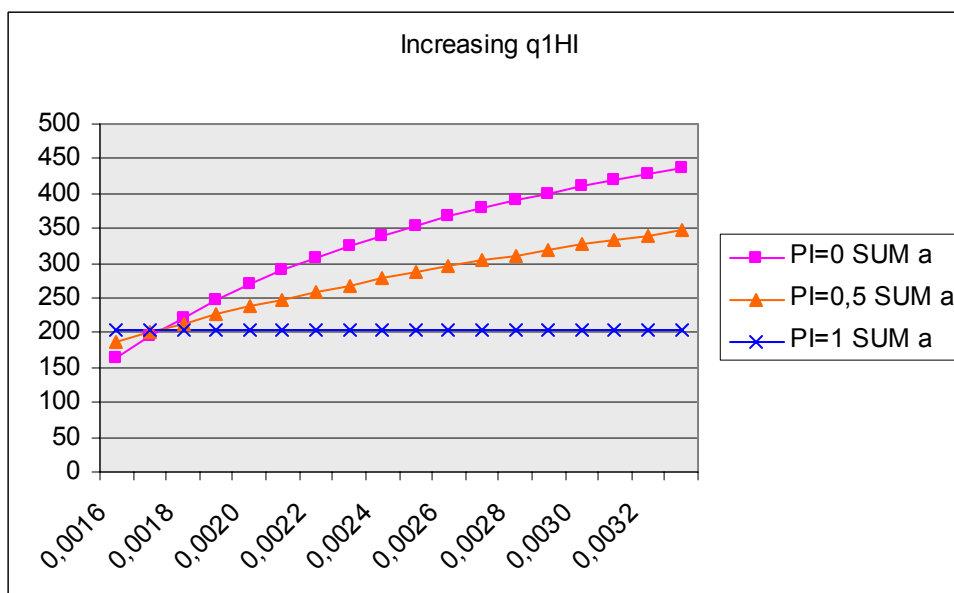
q0alpha0	q1alpha1A	q1alpha1B	pi	r
0.001	0.0015	0.0019	0.5	1.75

To show how sensitive the abatement decisions are to changes in the exogenous variables, we have made some calculations based on variations in one variable at the time illustrated in the figures below. First we examine the implication of increasing  $q_1\alpha_1$  in the damage function. Figure A shows how the sum of the abatement in the two periods change with changes in  $q_1\alpha_1$  for state A, which is the low damage scenario, for three different values of  $\pi$ . Recall that  $\pi$  is the probability of state A.



**Figure A**

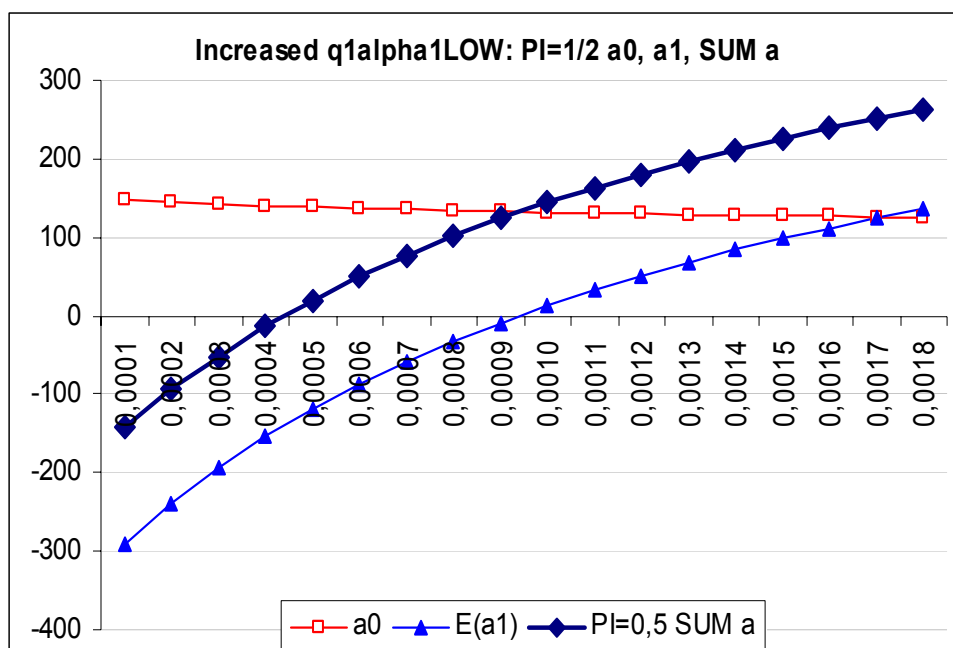
Similarly, Figure B shows the development in total abatement when  $q_1\alpha_1$  for state B increases. State B is the high damage cost scenario.



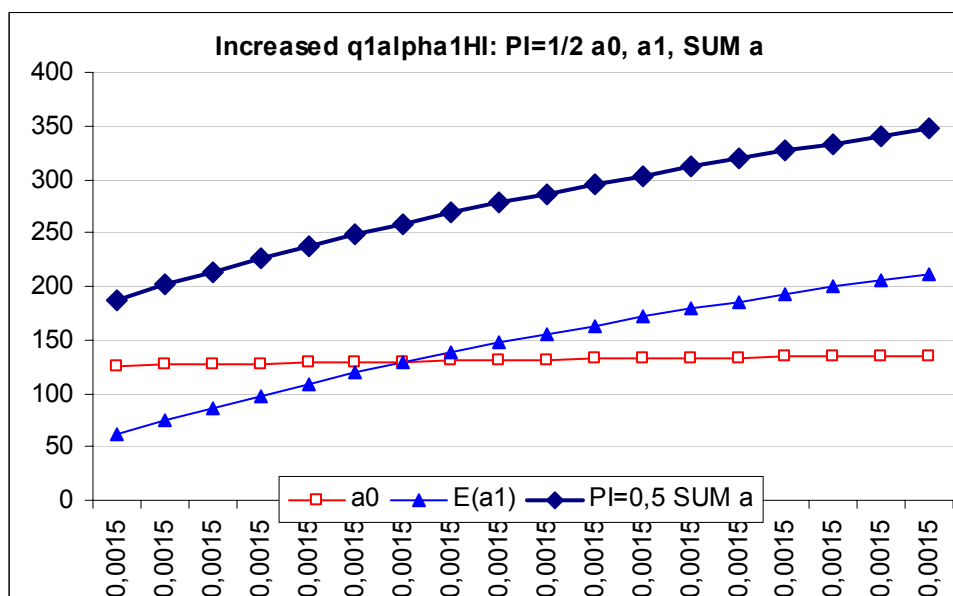
**Figure B**

The next two figures show the how the allocation of abatement between the two periods, as well as the total amount of abatement, changes with  $q_1\alpha_1$  for state A (Figure C) and state B (Figure D) respectively, when the probability of state A is 0.5.



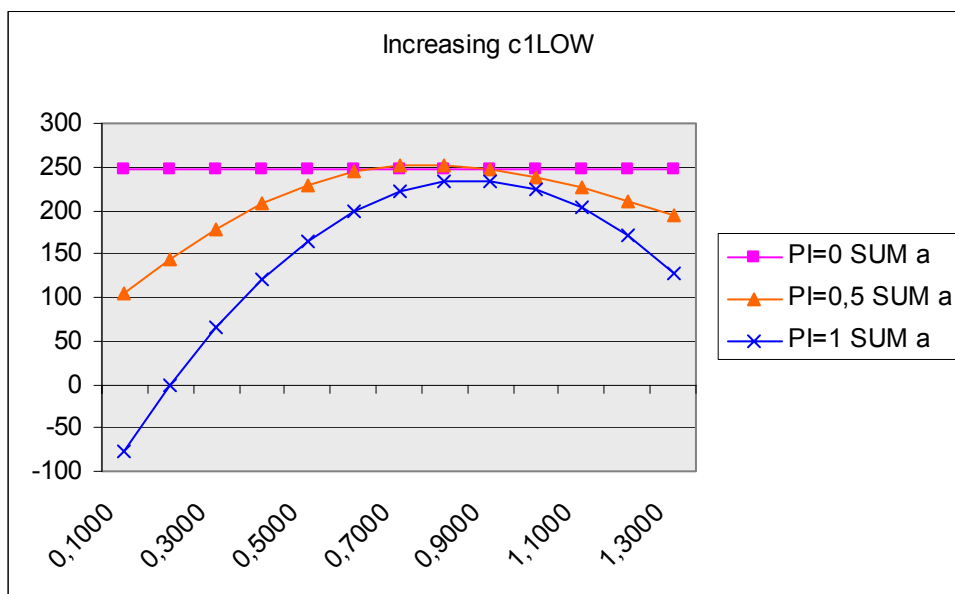


**Figure C**

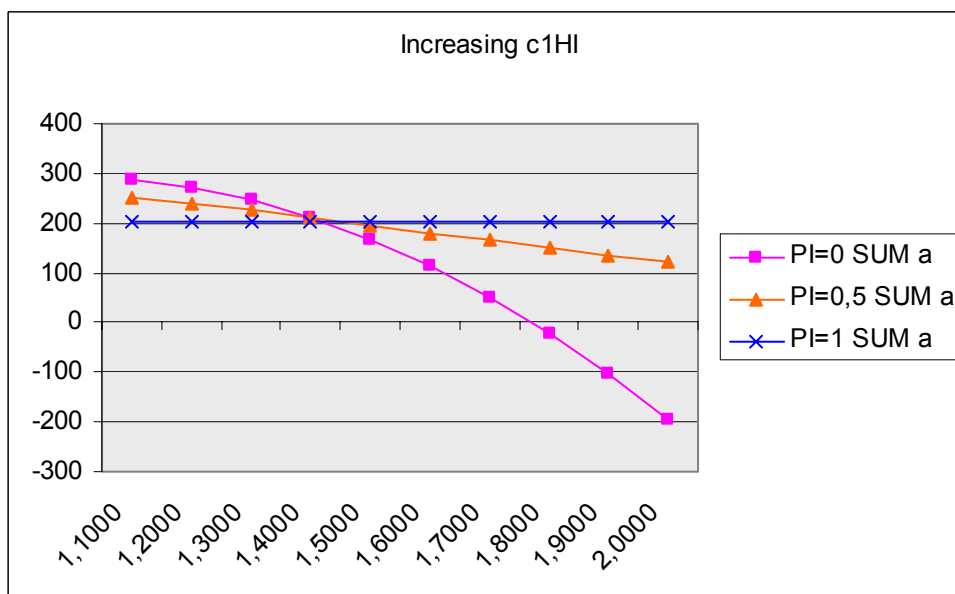


**Figure D**

In the same fashion, we have computed changes in abatement when the abatement cost in period 1 increases, first in the low-cost scenario (A), and second for the high-cost scenario (B). Figures E and F show the development in total abatement in the two cases for three different values of  $\pi$ .

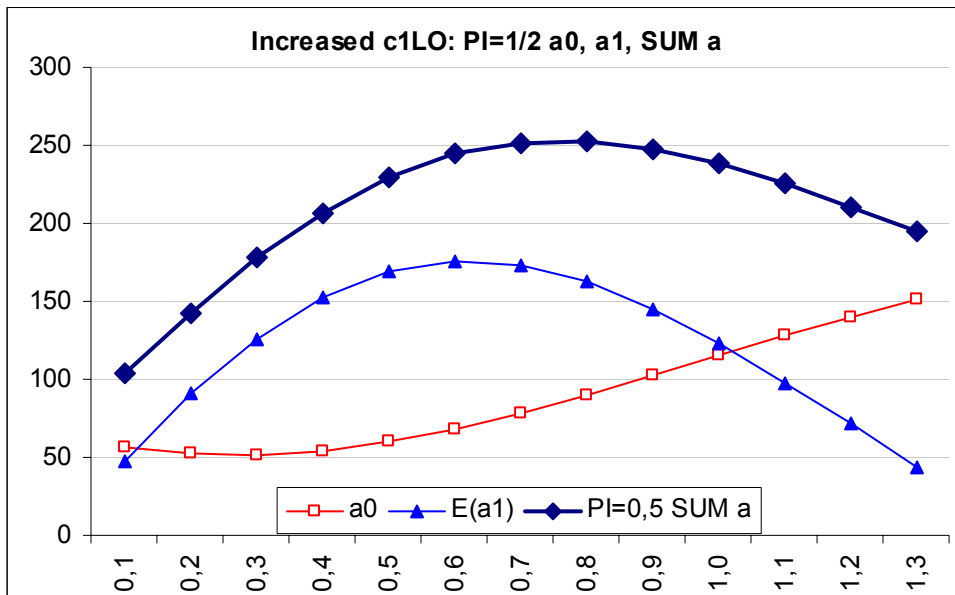


**Figure E**

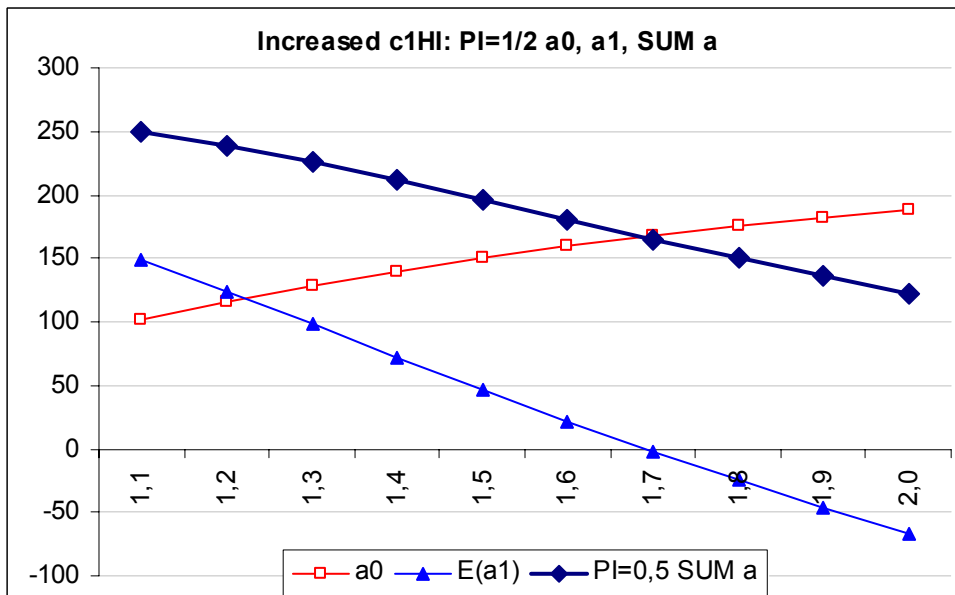


**Figure F**

Figures G and H show the changes in total abatement and how abatement is allocated from period 1 to period 0 when the future abatement costs increase and the probability of state A is 0.5.

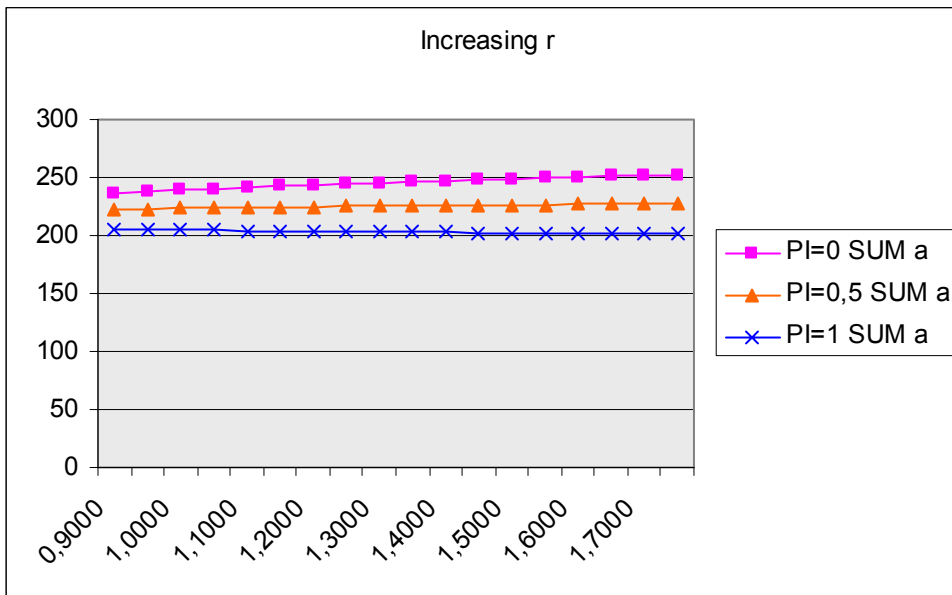


**Figure G**



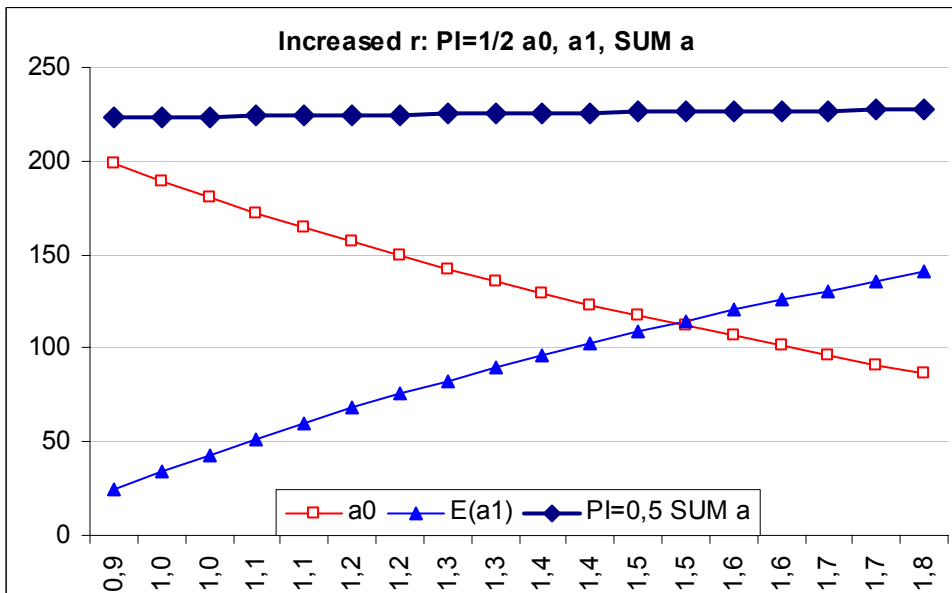
**Figure H**

Lastly we will show the implications on abatement decisions of changes in the discount rate. Figure I shows changes in total abatement for three different values of  $\pi$ .



**Figure I**

Figure J shows the allocation abatement for  $\pi=0.5$ . When the discount rate increases abatement is allocated from the present to the future, while the total amount is constant.



**Figure J**