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Abstract

A three-generation planning model incorporating uncertain climate change is developed. Each generation features a production activity based on capital and an exhaustible resource. An irreversible climate change may occur in period two or three, reducing the productivity for this and the remaining generation. The model is solved by stochastic dynamic programming. If the climate impact and climate change probability is constant, the optimal period one (and two) resource extraction is larger than for the reference case of climate stability. If, however, climate impact and climate change probability increases with increased aggregate resource use, this result is reversed.

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1 Introduction

The aim of this paper is to study some aspects of the long-term planning problem facing a society when there is a probability of climate change and production activities affect the probability and scale of future climate change.

Through various economic activities greenhouse gases (GHG) are released to the atmosphere. The major GHG are carbon dioxide, methane, nitrous oxide and chlorofluorocarbons. Over the last century the atmospheric concentration of these gases has increased, even if some amount of the gases are removed from the atmosphere through GHG sinks like the biosphere and the oceans (Houghton et al. (1990), (1992)). According to the greenhouse hypothesis increased concentration of these gases will induce global warming. The available climate models predict an increase in global average temperature of 2.2 to 4.8 °C from the pre-industrial level in the second half of the next century if the present fossil fuel utilization trend continues. There will be large regional variations in temperature change. In addition weather variability may change, for instance with an increased frequency of hurricanes. There is some possibility of passing a threshold, entering a new global climate regime, or new regional climate regimes. A new climate regime may be irreversible, at least for the foreseeable future. A major climate change may therefore induce sizeable cost increases for several human economic activities, for instance agriculture and water supply. In general these costs are likely to increase with the speed of climate change, but can be reduced through long-term planning and investments based on the best available knowledge of future climate change. Thus there is a value to collecting data and doing research on future climate change and socioeconomic consequences, constantly updating long-term economic plans, e.g. plans for investment in infrastructure. Even if the probability of a major and costly climate change is small, it may make sense to spend some money reducing the probability of such an event. This is equivalent to buying an insurance reducing the costs of a major future climate change, in the event that such a climate change should occur. The two main categories of policy measures are abatement measures, i.e. reducing GHG emissions, and adaptation to future climate change. Since climate change is a long-term problem, it should be addressed in an intergenerational frame. A climate policy is a matter both of intergenerational equity and of resource allocation efficiency.

The scope of the present paper is more limited than the range of policy questions involved in the climate change problem mentioned above. The focus of this paper is on the optimal planning of intergenerational resource use and real capital investment when accumulated resource use increases the probability of a major climate change. An obvious illustration is combustion of fossil fuels releasing carbon dioxide to the atmosphere.

The paper is inspired by the literature on irreversible development and option value (e.g. Arrow and Fisher (1974) and Conrad and Clark (1987)), and by the environmental and resource economics literature employing stochastic dynamic programming (e.g. Conrad and Clark (1987)). I would also like to mention the Solow (1974) paper on intergenerational optimal growth and exhaustible resources.

There are three generations or periods. In the first period there is climate stability. A major climate change may occur at the end of period one or two, affecting the productivity in period two or three, respectively. Thus there is some probability of having a major

climate change in period two or three. In addition there is a non-zero probability of climate stability throughout the planning horizon (from period one to three). Climate change is assumed to be irreversible. If there is climate change in period two this will persist in period three. Thus climate change is assumed to be a single, well-defined and irreversible state of nature.

There is only one production activity for each generation. A natural resource is used as input, and only the accumulated resource use influences the climate change probability. Since the emphasis in this study is on the economic consequences of climate change, a simplification to only one type of climate change should be acceptable. Either there is climate stability or a minor climate change, in which case the economic consequences are small, or there is a substantial climate change, in which case the economic costs are substantial. I am only concerned with the case of substantial economic costs. This approach could also be defended as giving emphasis to the possibility of an atmospheric GHG concentration threshold. If such a threshold is passed, a major (and possibly irreversible) climate change could take place. In the model there is an opening for different degrees of climate change in terms of economic costs. Accumulated resource use not only affects the probability of climate change, but also the degree of climate impact in terms of productivity reduction.

The paper focuses on the intertemporal planning problem, and therefore does not discuss the distribution of resource use and GHG emissions across economic sectors or nations.

In the beginning of each period (which is equivalent to one generation), the resource use and investment plans are revised based on the newest available information on climate change. At this time the climate state in the period is known, whereas the climate state in the next period is unknown. Climate change may occur at the end of the present period, prior to the next period. The optimal plan depends on the real capital stock and the remaining resource stock inherited from the previous generation. This is a Markov process, where the optimal policy only depends on the state variables, and where the history is relevant only through the state variables. The model is solved employing stochastic dynamic programming, where the objective is to maximize the discounted sum of expected utilities.

The optimal plan is influenced by the degree of risk aversion associated with the utility function (i.e. the degree of concavity). Furthermore there is an option value component in the optimal plan due to the assumption of irreversibility.

If there is no climate change in period two, the initial subjective probability parameter of climate change is reduced by a fixed fraction when the plan for the remaining planning horizon is made. Through this simple learning process the newest available information on climate change is made use of.

Since it is less realistic to assume constant technology in a model covering three generations a Hicks-neutral technical progress is specified.

There are a few studies addressing the same problem as the present paper, or related problems. d'Arge, Schulze and Brookshire (1982) employ a two-period model of climate change caused by combustion of fossil fuels. Depletion of fossil fuels by the first generation results in the possibility of losing some share of the capital stock of the future generation. For a fixed climate change probability for generation two they calculate the expected utility. Using different ethical objectives (Utilitarian, Rawlsian, Paretian and a few others)

they analyze and compare the outcomes. Under this approach the ethical objectives are entirely embodied in the discount rate. They conclude that the decisions on the climate change issue depend on the ethical objective employed.

In a more recent paper Spash and d'Arge (1989) present a two-period model of the greenhouse effect and intergenerational transfers. In the model fossil fuel depletion causes capital destruction in the second period due to climate change. There is a fixed probability of climate change in the second period. The objective function is the sum of the current period utility and the expected period two utility. Spash and d'Arge then discuss the potential of intergenerational transfers in terms of capital investment, technology investment or bequest in the form of final goods.

Manne and Richels (1990) analyze how an optimal hedging strategy against large greenhouse warming varies with the accuracy and timing of climate research, and with the prospects of new energy supply and conservation technologies. Specifically they discuss the value of improved information. They employ a probabilistic version of the Global 2100 model, which is an analytical framework for estimating the costs of carbon emissions limits, where five major geopolitical regions are specified. In each region a market or planned economy is simulated. Emphasis is on modelling different energy technology options. They find that the size of the hedge is sensitive to the quality and timing of the climate research results. There can be a large payoff to reducing climate uncertainty - in the region of \$100 billions for the U.S. alone. The optimal hedging strategy is also sensitive to the prospects for new technologies.

Finally, Howarth (1991) employs an overlapping generations framework to discuss allocative efficiency and intergenerational equity applied to the problem of uncertain climate change. The efficient level for a greenhouse gas emissions tax is derived for a competitive economy under rational expectations. Howarth shows that the efficiency and distributional criteria are complementary, making an integrated approach necessary. Consequently traditional cost-benefit techniques are of limited relevance to the analysis of climate policy.

One feature of the present paper that differs from the above-mentioned papers is the use of stochastic dynamic programming. In addition the climate change probability is endogenous, dependent on the accumulated use of an exhaustible resource.

The present paper is divided into two main parts. The model is presented in the first part. To keep the analysis simple I start out with a two-generation model with a fixed climate change probability and a fixed climate change impact. In this case explicit (closed form) solutions can be found. The optimal policy or plan is a function of the real capital stock and the resource stock, and is furthermore contingent on the climate state. Then the model is made more general and expanded to three generations. For this model closed form solutions are not attainable. Different degrees of risk aversion can be specified. The economic impact in the case of climate change, and the probability of climate change, now depend on accumulated resource use. Furthermore technical progress and the simple learning process described above are included.

In the second part of the paper the three-generation model is solved numerically. Some numerical examples are given with special emphasis on the model's sensitivity to parameter values.

2 A Planning Model with Uncertain Climate Change

2.1 A Two-generation Model

There are two generations, each of which lives for one period, $t = 1, 2$. Let the utility function for generation t be:

$$W_t = W(C_t) ; t = 1, 2 \quad (1)$$

where C_t is the consumption level of generation t . The utility function is assumed to be strictly concave.

A single representative firm produces a homogenous consumption/investment good Y_t in each period t using inputs of real capital stock (K_t) and an exhaustible resource (R_t). The production function for generation t is given by:

$$Y_t = f_t(K_t, R_t) \quad (2)$$

The production function is assumed to be strictly concave, differentiable, and strictly increasing with respect to each of the inputs. Both inputs are essential, which is formalized by assuming that the Inada conditions are satisfied ($f(0, R) = f(K, 0) = 0, f'_K(0, R) = f'_R(K, 0) = \infty, f'_K(\infty, R) = f'_R(K, \infty) = 0$, Blanchard and Fischer (1989), p. 38).

There is some probability of climate change at the end of the first period. If climate change occurs, production in period two will be affected. The stochastic variable Q_t represents uncertain climate change, and is equal to α with probability p , and otherwise equal to 1. In the case of climate change production is reduced to a fraction α of the production level with stable climate (no climate change), where $0 < \alpha < 1$. Hence the climate impact reduces the total output of the economy by some fraction. Since the marginal productivity is reduced, the climate impact loss in absolute terms is larger the larger the production. Net production y_t is thus given as:

$$y_t = Y_t Q_t = f_t(K_t, R_t) Q_t \quad (3)$$

The social planner makes a plan at the beginning of period one. At the beginning of period two he can observe if climate change has taken place or not. The climate state in period two will only influence the production level in this period, and thereby the consumption level. In both climate states it is optimal to use the rest of the resource stock in the last period.

The initial available stock of the exhaustible resource is \tilde{R}_1 :

$$\sum_{\tau=1}^2 R_\tau \leq \tilde{R}_1 \quad (4)$$

where τ is the period index. The remaining resource stock at the beginning of period two is \tilde{R}_2 :

$$\tilde{R}_2 = \tilde{R}_1 - R_1 \quad (5)$$

The budget constraints for the consumption/investment good in the two periods are given by:

$$C_1 = f_1(R_1, K_1) Q_1 + K_1 - K_2 \quad (6)$$

$$C_2 = f_2(R_2, K_2) Q_2 + K_2 \quad (7)$$

There are no resource extraction costs. The initial real capital stock K_1 is given. Saving is defined as production minus consumption, equal to $K_2 - K_1$ in period one. Since there is a two-generation horizon in the model, the rest of the real capital stock is consumed by generation two, making saving equal to $-K_2$.

2.2 Stochastic Dynamic Programming

An optimal feedback control policy is found by employing stochastic dynamic programming (Conrad and Clark (1987), Ravindran et al. (1987) and Sydsæter (1990)). The optimal action in each period is then chosen after observing the current state, which is the real capital stock available K_t , the remaining resource stock available \tilde{R}_t , and the climate state Q_t . Initial climate stability is assumed, so the climate state in the first period is given. The planning problem is then solved by backwards induction, finding the optimal action in period two as a function of the state at the beginning of period two. The optimal action in period two is then employed to find the optimal action in period one, taking care of the budget equations for the real capital stock and the resource stock.

Let $J_2(K_2, \tilde{R}_2, Q_2)$ be the maximum total utility for the last generation as a function of the real capital stock, the remaining resource stock, and the climate state. For *the case of climate stability* we then have:

$$J_2(K_2, \tilde{R}_2, Q_2 = 1) = \max_{R_2} W_2[K_2 + f_2(R_2, K_2)] \quad (8)$$

and for *the case of climate change prior to period two* (at the end of period one):

$$J_2(K_2, \tilde{R}_2, Q_2 = \alpha) = \max_{R_2} W_2[K_2 + \alpha f_2(R_2, K_2)] \quad (9)$$

Using the standard maximization technique we find that the optimal action is to use the remaining resource stock \tilde{R}_2 and consume the rest of the real capital stock for both climate states. Thus:

$$J_2(K_2, \tilde{R}_2, Q_2 = 1) = W_2[f_2(\tilde{R}_2, K_2) + K_2] \quad (10)$$

$$J_2(K_2, \tilde{R}_2, Q_2 = \alpha) = W_2[\alpha f_2(\tilde{R}_2, K_2) + K_2] \quad (11)$$

The objective of the social planner is to maximize the utility of generation one plus the expected utility of generation two. The latter is discounted by a factor equal to ρ_2 , where $0 < \rho_2 < 1$. Based on an optimal action in the second period, the problem in the first period can be stated as finding

$$\begin{aligned} J_1(K_1, \tilde{R}_1, Q_1 = 1) &= \max_{C_1, R_1} \{W_1(C_1) \\ &+ \rho_2(1-p)W_2[K_1 + f_1(R_1, K_1) - C_1 + f_2(\tilde{R}_1 - R_1, K_1 + f_1(R_1, K_1) - C_1)] \\ &+ \rho_2 p W_2[K_1 + f_1(R_1, K_1) - C_1 + \alpha f_2(\tilde{R}_1 - R_1, K_1 + f_1(R_1, K_1) - C_1)]\} \end{aligned} \quad (12)$$

where

$$K_2 = K_1 + f_1(R_1, K_1) - C_1 \quad (13)$$

$$R_2 = \tilde{R}_2 = \tilde{R}_1 - R_1 \quad (14)$$

The first-order condition for the consumption level C_1 can be expressed as:

$$\frac{1}{\rho_2} \frac{\partial W_1 / \partial C_1}{\partial W_2 / \partial C_{2:N}} \equiv \frac{1}{\rho_2} MRS_{1,2:N} = 1 - p(1 - MRS_{2:A2,2:N}) + [1 - p(1 - \alpha MRS_{2:A2,2:N})] \frac{\partial f_2}{\partial K_2} \quad (15)$$

$MRS_{1,2:N}$ is the marginal rate of substitution between consumption in period one and period two in the case of *climate stability in period two*, whereas $MRS_{2:A2,2:N}$ is the marginal rate of substitution between consumption in period two in the case of climate change prior to period two and the case of climate stability. $MRS_{2:A2,2:N} > 1$ due to the assumption of a strictly concave utility function and the second period consumption being lower in the case of climate change. The first right hand side parenthesis of eq. (15) corrects for the expected change in marginal utility. The last right hand parenthesis is the shadow price of real capital corrected for expected climate change and marginal utility change.

If the probability of climate change in period two is zero or the potential climate impact is zero, $p(1 - \alpha) = 0$ and eq. (15) simplifies to

$$\frac{1}{\rho_2} MRS_{1,2:N} = 1 + \frac{\partial f_2}{\partial K_2} \quad (16)$$

For the utility function specified as $W_t = \ln C_t$ (confer eq. (21)) and in the case of $p(1 - \alpha) = 0$, $MRS_{1,2:N} = C_2/C_1$ and eq. (16) can be expressed as

$$\frac{1}{\rho_2} \frac{\Delta C_1}{C_1} + \delta_2 = \frac{\partial f_2}{\partial K_2} \quad (17)$$

where $\Delta C_1 = C_2 - C_1$ and δ_2 is the discount rate ($\delta_2 = 1/\rho_2 - 1$). Thus the discounted consumption growth rate plus the discount rate should in optimum equal the period two real capital marginal product. This is the *Ramsey Rule*, which characterizes the optimum rate of capital accumulation (Dasgupta and Heal (1979), pp. 296-7).

The first-order condition for the resource use R_1 can be expressed as:

$$\frac{\partial f_2 / \partial R_2}{\partial f_1 / \partial R_1} = \frac{\partial f_2}{\partial K_2} + \frac{1 - p(1 - MRS_{2:A2,2:N})}{1 - p(1 - \alpha MRS_{2:A2,2:N})} \quad (18)$$

In optimum the marginal rate of transformation between production in period two and one should equal the second period real capital marginal product corrected for expected climate change and marginal utility change.

In the case of $(1 - \alpha)p = 0$ eq. (18) can be expressed as:

$$q = \frac{\partial f_2}{\partial K_2} \quad (19)$$

where q is the growth rate of the marginal resource product, which can be expressed as $(\partial f_2 / \partial R_2 - \partial f_1 / \partial R_1) / (\partial f_1 / \partial R_1)$. In optimum the growth rate of the marginal resource product should equal the period two real capital marginal product. This is the *Hotelling Rule*, characterizing the optimum resource use (extraction) rate (Dasgupta and Heal (1979), p. 156, 291).

Checking out the partial derivatives we find that decreasing α will lead to reduced resource use and consumption for the last generation. If the climate impact is increased the expected last period resource productivity is reduced, making a higher first period resource use optimal. Increasing the relative weight of the last generation in terms of a higher discount factor ρ_2 leads to higher consumption in the last period. If the second period capital productivity is increased the second period resource use is reduced, whereas the consumption is increased. In this case a higher consumption level can be supported in period two, and less resource transfer makes a higher consumption level attainable for generation one. If the concavity of the utility function is increased, i.e. $MRS_{2:A2,2:N}$ is increased, the resource use is reduced and the consumption level increased in period two. Since the marginal utility in period two in the case of climate change is increased, the consumption level is increased. Furthermore, capital transfer is substituted for resource transfer since a fraction α of production in period two is lost in the case of climate change, whereas capital is consumed without any loss. For an increase in the probability of climate change the resource use in period two is reduced due to the reduction in expected capital productivity. On the other hand the effect from an increased climate change probability p on period two consumption depends on the net of two opposite effects. A lower expected capital productivity in period two favors lower capital transfer to period two, whereas a higher marginal utility in the case of climate change favors a higher capital transfer and higher consumption in period two. The net effect depends on which is the dominating factor, confer inequalities (20):

$$\begin{aligned}
1 &\leq \frac{1 + \partial f_2 / \partial K_2}{1 + \alpha \partial f_2 / \partial K_2} < MRS_{2:A2,2:N} \wedge p \uparrow \Rightarrow C_2 \uparrow & (20) \\
1 &< MRS_{2:A2,2:N} < \frac{1 + \partial f_2 / \partial K_2}{1 + \alpha \partial f_2 / \partial K_2} \wedge p \uparrow \Rightarrow C_2 \downarrow
\end{aligned}$$

2.3 A Three-generation Model

In the expanded model there are three periods (and generations). Different degrees of risk aversion can be specified, and the climate change probability and impact parameters are dependent on accumulated resource use. Furthermore a simple learning process and technical progress are specified. The model structure is shown in Figure 1.

The following specification of the utility function is chosen:

$$\begin{aligned}
W_t &= \frac{C_t^{1-\sigma} - 1}{1 - \sigma} \quad \text{for } \sigma > 0, \sigma \neq 1 \\
\lim_{\sigma \rightarrow 1} W_t &= \ln C_t
\end{aligned} \tag{21}$$

This is an isoelastic utility function, where the elasticity of substitution between consumption at two points in time is constant and equal to $1/\sigma$. Thus σ depends on the planner's rate of pure time preference. In the present model including uncertainty, σ has an alternative interpretation as the coefficient of relative risk aversion, which is defined as $-W''(C)C/W'(C)$. The specification implies risk aversion since the function is strictly concave. The larger σ is, the more risk averse each generation is.

The production function is specified as a Cobb-Douglas type:

$$y_t = f_t(K_t, R_t)Q_t = (K_t R_t)^{1/2} Q_t \quad ; \quad t = 1, 2, 3 \quad (22)$$

where Q_t is either equal to 1 or α . Thus constant returns to scale are assumed in the production function. The Inada conditions (explained in section 2.1) are satisfied for this specification.

The maximum utility functions (10) - (11), and (12) still apply, but now for period three and two, respectively. In addition there is a new maximum utility function in the second period for the case of climate change, since climate change may now occur prior to the second period (which is comparable to the first period in the earlier two-generation model). Finally there is a new maximum utility function in the first period.

The new second period maximum utility function in *the case of climate change prior to period two* is:

$$\begin{aligned} J_2(K_2, \tilde{R}_2, Q_2 = \alpha) &= \max_{C_2, R_2} \{W_2(C_2) + \rho_3 W_3[K_2 + \alpha f_2(R_2, K_2) - C_2 \\ &+ \alpha f_3(\tilde{R}_2 - R_2, K_2 + \alpha f_2(R_2, K_2) - C_2)]\} \end{aligned} \quad (23)$$

and

$$K_3 = K_2 + \alpha f_2(R_2, K_2) - C_2 \quad (24)$$

$$R_3 = \tilde{R}_2 - R_2 \quad (25)$$

where ρ_3 is the discount factor pertaining to period three. The second period saving is equal to $K_3 - K_2$.

Using the standard maximization technique we find that the first-order conditions for consumption C_2 and resource use R_2 , respectively, can be expressed as:

$$\frac{1}{\rho_3} \frac{\partial W_2 / \partial C_{2:A2}}{\partial W_3 / \partial C_{3:A2}} = \frac{1}{\rho_3} MRS_{2:A2,3:A2} = 1 + \alpha \frac{\partial f_3}{\partial K_3} \quad (26)$$

$$\frac{\partial f_3 / \partial R_3}{\partial f_2 / \partial R_2} = 1 + \alpha \frac{\partial f_3}{\partial K_3} \quad (27)$$

Comparing these equations with (15) and (18) we see that they differ since we *now have climate change prior to period two* according to eq. (23), which will persist with certainty in the last period due to the assumption of irreversibility. If the potential climate impact is zero ($\alpha = 1$) eqs. (26) and (27) are identical to eqs. (15) and (18) in the earlier two-generation model for the case $\alpha = 1$ or $p = 0$, noticing the change in the period index.

The partial derivatives of the last period consumption and resource use with respect to the discount factor, the climate impact α and the last period marginal capital product are the same as in the two-generation model except for the resource use with respect to climate impact. An increase in climate impact in terms of a lower α leads to higher last period resource use. If some of the resource is used in the second period a fraction of the production is lost. For real capital transferred to the last period another fraction is lost through the production process due to the climate impact. However, if the resource is used in the last period a fraction of production will be lost only in this period.

So far the potential climate impact has been constant, equal to α . A more realistic assumption is to let the impact depend on accumulated resource use, and thus on accumulated greenhouse gas release to the atmosphere. Let the climate impact function be:

$$\alpha_t = g(\sum_{\tau=1}^t R_\tau) = \theta_0 - \theta_1(\sum_{\tau=1}^t R_\tau)^\nu \quad (28)$$

where $0 < \theta_0 \leq 1$, $0 < \theta_1$, $0 < \nu$, and these parameters are chosen so as to make $0 < g(\sum_{\tau=1}^t R_\tau) \leq 1$. The potential climate impact we already are committed to is represented by θ_0 . Even if there is no resource use, a climate change would have an impact on production equal to θ_0 .

Furthermore the climate change probability has been fixed at p . It may be argued that the probability of climate change depends on accumulated resource use. Resource use leads to greenhouse gas emissions which are added to the stock of greenhouse gases in the atmosphere. The exhaustible resource may be thought of as fossil fuels, where combustion releases carbon dioxide to the atmosphere, increasing the probability of a climate change. Let the probability function be:

$$p_t = h(\sum_{\tau=1}^t R_\tau) = p_0(\sum_{\tau=1}^t R_\tau)^\gamma \quad (29)$$

Thus it is assumed that the probability of a major climate change depends on accumulated resource use at an elasticity equal to γ , which is assumed to be positive. The parameters are chosen so as to make $0 \leq h(\sum_{\tau=1}^t R_\tau) \leq 1$.

The parameter p_0 can be interpreted as a subjective probability of climate change in the first period. The true value of p is not known by the social planner, so he chooses an initial subjective value which is updated according to a simple learning process, where the planner observes if climate change has taken place prior to the second period or not. If there is no climate change, he takes this as an indication that the subjective p_0 is too high. Thus the parameter is reduced by some factor β , where $0 < \beta < 1$. The reduction of the probability parameter may consequently influence the optimal strategy for the second and third period if there is climate stability up to the second period. This updating process is consistent with Bayes's theorem.

In a model covering many decades (and generations) it is less realistic to assume a given technology. Let us now assume that there is a Hicks-neutral technical progress at a growth rate equal to ω . The production function is thus specified as:

$$y_t = (1 + \omega)^{t-1} (K_t R_t)^{1/2} Q_t \quad ; \quad t = 1, 2, 3 \quad (30)$$

The new first period maximum expected utility function becomes:

$$\begin{aligned} J_1(K_1, \tilde{R}_1, Q_1 = 1) &= \max_{C_1, R_1} \{W_1(C_1) + \rho_2 E_{Q_2} J_2(K_2, \tilde{R}_2, Q_2)\} \\ &= \max_{C_1, R_1} \{W_1(C_1) + \rho_2(1 - p_2) J_2[K_1 + f_1(R_1, K_1) - C_1, \tilde{R}_1 - R_1, Q_2 = 1] \\ &\quad + \rho_2 p_2 J_2[K_1 + f_1(R_1, K_1) - C_1, \tilde{R}_1 - R_1, Q_2 = \alpha_2]\} \end{aligned} \quad (31)$$

where E_{Q_2} is the expectation operator applied on the climate state variable Q_2 . The optimal saving and resource use path is found as the path that maximizes the maximum

expected utility function. The first-order conditions for consumption C_1 and resource use R_1 are, respectively:

$$\frac{1}{\rho_2} \frac{\partial W_1}{\partial C_1} = (1 - p_2) \frac{\partial J_{2:N}}{\partial K_2} + p_2 \frac{\partial J_{2:A2}}{\partial K_2} \quad (32)$$

$$\begin{aligned} \left\{ (1 - p_2) \frac{\partial J_{2:N}}{\partial K_2} + p_2 \frac{\partial J_{2:A2}}{\partial K_2} \right\} \frac{\partial f_1}{\partial R_1} + \{ J_{2:A2} - J_{2:N} \} \frac{\partial p_2}{\partial R_1} \\ = (1 - p_2) \frac{\partial J_{2:N}}{\partial R_2} + p_2 \frac{\partial J_{2:A2}}{\partial R_2} \end{aligned} \quad (33)$$

where $J_{2:N}$ is the second period maximum expected utility function in the case of climate stability and $J_{2:A2}$ is the second period maximum expected utility function in the case of climate change prior to period two.

The model is solved numerically applying backwards induction (confer section 2.2).

2.4 A Model Version Featuring Reversibility

Climate change is assumed to be irreversible, and this will contribute to an option value component in the optimal plan. The importance of the irreversibility assumption can be evaluated by analyzing a variant of the earlier model incorporating a positive probability of reversibility. Thus λ is the probability that the initial climate situation again arises in period three given climate change prior to period two. Eq. (23) is now replaced by:

$$\begin{aligned} J_2^*(K_2, \tilde{R}_2, Q_2 = \alpha) &= \max_{C_2, \tilde{R}_2} \{ W_2(C_2) \\ &+ \rho_3(1 - \lambda)W_3[K_2 + \alpha f_2(R_2, K_2) - C_2 + \alpha f_3(\tilde{R}_2 - R_2, K_2 + \alpha f_2(R_2, K_2) - C_2)] \\ &+ \rho_3\lambda W_3[K_2 + \alpha f_2(R_2, K_2) - C_2 + f_3(\tilde{R}_2 - R_2, K_2 + \alpha f_2(R_2, K_2) - C_2)] \} \end{aligned} \quad (34)$$

and the new first-order conditions are:

$$\frac{1}{\rho_3} \frac{\partial W_2 / \partial C_{2:A2}}{\partial W_3 / \partial C_{3:A2}} = 1 - \lambda(1 - MRS_{3:N3,3:A2}) + [\alpha - \lambda(\alpha - MRS_{3:N3,3:A2})] \frac{\partial f_3}{\partial K_3} \quad (35)$$

$$\frac{\partial f_3 / \partial R_3}{\partial f_2 / \partial R_2} = \alpha \frac{\partial f_3}{\partial K_3} + \frac{1 - \lambda(1 - MRS_{3:N3,3:A2})}{1 - \lambda(1 - \frac{1}{\alpha} MRS_{3:N3,3:A2})} \quad (36)$$

where $MRS_{3:N3,3:A2}$ is the marginal rate of substitution between consumption in period three in the case of climate stability following climate change in period two, and the case of climate change prior to period two. $MRS_{3:N3,3:A2} < 1$ due to the concavity assumption for the utility function. If the reversibility probability λ is zero the first-order conditions are equal to eqs. (26) and (27).

The net effect of a decreased reversibility probability on last period consumption depends on the relative strength of two opposing effects. A lower expected last period capital productivity favors lower capital transfer to the last period, and lower consumption, whereas a higher last period consumption is favored due to a higher marginal utility

in the case of climate change in the last period. Thus:

$$\begin{aligned} \frac{1 + \alpha \partial f_3 / \partial K_3}{1 + \partial f_3 / \partial K_3} < MRS_{3:N3,3:A2} < 1 \wedge \lambda \downarrow &\Rightarrow C_3 \downarrow \\ MRS_{3:N3,3:A2} < \frac{1 + \alpha \partial f_3 / \partial K_3}{1 + \partial f_3 / \partial K_3} \leq 1 \wedge \lambda \downarrow &\Rightarrow C_3 \uparrow \end{aligned} \quad (37)$$

Reformulating the first-order condition for consumption (35) we find that the effect of a possible reversibility compared to the irreversibility case given in eq. (26) is proportional to the absolute value of $-[(\alpha - MRS_{3:N3,3:A2})\partial f_3 / \partial K_3 + 1 - MRS_{3:N3,3:A2}]\lambda$. This term is positive if the concavity effect is stronger than the effect from the reduced productivity, confer the last inequality in (37), and negative in the opposite case, confer the first inequality in (37). If the term is positive the last period consumption is increased compared to the irreversibility case, and vice versa for a negative term.

Checking out the partial derivatives in the case of a positive term we find that the reversibility effect, i.e. the increase in last period consumption, is strongest for a large climate change impact (i.e. small α), a large capital productivity ($\partial f_3 / \partial K_3$), for a constant marginal utility (i.e. $MRS_{3:N3,3:A2} = 1$), and for a large reversibility probability (λ). If the term is negative the reversibility effect, i.e. the reduction in last period consumption compared to the irreversibility case, is strongest when the climate impact is small, the capital productivity large, the concavity strong (i.e. $MRS_{3:N3,3:A2}$ close to zero), and the reversibility probability large.

If the reversibility probability is reduced the last period resource use is reduced due to the reduction in expected resource productivity. From the first-order condition (36) the reversibility effect on last period resource use compared to the irreversibility case depends on the term $[1 - \lambda(1 - MRS_{3:N3,3:A2})]/[1 - \lambda(1 - \alpha^{-1}MRS_{3:N3,3:A2})]$, which is positive and less than one. The reversibility effect in terms of a larger last period resource use is then larger the smaller the value of this term. The value of the term is smallest for a large climate impact, a weak concavity of the utility function, and a high reversibility probability.

The numerical illustrations in the next section are based on the earlier irreversibility version of the model.

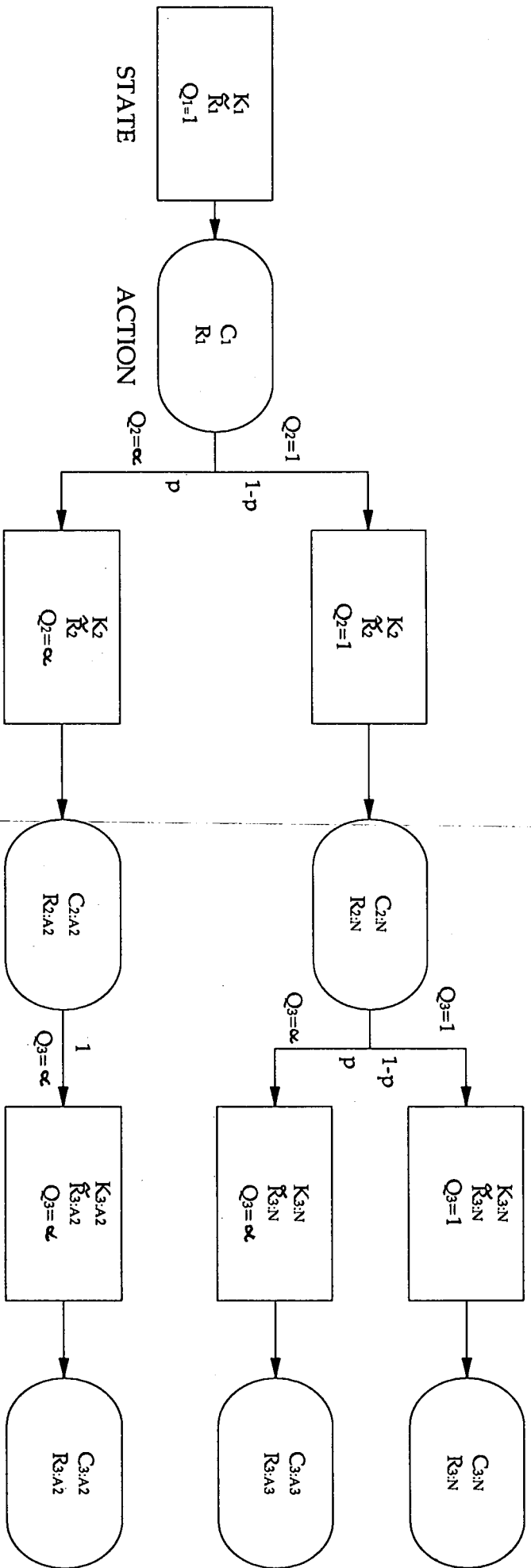


FIGURE 1

3 Numerical Illustrations and Parameter Sensivity

To solve the model numerically, the following initial stock and parameter values are chosen: $\tilde{R}_1 = 1.5, K_1 = 1, \sigma = 1, \rho_2 = \rho_3 = 0.9$.

A reference case with no climate change uncertainty (climate stability) is chosen by setting $p_0 = 0$. The optimal consumption, saving and resource extraction paths for different parameter values (in the case of climate stability) are compared to this reference case.

In another base case the parameter values are chosen to give a constant climate impact equal to 0.7, a constant climate change probability equal to 0.3, no learning and no technical progress. The relevant parameter values are: $\gamma = 0, p_0 = 0.3, \theta_0 = 1, \theta_1 = 0.3, \nu = 0, \beta = 1, \omega = 0$.

The initial stocks \tilde{R}_1 and K_1 , and the parameters p_0 and γ are calibrated to make $h(\sum_{\tau} R_{\tau})$ defined in (29) positive and less than one for R_1 and $R_1 + R_2$. In addition the parameters θ_0, θ_1 and ν are calibrated to constrain $g(\sum_{\tau} R_{\tau})$ defined in (28) to positive values less than or equal to one for R_1 and $R_1 + R_2$.

3.1 The Reference Case: Climate Stability

In the reference case there is no uncertainty, and thus climate stability. The optimal paths are shown in Table 1.

	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 2$
C_1	0.723	0.843	0.908
C_2	1.034	1.014	0.997
C_3	1.331	1.155	1.065
K_2	1.151	1.057	1.005
K_3	0.875	0.754	0.693
R_1	0.764	0.809	0.834
R_2	0.498	0.478	0.466
R_3	0.238	0.213	0.200
Y_1	0.874	0.899	0.913
Y_2	0.757	0.711	0.685
Y_3	0.456	0.401	0.372

Table 1: The reference case for different levels of elasticity of substitution between consumption in any two periods.

Choosing an elasticity of substitution between consumption in any two periods equal to one as the base case (equivalent to $\sigma = 1$), yields the results in the second column of the table. There is a steady consumption growth and a steady reduction in the resource use or extraction. Likewise the production level shows a steady decrease. The capital transfer (or saving) is lower in period two than one. If the elasticity of substitution is halved ($\sigma = 2$), the first period consumption level is increased and the consumption growth rate reduced. The saving is reduced in both periods. The resource extraction rate is increased, as also is the first period production. Production in period two and three are reduced.

For an elasticity of substitution increased to 2 ($\sigma = 0.5$), the deviations from the base case are reversed.

For the rest of the analysis the elasticity of substitution between consumption in any two periods is kept constant at one ($\sigma = 1$).

3.2 Different Specifications of the Climate Impact and Climate Change Probability Functions

Three different specifications of the climate impact and climate change probability functions are chosen: constant, linear and quadratic. The optimal paths are shown in Table 2, where the reference case is included in the first column for comparison. The three cases shown are climate stability in all periods (N), climate change prior to period three (A3), and climate change prior to period two (A2).

In the first specification the climate impact is constant at 0.7 ($\alpha = 0.7$), meaning that output is reduced by 30% in the event of climate change. Furthermore the climate change probability is constant at 30% ($p_0 = 0.3$). The optimal paths are shown in column two to four in the table. In the second specification climate impact and climate change probability is a linear function of aggregate resource extraction. The optimal paths are shown in column five to seven in Table 2. In the third specification climate impact and climate change probability is a quadratic function of aggregate resource extraction. In column eight to ten in Table 2 the optimal paths are presented. Thus the parameter values of θ_0 , θ_1 and p_0 are kept constant at their base case value (refer to the beginning of this section), whereas ν and γ are given the values 0, 1 and 2 in the three different specifications of the functions.

	Ref. case	Constant impact and prob.			Linear impact and prob.			Quadratic impact and prob.		
		N	A3	A2	N	A3	A2	N	A3	A2
C_1	0.843	0.839	0.839	0.839	0.818	0.818	0.818	0.809	0.809	0.809
C_2	1.014	1.018	1.018	0.886	1.013	1.013	0.906	0.988	0.988	0.912
C_3	1.155	1.152	1.040	0.947	1.193	1.031	0.966	1.224	1.022	0.998
K_2	1.057	1.088	1.088	1.088	1.034	1.034	1.034	1.011	1.011	1.011
K_3	0.754	0.778	0.778	0.688	0.765	0.765	0.705	0.733	0.733	0.698
R_1	0.809	0.859	0.859	0.859	0.726	0.726	0.726	0.672	0.672	0.672
R_2	0.478	0.460	0.460	0.442	0.536	0.536	0.525	0.498	0.498	0.474
R_3	0.213	0.181	0.181	0.198	0.238	0.238	0.249	0.329	0.329	0.353
Y_1	0.899	0.927	0.927	0.927	0.852	0.852	0.852	0.820	0.820	0.820
Y_2	0.711	0.707	0.707	0.486	0.744	0.744	0.576	0.710	0.710	0.599
Y_3	0.401	0.375	0.262	0.259	0.427	0.266	0.262	0.491	0.289	0.301

Table 2: The optimal paths' sensivity to climate impact elasticity and climate change probability elasticity.

Comparing *the climate stability* paths (the N case) to the reference case, we find that the first generation consumption is reduced for all specifications of the climate impact and

climate change probability functions, and more so for the linear case and the quadratic case. The higher the elasticities, the lower the first generation consumption. Even if the first generation consumption is reduced in the constant impact and probability case, the consumption growth rate from period one to three is similar to the reference case, whereas the consumption growth rate is increased in the linear and quadratic cases.

The first generation saving (real capital transfer) is increased in the constant case compared to the reference case, whereas it is reduced for the two other specifications. A higher first generation saving in the constant case is related to a higher first generation resource use due to lower expected future resource productivity - refer to the discussion later in this section. The higher first period resource use leads to higher first period production and consumption, and makes increased saving optimal. Some of this saving is related to hedging future generations against a potential consumption and utility loss due to climate change. In the second period the saving is higher in the constant and linear cases than the reference case, whereas the quadratic case saving is lower.

If *climate change occurs prior to period two* (the A2 case) consumption is reduced in period two and three compared to the climate stability case, and the consumption growth rate from period one to three is higher for the quadratic specification than the linear specification, which again is higher than for the constant specification. The second period saving is lower than the climate stability case for all three specifications of the impact and probability functions, and is highest for the linear case and lowest for the constant case when comparing the three specifications.

Finally, if *climate change occurs prior to period three* (the A3 case) third generation consumption is lower than for the climate stability case, and consumption is lowest for the quadratic case and highest for the constant case when comparing the different specifications.

When it comes to optimal resource extraction for the *climate stability case* (the N case) the resource extraction rate is fastest for the constant case, followed by the reference case, the linear case and the quadratic case. (By a faster resource extraction rate I mean a higher first period resource use and/or a lower last period resource use).

From the resource extraction and saving path it follows that first generation production is highest for the constant case, followed by the reference case, the linear case and the quadratic case. This order is reversed for the third generation production.

Given the event that *climate change occurs prior to period two* (the A2 case) resource extraction is slower than for the climate stability case. (By a slower resource extraction I mean a lower first period resource use and/or a higher last period resource use). Furthermore resource extraction is fastest for the constant case and slowest for the quadratic case when the different specifications are compared. Comparing the different specifications of the impact and probability functions we find that production is highest for the quadratic case and lowest for the constant case in period two and three.

Comparing the different specifications if *climate change occurs prior to period three* (the A3 case), the resource extraction is fastest for the constant case, and slowest for the quadratic case.

From this analysis we find that the optimal resource extraction rate is faster when there is a *constant climate impact and climate change probability* compared to the reference case of no uncertainty and climate stability. Since there is a future probability of output loss

due to climate change, it is optimal to extract more of the resource today. If *climate impact and climate change probability are made a function of aggregate resource extraction* this finding is reversed, and the optimal resource extraction rate is slower than for the reference case. The latter conclusion depends on the elasticities of climate impact and climate change probability being sufficiently large, confer section 3.4. In terms of expected future output and utility there is a cost to a faster resource extraction rate, since the impact and probability of future climate change will increase. Thus it is optimal to reduce the first period resource extraction to a level lower than the case of no uncertainty and stable climate. This effect is stronger for the quadratic than linear version of the functions since the marginal cost of increased resource extraction is higher for the quadratic functions.

If climate change occurs prior to period two, a fraction of the production will be lost in period two and another fraction will be lost in period three. In addition there is a larger climate impact in the last period. Thus it is optimal to slow down the resource extraction relative to the case of no climate change prior to period two, making more of the resource available for production in period three.

Since the first period resource use is higher when the impact and climate change probability is constant than in the reference case of climate stability, production is higher and it is optimal to save more for the next generation. It is also optimal to save more for the next generation to hedge against the possible loss of utility due to climate change. For the same reason higher saving is found in the second period. For the linear and quadratic cases, however, first period production is lower than in the reference case, and it is optimal to save less for the next generation. On the other hand saving in period two is higher for the linear case than the reference case. This is caused by a higher period two resource use for the linear case, and thus a higher production level making a higher saving optimal. For all specifications of the impact and probability functions, period two saving is lower if climate change occurs prior to period two. There is a relative gain in delaying resource extraction since a fraction of the output will be lost in both remaining periods. By delaying resource extraction and making more of the resource available for the last generation, a fraction of the output based on this resource use is only lost in the last period, reducing the total loss due to climate impact. The second generation is compensated for a lower resource use by a relatively higher consumption, and consequently a lower real capital transfer to the last generation.

3.3 The Effect of Reducing the Probability Function Parameter (Learning) and Technical Progress

If the initial climate change probability parameter p_0 is unknown and based on the beliefs of the social planner, it may be argued that it should be reduced by some factor $0 < \beta < 1$, in the case of climate stability until period three. This is a simple updating or learning process. Let us assume that p_0 prior to period three is reduced by 30% in this case ($\beta = 0.7$), and check the effect on the optimal policy.

Over three generations it is less realistic to assume a given technology. Let us assume a Hicks-neutral technical progress at a 20% growth rate ($\omega = 0.2$). In period two and three the output level will be increased 20% for the same resource and capital input relative to the previous period. The effect on optimal policy of this type of technical progress is

analyzed.

The results for this version of the model are shown in Table 3 for the case of linear climate impact and climate change probability functions, and in Table 4 for the case of quadratic climate impact and climate change probability functions. In the first three columns of the tables the probability parameter is kept constant (no learning), and the technology is constant. In the next part of the tables the probability parameter is reduced (learning), whereas technology is still constant. In the last part of the tables the probability parameter is constant and technical progress assumed.

	Constant prob. parameter. Constant technology			Learning $\beta = 0.7$			Technical progress $\omega = 0.2$		
	N	A3	A2	N	A3	A2	N	A3	A2
C_1	0.818	0.818	0.818	0.819	0.819	0.819	0.823	0.823	0.823
C_2	1.013	1.013	0.906	1.015	1.015	0.905	1.086	1.086	0.968
C_3	1.193	1.031	0.966	1.188	1.027	0.966	1.471	1.220	1.160
K_2	1.034	1.034	1.034	1.031	1.031	1.031	0.969	0.969	0.969
K_3	0.765	0.765	0.705	0.761	0.761	0.704	0.760	0.760	0.697
R_1	0.726	0.726	0.726	0.722	0.722	0.722	0.626	0.626	0.626
R_2	0.536	0.536	0.525	0.538	0.538	0.527	0.553	0.553	0.527
R_3	0.238	0.238	0.249	0.240	0.240	0.250	0.321	0.321	0.347
Y_1	0.852	0.852	0.852	0.850	0.850	0.850	0.791	0.791	0.791
Y_2	0.744	0.744	0.576	0.745	0.745	0.577	0.878	0.878	0.696
Y_3	0.427	0.266	0.262	0.427	0.266	0.262	0.711	0.460	0.463

Table 3: The optimal paths' sensitivity to reduction of the subjective climate change probability parameter p_0 (learning), and to Hicks-neutral technical progress. The case of linear climate impact and climate change probability functions.

The optimal resource extraction paths are very similar for the cases of no learning and learning in the linear case, with a slightly slower resource extraction rate in the latter case. However, in the quadratic case resource extraction is faster when the subjective probability parameter is reduced (learning). A reduced parameter means a reduced cost to increased resource use in terms of reduced future probability of climate change. Thus a faster extraction rate is optimal. The marginal reduction in potential future climate impact cost due to learning is larger for the quadratic case than the linear case. Consequently the effect of learning is larger in the quadratic case since each generation is assumed to be risk averse. Also the optimal policy in the case of climate change prior to period two is slightly influenced by learning since the real capital transfer to and resource stock for generation two is influenced by the potential probability parameter reduction.

The optimal first period saving is similar in both the linear and quadratic specification cases when comparing probability parameter reduction to the constant parameter case. However, the second period saving is reduced in the linear case and increased in the quadratic case. A faster resource extraction in the quadratic case leads to higher production in period two, and the last generation is compensated for this through an increase of the real capital transfer.

Due to the reallocation of resources between periods caused by the potential reduction in the probability parameter, consumption is increased for generation one and two in all cases, whereas consumption for generation three is reduced.

	Constant prob. parameter. Constant technology			Learning $\beta = 0.7$			Technical progress $\omega = 0.2$		
	N	A3	A2	N	A3	A2	N	A3	A2
C_1	0.809	0.809	0.809	0.811	0.811	0.811	0.816	0.816	0.816
C_2	0.988	0.988	0.912	0.998	0.998	0.911	1.058	1.058	0.974
C_3	1.224	1.022	0.998	1.212	1.009	0.995	1.507	1.227	1.227
K_2	1.011	1.011	1.011	1.012	1.012	1.012	0.952	0.952	0.952
K_3	0.733	0.733	0.698	0.740	0.740	0.697	0.723	0.723	0.682
R_1	0.672	0.672	0.672	0.679	0.679	0.679	0.590	0.590	0.590
R_2	0.498	0.498	0.474	0.519	0.519	0.471	0.500	0.500	0.449
R_3	0.329	0.329	0.353	0.302	0.302	0.350	0.410	0.410	0.461
Y_1	0.820	0.820	0.820	0.824	0.824	0.824	0.768	0.768	0.768
Y_2	0.710	0.710	0.599	0.725	0.725	0.595	0.828	0.828	0.703
Y_3	0.491	0.289	0.301	0.473	0.269	0.298	0.784	0.504	0.545

Table 4: The optimal paths' sensitivity to reduction of the subjective climate change probability parameter p_0 (learning), and to Hicks-neutral technical progress. The case of quadratic climate impact and climate change probability functions.

When a Hicks-neutral technical progress is specified the output level for the same level of inputs increases over time. Future generations will be better off and the saving is consequently reduced. Especially the first generation saving is reduced. A slower resource extraction rate becomes optimal since future use leads to higher production. Taken together the effect is a slight increase in first generation consumption and a higher consumption growth rate, making especially generation three much better off.

3.4 Parameter Sensivity

We have found that the optimal resource extraction rate is faster for a constant climate impact and climate change probability than for the reference case with no uncertainty and climate stability. On the other hand the optimal resource extraction rate is slower for a linear or quadratic climate impact and climate change probability than for the reference case. To test the parameter sensivity of the model we now ask what climate impact elasticity (ν) is necessary to keep first period resource extraction equal to the reference case when a constant climate change probability at 0.3 is assumed. The same question in terms of climate change probability elasticity (γ) can be asked for the opposite case of a constant climate impact at 0.7. Finally we choose the linear climate impact and climate change probability case and find the necessary coefficient of relative risk aversion (σ) to make the first generation consumption equal to the reference case. Similar analyses can be done for the constant and quadratic climate impact and climate change probability cases. The results are shown in Table 5.

	Ref. case	Constant prob. and $\nu = 0.7$			Constant impact and $\gamma = 0.6$			Linear impact and prob., and $\sigma = 1.25$		
		N	A3	A2	N	A3	A2	N	A3	A2
C_1	0.843	0.830	0.830	0.830	0.831	0.831	0.831	0.842	0.842	0.842
C_2	1.014	1.019	1.019	0.897	1.019	1.019	0.884	1.006	1.006	0.895
C_3	1.155	1.168	1.026	0.951	1.167	1.049	0.951	1.161	1.003	0.943
K_2	1.057	1.068	1.068	1.068	1.067	1.067	1.067	1.015	1.015	1.015
K_3	0.754	0.776	0.776	0.701	0.775	0.775	0.684	0.745	0.745	0.687
R_1	0.809	0.808	0.808	0.808	0.808	0.808	0.808	0.735	0.735	0.735
R_2	0.478	0.494	0.494	0.476	0.495	0.495	0.478	0.533	0.533	0.521
R_3	0.213	0.198	0.198	0.216	0.197	0.197	0.214	0.232	0.232	0.244
Y_1	0.899	0.899	0.899	0.899	0.899	0.899	0.899	0.857	0.857	0.857
Y_2	0.711	0.727	0.727	0.529	0.727	0.727	0.500	0.735	0.735	0.567
Y_3	0.401	0.392	0.250	0.250	0.391	0.274	0.268	0.416	0.258	0.256

Table 5: Parameter combinations making the first generation resource extraction or consumption equal to the reference case.

If the climate change probability is fixed, a climate impact elasticity at 0.7 makes the first period resource use equal to the reference case. A climate change probability at 0.6 is sufficient to make the first period resource use equal to the reference case. The optimal paths for the two cases are almost identical. Compared to the reference case there is a faster resource extraction rate, a lower first generation consumption but higher consumption growth rate, and a larger saving. Thus the effect of the constant climate impact or the constant climate change probability favoring a relative faster resource extraction dominates in period two and three.

Finally, a coefficient of relative risk aversion at 1.25 is sufficient to make the first generation consumption equal to the reference case. The coefficient of relative risk aversion has so far been equal to 1 in the reference case and all versions of the model. Increasing the coefficient to 1.25 makes a higher first period consumption, a lower consumption growth rate and a faster resource extraction rate optimal. For the first generation consumption this offsets the effect of climate impact and climate change probability being a linear function of aggregate resource use, which compared to the reference case favors a lower first period consumption, a higher consumption growth rate and a slower resource extraction rate. The net of the two conflicting effects on period one consumption is zero, making the first generation consumption equal to the reference case.

Compared to the reference case generation two consumption is reduced, whereas last generation consumption is increased. The saving is reduced and the resource extraction rate slower.

4 Conclusions

Summing up, the optimal resource extraction path compared to a reference case of climate stability will depend on the assumed relation between aggregate resource extraction and climate impact, and climate change probability. If the climate impact and climate change probability are constant and independent of the resource extraction path, a faster extraction rate (i.e. higher first period resource use and/or lower last period resource use) than the reference case is optimal since the expected future resource productivity is lower than the present productivity. If, on the other hand, climate impact and climate change probability increases with increased aggregate resource use, this result is reversed and the optimal resource extraction rate is slower than the reference case due to the increased climate change probability and potential climate impact from a faster resource extraction path.

If climate change occurs prior to period two, the optimal resource extraction rate is even slower since a fraction of the output will be lost in each of the remaining periods, and since the climate impact may increase with increased aggregate resource use.

Due to the differing effects on optimal resource extraction from different assumptions about the relation between aggregate resource extraction and climate impact, and climate change probability, there are combinations of parameter values in these functions that make the first period resource extraction equal to the reference case of climate stability. By increasing the coefficient of relative risk-aversion, the first-generation consumption in the linear climate impact and climate change probability case can be lifted up to the consumption level of the reference case. For the constant and quadratic impact and probability function cases other values for the coefficient of relative risk aversion would make the first generation consumption equal to the reference case.

A faster resource extraction leads to higher production for the first generation(s), which leads to higher consumption and higher saving or capital transfer to the next generation. Some of this saving is related to hedging future generations against a potential consumption and utility loss due to climate change, caused by risk aversion and an option value component due to irreversibility.

There is a tradeoff between resource and capital allocation efficiency and intergenerational equity since saving and resource use determines the capital stock and resource stock available for the next generation(s), and through this the capital and resource productivity. Thus allocation efficiency and intergenerational equity are interdependent, and it will not be possible to use capital transfer to secure some distribution of utility (consumption) without affecting the allocation efficiency. A further difficulty is caused by the inability to transfer capital to earlier generations - capital can only be transferred forward in time.

If there is climate stability until period two, the climate change probability parameter may be reduced due to some simple updating or learning process. This leads to a faster resource extraction rate since the cost from a faster extraction in terms of future climate change probability is reduced.

A Hicks-neutral technical progress makes future generations relatively better off, and leads to a lower optimal saving. The optimal resource extraction rate is slower since more output is to be gained from delaying extraction.

The present model can be expanded in various directions. One obvious suggestion is

to increase the number of periods to check if there is a significant 'horizon effect' affecting the results. Another suggestion is to try more flexible forms for the production function. A more elaborate updating process for the climate change probability parameters may be incorporated in the model, making the social planner able to discriminate between for instance a linear and quadratic relation between resource extraction and the climate change probability. The numerical part of the study can be expanded by calibrating the parameters on GNP and energy data, and on potential global warming impact taken from the relevant literature.

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