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EFFICIENT INCOMPLETE INTERNATIONAL CLIMATE AGREEMENTS[a]

by

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Abstract

We study the optimal design of a carbon tax when a group of countries seeks to maximize its net income minus its environmental costs, which depend on the sum of CO_2 emissions from all countries. When both production and consumption of internationally traded fossil fuels are taxed, there exists a particular combination of producer and consumer taxes which is optimal. It is also shown that with this tax the sum of the consumer tax and producer tax should be equal across all fossil fuels per unit of carbon. On the other hand, when the cooperating countries use a tax on consumption (or production) of fossil fuels as the only policy instrument, the tax per unit of carbon should in general be differentiated across fossil fuels. We close the paper by giving an empirical illustration of the theoretical analysis, assuming the cooperating countries consist of the OECD countries.

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1. Introduction

In discussions of the greenhouse problem, some kind of carbon tax is often proposed as part of an international climate policy, see e.g. Pearce (1991), Poterba (1991) and Hoel (1992). Global cost effectiveness suggests that a carbon tax should be equalized across types of fossil fuels, across sectors within each country, and across countries[1]. However, it seems unlikely that an international climate agreement in the next couple of decades will imply harmonization of domestic carbon taxes across all countries in the world. One reason is that with such an agreement the costs will be very inequitable, with LDCs bearing the main burden (in terms of percentage GDP loss), see e.g. Kverndokk (1993) and Burniaux, Martin, Nicoletti and Martins (1992) for a more general discussion on distribution of costs. Moreover, even if this distributional issue could be "solved" through appropriate side payments, universal participation in an international climate agreement is unlikely due to the free rider problem: If many countries are cooperating to reduce emissions, any individual country will be better off if the other countries cooperate, while the country does not take part in the agreement and pursues its self-interest. For a further discussion of the free rider problem in the context of international environment problems, see e.g. Barrett (1991), Carraro and Siniscalco (1991) and Mäler (1990).

Limited participation in a climate agreement has important consequences: When the participating countries reduce their demand for fossil fuels, international fuel prices may fall. This gives the non-participating countries an incentive to increase their use of fossil fuels, thus partly offsetting the reduced emissions from the participating countries. Studies by e.g. Bohm (1993) and Pezzey (1992) demonstrate that this effect might be quite strong. An important question is therefore how the specific design of a carbon tax might influence international fuel prices, and thereby CO_2 emissions from the non-signatories. In particular, it is of interest to see whether a differentiation of the carbon tax (per unit of carbon) between different types of fossil fuels can be used as a way to influence CO_2 emissions from non-participating countries. This is the topic of the present paper.

[1] For a further discussion and some qualifications, see e.g. Hoel (1992).

In our analysis, both the cooperating and the non-cooperating countries are assumed to consume and produce oil, coal and natural gas. All three fuels are traded internationally. The objective of the group of cooperating countries is to maximize its net income minus its environmental costs, where environmental costs depend on the sum of CO_2 emissions from all countries.

In a situation where all countries participate in an agreement to reduce CO_2 emissions, taxes on consumption of fossil fuels and taxes on production of fossil fuels have identical economic consequences, provided the use of tax revenues are identical in the two cases, see e.g. Hoel (1993) and Whalley and Wigle (1991). This is no longer true when there is limited participation in the international agreement, as in the present context. In this case the international fuel prices, and therefore consumption and production of fossil fuels in the non-cooperating countries, depend on whether it is consumption or production of fossil fuels (or both) which are taxed in the cooperating countries.

The case with no restrictions on the tax policy is studied in Section 3. We show that there is a particular combination of taxes on consumption and production of fuels which is optimal[2]. Furthermore, when the taxes are optimally divided between a consumption and production tax, the total tax per unit of carbon should be equal for the three fossil fuels. In Section 4 the optimal design of a carbon tax is derived, under the assumption that only consumption of fossil fuels is taxed. It is shown that in general, the tax per unit of carbon should be differentiated across the fossil fuels. Section 5 gives an empirical illustration of the theoretical analysis of the previous sections, assuming that the cooperating countries consist of the OECD countries. We restrict the empirical analysis to the case in which global emissions are given. Finally, in Section 6 we sum up our results.

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[2] This has been discussed by Hoel (1993) for the case in which the fossil fuel market is modeled as one aggregate market, and by Golombek et al. (1993) for the case in which all fuel markets are competitive and only oil is traded internationally.

2. The model

Henceforth, countries participating in the agreement are called the cooperating countries, whereas countries not participating are termed the non-cooperating countries. The purpose of the climate agreement is to design carbon taxes such that the difference between net income and environmental costs of the cooperating countries is maximized.

In the cooperating countries, production of the three fossil fuels oil, coal and natural gas are denoted by x_1, x_2, and x_3, respectively. Moreover, consumption of oil, coal and natural gas is termed y_1, y_2, and y_3. All fuels are traded internationally at the prices P_1, P_2 and P_3. In the non-cooperating countries, production of fossil fuels are denoted by X_i, i=1,2,3, and consumption by $D_i(P_1,P_2,P_3)$.

The objective function of the cooperating countries is given by

(1) $$W = u(y_1,y_2,y_3) - \sum_{i=1}^{3} c_i(x_i) - \sum_{i=1}^{3} P_i \cdot (y_i - x_i) - H(E)$$

where u is the utility derived from consumption of oil, coal and natural gas, while the c_i-functions give production costs of the three fuels. We ignore the fact that oil, coal and gas are exhaustible resources, which implies that extraction costs may increase as these fuels are depleted. The term $P_i \cdot (y_i - x_i)$ represents the import bill of fuel i (or the export revenues if the cooperating countries are net exporters). Finally, environmental costs of the cooperating countries, $H(\cdot)$, depend on global emissions (E). Measuring all quantities of fossil fuels in units of their carbon content, global emissions can be expressed either by total consumption of fossil fuels or by total production of fossil fuels;

(2) $$\sum_{i=1}^{3} y_i + \sum_{i=1}^{3} D_i(P_1,P_2,P_3) = E = \sum_{i=1}^{3} x_i + \sum_{i=1}^{3} X_i$$

Henceforth, we use the first equation to state the necessary conditions for the optimal solution, while the second equation is helpful when deriving some of the characteristics of the optimal solution.

Turning to the international fuel markets, in each market total supply from the non-cooperating countries (X_i) equals the sum of demand from the non-cooperating countries (D_i) and the net demand from the cooperating countries (y_i - x_i):

(3) $$X_i = D_i(P_1,P_2,P_3)+y_i-x_i$$

If all producers in the non-cooperating countries are price takers, production in these countries is given by a set of aggregate supply functions $X_i=S_i(P_i)$. Inserting these functions into (3) gives all prices as functions of net imports, i.e.

(4) $$P_i = f_i(y_1-x_1,y_2-x_2,y_3-x_3)$$

In Appendix A it is shown that with perfect competition, increased net imports leads to higher international prices (under weak assumptions). In other words, using the notation $f_{ij}'=\partial f_i/\partial(y_j-x_j)$, we have

(5) $$f_{ij}'(y_1-x_1,y_2-x_2,y_3-x_3)>0 \qquad \forall i,j$$

If some of the producers in the non-cooperating countries are not price takers, we no longer have simple aggregate supply functions of the type $S_i(P_i)$. However, also for more complex market structures there will usually exist equilibrium prices. The prices will depend on several factors. In particular, the equilibrium prices will depend on the demand functions facing the non-cooperating countries. Since the net imports y_i-x_i are exogenous variables affecting the relationship between (P_1, P_2, P_3) and (X_1, X_2, X_3), see (3), these net imports will affect the equilibrium prices of fossil fuels. Hence, there are equations of type (4) also for more general market structures than perfect competition. The case in which the non-cooperating countries take steps - after the carbon taxes of the cooperating countries have been determined - to counteract the climate policy, is also covered by (4). It is, however, not obvious that the properties (5) hold. Nevertheless, (5) are likely to hold for a broad class of market structures: Conventional wisdom says that (under most market structures) increased demand leads to higher price and larger quantity. Hence, if the cooperating countries' net demand for e.g. oil rises, price and quantity of oil increase as well. As the price of oil increases, demand for coal and gas increase, i.e. price and quantity of coal and gas increase, etc. To sum up, conventional wisdom justifies the assumption that fuel prices are increasing in net demand, as expressed by (5).

From (3) and (4) it follows that production in the non-cooperating countries depends on the net demand of the cooperating countries:

$$X_i = h_i(y_1 - x_1, y_2 - x_2, y_3 - x_3) \equiv D_i(f_1(\cdot), f_2(\cdot), f_3(\cdot)) + y_i - x_i \qquad (6)$$

Under perfect competition, it is clear that (6) implies that all h_i-functions are increasing in all their arguments, provided that all aggregate supply functions $S_i(P_i)$ are increasing. We argued above that also for more general market structures than perfect competition it seems likely that all quantities are increasing in net imports. However, the assumption that all $h_{ij}' = \partial h_i / \partial (y_j - x_j) > 0$ is stronger than necessary to prove our propositions. We therefore make the somewhat weaker assumption that higher net demand increases total production of fossil fuels in the non-cooperating countries, i.e.

$$\sum_{i=1}^{3} h_{ij}' > 0 \quad \forall j \qquad (7)$$

Note that we do not use (6) and (7) to prove our results concerning the relationship between the total carbon tax of different fuels (proposition 1 and 5). We only use condition (5) and (7) to determine the sign of the tax rates on consumption and production of fossil fuels.

The cooperating countries maximize W (given by (1)) subject to the environmental constraint (2) and the price functions (4). The solution of the optimization problem gives the quantities y_1, y_2, y_3, x_1, x_2 and x_3. With standard demand functions in the cooperating countries, the consumer prices for oil, coal and natural gas are given by $\partial u/\partial y_1$, $\partial u/\partial y_2$ and $\partial u/\partial y_3$, respectively. To determine the producer prices and the optimal carbon taxes, it is necessary to specify the market structure in the cooperating countries. Assuming competitive markets, producer prices in the cooperating countries are simply $c'_i(x_i)$, i=1,2,3. Total carbon taxes for the three fossil fuels are thus given by

$$(8) \qquad t_i = \frac{\partial u}{\partial y_i} - c_i'(x_i)$$

Furthermore, the total carbon taxes can be split up in a consumer tax t^c and a producer tax t^p, where

$$(9) \qquad \begin{aligned} t_i &= t_i^c + t_i^p \\ t_i^c &= \frac{\partial u}{\partial y_i} - P_i \\ t_i^p &= P_i - c_i'(x_i) \end{aligned}$$

In the next two sections, we derive the optimal quantities for the cooperating countries, and thereby implicitly (through (8)) the optimal carbon taxes. In Section 3 there are no restrictions on how the carbon taxes should be designed. Section 4 analyzes the case with only consumer taxes, i.e. with the restrictions $t_i^p=0$.

3. Optimal carbon taxes without policy restrictions

In this case there are no restrictions on the design of the tax policy. Hence, it may be optimal to tax both consumption and production of fossil fuels. The optimization problem is simply to maximize the objective function W subject to the price functions (4). The Lagrangian to this problem is

$$L = u(y_1,y_2,y_3) - \sum_{i=1}^{3} c_i(x_i) - \sum_{i=1}^{3} f_i(\cdot)\cdot(y_i - x_i) - H\left(\sum_{i=1}^{3} y_i + \sum_{i=1}^{3} D_i(f_1(\cdot),f_2(\cdot),f_3(\cdot))\right) \tag{10}$$

which gives the first order conditions

$$\begin{aligned} u_i' &= P_i + R_i + H'\cdot C_i \\ c_i' &= P_i + R_i - H' + H'\cdot C_i \end{aligned} \tag{11}$$

where H'> 0 and the terms R_i and C_i are given by (using the notation $f_{ji}'=\partial f_j/\partial(y_i-x_i)$ and $D_{jh}'=\partial D_j/\partial P_h$)

$$R_i = \sum_{j=1}^{3} f_{ji}'\cdot(y_j - x_j) \tag{12}$$

$$C_i = 1 + \sum_{j=1}^{3}\sum_{h=1}^{3} D_{jh}' f_{hi}' \tag{13}$$

The sign of R_i depends partly on the derivatives of the price (f_j) functions and partly on the net import of each fuel. Inserting (11) into (8) and (9) gives

(14) $$t_i = H' \qquad i=1,2,3$$

and

(15) $$t_i^c = R_i + H' \cdot C_i$$
$$t_i^p = -R_i + H' \cdot (1-C_i)$$

Condition (14) may be formulated as the following proposition:

<u>Proposition 1:</u> **If there are no constraints on the carbon tax policy, the total tax per unit of carbon should be the same for all fossil fuels, and equal to the marginal environmental cost of total emissions (in the cooperating countries).**

To determine more characteristics of the optimal solution, it is necessary to identify the size of C_i. From (13) it is clear that $C_i < 1$ if and only if

(16) $$\sum_{j=1}^{3}\sum_{h=1}^{3} D'_{jh} f'_{hi} = \sum_{h=1}^{3}\left(\sum_{j=1}^{3} D'_{jh}\right) f'_{hi} < 0$$

Thus, a sufficient condition for $C_i < 1$ is

(17) $$\sum_{j=1}^{3} D'_{jh} < 0 \ \forall h \ \wedge \ f'_{hi} > 0 \ \ \forall h,i$$

The first condition holds if an increase in the price of any fossil fuel reduces the sum of the demand of all fuels, measured in carbon content. This is the case if the cross derivatives are not too large[3]. The second condition is identical to (5).

[3] The inequalities hold in the reference case of our numerical illustration in Section 5.

According to (2) and (6) C_i can be rewritten as

(18) $$C_i = \sum_{j=1}^{3} h'_{ji}$$

Hence, $C_i > 0$ when condition (7) holds.

It is now easy to obtain the following propositions:

Proposition 2: **Assume that (7) and (17) hold. Then a sufficient condition for positive optimal taxes on consumption is non-negative net import of each fuel and strictly positive net import of at least one fuel. In this case, the tax on production will be positive or negative depending on the structure of the economy.**

Proof: From the discussion and assumptions above, we know that $0 < C_i < 1$. This implies that the second term in both t_i^c and t_i^p are positive. Hence, if the cooperating countries have non-negative import of all fuels, and strictly positive net import of at least one fuel (i.e. $y_i \geq x_i$, i=1,2,3, and $y_i > x_i$ for at least one i), t_i^c is positive as $R_i > 0$. However, this means that the sign of t_i^p is ambiguous.

□

To interpret proposition 2, it is useful to examine each of the two terms of t_i. The first term, R_i, is the "optimal tariff". An increase in the demand for fuel i by one unit will increase the international prices by f_{1i}', f_{2i}' and f_{3i}'. If the cooperating countries are net importers, the total cost following from this increase is equal to the sum of the increase in the initial import bill (R_i) and the cost of the marginal unit of fuel i (P_i). In equilibrium, the costs of increasing consumption by one unit should equal the benefit, i.e. the marginal utility of fuel i (u'_i). Hence, the consumer tax u'_i-P_i should equal R_i. If the cooperating countries are net exporters of fuel i, i.e. if R_i<0, the countries' initial export revenue increases by -R_i if consumption of fuel i increases by one unit. In this case, the benefit of the group increases by the sum of the marginal utility of fuel i and the increased export revenues, whereas the cost of the policy equals the fuel price. Hence, in equilibrium it is optimal to subsidize consumption at the rate -R_i.

The second term, $H'C_i$, can be interpreted as a "pure carbon tax". More precisely, C_i is the total increase in carbon consumption, i.e. the total increase in carbon emissions, when consumption of fuel i in the cooperating countries increases by one unit. The partial effect of the "pure carbon tax" is to impose a tax on consumption of fuel i. Hence, it is only optimal to subsidize consumption of fuel i if the "pure carbon tax of fuel i" is lower than a negative optimal tariff.

Proposition 3: **Assume that (7) and (17) hold. Then a sufficient condition for positive optimal taxes on production is non-negative net export of each fuel and strictly positive net export of at least one fuel. In this case, the tax on consumption will be positive or negative depending on the structure of the economy.**

Proof: Similar to the proof of proposition 2, except that $y_i \leq x_i$, i=1,2,3, and $y_i < x_i$ for at least one i, i.e. R_i is negative.

□

Note that proposition 2 and 3 only consider the case in which the country is either a net importer or a net exporter of *all* fossil fuels. If, however, the country is a net importer of one fuel, but a net exporter of another fuel, the sign of the tax rates are in general ambiguous. Our next proposition deals with the case in which the cooperating countries' net export of each fuel is zero:

Proposition 4 **Assume that (7) and (17) hold. Then if the cooperating countries' net export of all fossil fuels is zero in equilibrium, both consumption and production of each fossil fuel will be taxed.**

Proof: Zero net export of all fuels, i.e. $y_i = x_i$, i=1,2,3, implies that $R_i = 0$. In this case we have $t_i^c = H'C_i$ and $t_i^p = H'(1-C_i)$, i.e. both tax rates are positive as $0 < C_i < 1$.

□

4. Optimal taxes on the use of carbon

In this section it is assumed that only the use of carbon is taxed, i.e. we impose the restrictions $t_i^P=0$, i=1,2,3. From (9), this means that the producer price of a fossil fuel in the cooperating countries must be equal to the international fuel price P_i, i.e.

$$c_i'(x_i) = P_i. \tag{19}$$

Equation (19) defines the supply function $x_i=s_i(P_i)$, which is an increasing function of P_i. Inserting this function into (4) gives

$$P_i = f_i(y_1-s_1(P_1),y_2-s_2(P_2),y_3-s_3(P_3)) \tag{20}$$

which may be solved to give

$$P_i = g_i(y_1,y_2,y_3) \tag{21}$$

The optimization problem of the cooperating countries is to maximize the objective function W subject to the supply functions $x_i=s_i(P_i)$ (defined implicitly by (19)), and the price functions (21). The Lagrangian is

$$\begin{aligned} L = {} & u(y_1,y_2,y_3)-\sum_{i=1}^{3} c_i(s_i(g_i(\cdot)))-\sum_{i=1}^{3} g_i(\cdot)\cdot(y_i-s_i(g_i(\cdot))) \\ & -H(\sum_{i=1}^{3} y_i+\sum_{i=1}^{3} D_i(g_1(\cdot),g_2(\cdot),g_3(\cdot))) \end{aligned} \tag{22}$$

Using (19) and (21), the first order conditions are found to be

(23) $$u_i' = P_i + T_i + H' \cdot B_i$$

where the terms T_i and B_i are given by

(24) $$T_i = \sum_{j=1}^{3} g_{ji}' \cdot (y_j - s_j(P_j))$$

and

(25) $$B_i = 1 + \sum_{j=1}^{3} \sum_{h=1}^{3} D_{jh}' \cdot g_{hi}'$$

Inserting (23) into (9) gives

(26) $$t_i = T_i + H' \cdot B_i$$

From the formulas above we obtain the following proposition:

Proposition 5: **When only the use of carbon is taxed, the optimal tax per unit of carbon should generally differ between the fossil fuels.**

Proof: Proposition 5 follows directly from (26), as the size of T_i and B_i generally differ between the fossil fuels.

□

Note that proposition 5 differs from proposition 1; the three tax rates should only be equal when there are no constraints on the tax policy. The tax rates should in general differ if either consumption or production of fossil fuels cannot be taxed, cf. Appendix B for a discussion of the latter case.

As seen from (26) the sign of the tax rates depend on the terms T_i and B_i, which correspond closely to R_i and C_i in Section 3. By comparing (12), (13), (24) and (25) it is clear that if all g'_{ij} are positive (remember that all f'_{ij} are positive, cf. (17)), then the results of propositions 2-4 concerning the sign of the consumption taxes still apply. Under perfect competition, all g'_{ij} are positive, cf. Appendix A. However, as the properties of the g_i functions in general depend on the market structure, propositions 2-4 cannot easily be generalized[4].

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[4] As long as there is international trade in only two goods, all g'_{ij} are positive if $(f'_{11}f'_{22}-f'_{12}f'_{21}) > 0$. Unfortunately, we have not obtained a sufficient condition for positive derivatives for the case with international trade in all fuels that has a clear economic interpretation.

5. An empirical illustration

In Section 3 and 4 we derived the general properties of the carbon taxes when the group of cooperating countries maximized its net income minus environmental costs. Assume, however, that the cooperating countries maximize net income subject to a constraint on global carbon emissions (which may have been set by politicians). From standard economic theory we know that the structure of the carbon taxes are identical under the two approaches[5]. Moreover, if the carbon target in the second approach is equal to the (endogenous) emission level in the first approach, the tax levels are identical as well. In this section we illustrate the theoretical analysis when global emissions of CO_2 from fossil fuels are reduced by 15 per cent.

We shall assume that the cooperating countries consist of the OECD countries and that all fuel markets are competitive. We compare two long-run equilibria: The observed 1990 outcome, which we assume is an equilibrium of our model without any constraints on CO_2 emissions (and no optimal tariffs) and an (hypothetical) equilibrium where global emissions are 15 per cent lower than the 1990 level. The emission reduction is due to an optimal tax policy in the OECD countries. We close this section by reporting the results from a sensitivity analysis which indicates how the tax rates depend on the key parameters of the model.

Table 1 shows the key parameters of the empirical model. Our intention was to construct marginal cost curves for each fossil fuel. However, due to lack of data we use a simpler approach in which all marginal cost curves are assumed to be linear. The level and slope of each curve are determined by the point of production in 1990 (level of production and price of the fuel) and by the supply elasticity at this point. The two supply elasticities of coal are equal to 2.00, which is greater than the common value of the supply elasticities of oil and gas (0.75). This is due to the enormous reserves of coal; an increase in the price of coal by one per cent may trigger a large increase in the supply of solids.

[5] The new first order conditions are identical to the initial ones, except that the marginal environmental cost, H', is replaced by the shadow price of the emission constraint.

In OECD, demand for (manufactured) fuels is derived from a quadratic utility function. The parameters of the utility function are determined such that (i) all direct price elasticities are equal to -0.90, and (ii) three cross price elasticities are equal to 0.15. The three other cross price elasticities, which now follow from the parameters of the utility function, are on average (approximately) 0.05. Demand for oil, coal and gas in non-OECD are assumed to be linear in all fossil fuel prices. The level and slope of these functions are determined by the observed point of consumption in 1990 and the elasticities at these points. The direct price elasticities are all equal to -0.75. Moreover, all cross price elasticities are equal to 0.10. The choice of elasticities is based on the conventional wisdom that demand is more inelastic in developing countries as fuels are here used to satisfy basic needs. For a more detailed discussion about costs and elasticities, see Appendix C in Golombek, Hagem and Hoel (1993).

Table 1: Key parameters in the empirical model

	OECD	NON-OECD
Production		
- Elasticities of supply		
oil	0.75	0.75
coal	2.00	2.00
gas	0.75	0.75
Consumption		
- Direct price elasticities	-0.90	-0.75
- Cross price elasticities	0.10 on average	0.10

In our reference scenario, global emissions of CO_2 from oil, coal and gas are reduced by 15 per cent relative to the actual level of emissions in 1990. The emission abatement is due to an optimal carbon tax policy in OECD. With no constraints on the tax policy, the total carbon tax in OECD should equal 101 USD per tonne carbon (in 1990 prices), cf. table 2. To reach this target the tax structure must be designed such that the consumer taxes on oil, coal and gas are 135, 65 and 96 USD per tonne carbon, respectively. Moreover, production of oil, coal and gas in OECD should be taxed by -34, 36 and 5 USD per tonne carbon.

Compared to the actual 1990 equilibrium net demand for oil and gas are lower, whereas net demand for coal is higher. As increased net demand increases all fuel prices, the international prices on oil and gas decrease (because the direct price effect is stronger than the two cross price effects). Moreover, the price on coal increases. It turns out that the changes in the international prices yield lower non-OECD production (by 6 per cent), whereas non-OECD consumption is almost unchanged.

To reduce global emissions by 15 per cent, OECD consumption must decrease by 32 per cent, i.e. consumer prices in OECD have increased. In our case, consumer prices on oil, coal and gas increase by 23, 83 and 20 per cent, respectively. Hence, the drop in consumption of coal is much larger than the decline in consumption of oil and gas. This is partly due to the differences in the emission coefficients of the fossil fuels. The producer prices of oil and gas in OECD are almost unchanged, whereas the producer price of coal decreases by 39 per cent. In total, production of fossil fuels declines by 35 per cent in OECD.

Compared to the initial 1990 situation, the result of our reference scenario consists of two effects. First, the OECD countries cooperate to maximize their net income, subject to the international trade in fossil fuels. This gives the optimal tariff effects. In our case, the optimal tariff effects are for sure positive because OECD is net importer of all fossil fuels. As seen from (11), the optimal tariff terms then correspond to an increase in both the marginal utility of consumption and the marginal cost of production. Hence, it is optimal to decrease consumption and increase production, i.e. OECD should tax consumption and subsidize production. In the reference scenario, the optimal taxes on consumption of oil, coal and gas (which are also the optimal subsidies on production of these fuels) turn out to be 54, 4 and 16 USD.

Secondly, global emissions should not exceed 85 per cent of the emission level in 1990. As OECD is net importer of all fossil fuels, marginal utility of consumption should increase whereas marginal cost of production should decrease, cf. (11). Hence, there should be positive (pure carbon) taxes on both consumption and production of each fuel. In the reference scenario, the optimal pure carbon taxes on consumption of oil, coal and gas are 81, 61 and 80 USD, respectively. The corresponding taxes on production are 20, 40 and 21 USD. To sum up, the total consumer taxes are positive, whereas the sign of the total producer taxes is in general ambiguous, cf. proposition 2. In our reference scenario, the optimal tariff effect on e.g. production of oil is larger than the pure carbon term of oil. Hence, total tax on production of oil should be negative (=-54+20=-34).

To test the robustness of our results, we first changed the level of the emission target. In the case without restrictions on the tax policy, the total carbon tax should be 64 USD when global emissions are reduced by 10 per cent (instead of 15 per cent), i.e. 37 per cent lower than in the reference scenario. Turning to the case in which demand becomes less elastic, the total carbon tax increases by 23 per cent when all direct price elasticities in OECD are changed from -0.90 to -0.70 and all direct price elasticities i non-OECD are -0.55 (in stead of -0.75). When the cross price elasticities are (approximately) doubled, the total carbon tax increases from 101 USD in the reference scenario to 111 USD. Finally, we find that less elastic supply leads to higher taxes: When the supply elasticities of coal are reduced from 2 to 1, and all other supply elasticities are reduced from 0.75 to 0.375, the total carbon tax should equal 133 USD. In general, the qualitative characteristics of the reference scenario do not change much when the key parameters of the empirical model shift, cf. table 2.

Table 2: Reduced global emissions. Effects on OECD taxes (in 1990 USD/tcarbon), prices changes (in per cent) in OECD and changes in production and consumption of fossil fuels (in per cent). No restrictions on the tax policy.

	(1) base case	(2) higher global emissions	(3) lower direct price elast.	(4) higher cross price elast.	(5) lower supply elast.
Taxes in OECD					
total	101	64	124	111	133
oil					
- consumers	135	108	164	149	174
- producers	-34	-44	-40	-38	-41
coal					
- consumers	65	42	90	80	66
- producers	36	22	34	31	67
gas					
- consumers	96	67	129	129	114
- producers	5	-3	-5	-18	19
Prices in OECD					
oil					
- consumers	23	17	28	25	26
- producers	2	9	5	4	-6
coal					
- consumers	83	53	113	99	84
- producers	-39	-24	-38	-36	-75
gas					
- consumers	20	13	26	26	22
- producers	-9	-3	-4	3	-22
Production					
total	-15	-10	-15	-15	-15
- OECD	-35	-19	-33	-30	-36
- non-OECD	- 6	- 6	- 7	- 8	- 6
Consumption					
total	-15	-10	-15	-15	-15
- OECD	-32	-22	-32	-33	-34
- non-OECD	0	1	0	1	2

Scenarios:

(1) Global emissions are reduced by 15 per cent relative to 1990

(2) Global emissions are reduced by 10 per cent relative to 1990

(3) All direct price elasticities in OECD are changed from -0.90 to -070
All direct price elasticities in non-OECD are changed from -0.75 to -0.55

(4) All cross price elasticities in OECD are approximately increased by 100 per cent
All cross price elasticities in non-OECD are changed from 0.10 to 0.20

(5) All (six) supply elasticities are reduced by 50 per cent

6. Concluding comments

We have shown that with international trade in all fuels and no restrictions on the design of the carbon taxes in the cooperating countries, the total carbon tax per unit of carbon should be equal for all fossil fuels. We have also analysed cases when there are restrictions on the tax policy. If there are no taxes on production of fuels, the total carbon tax on each fuel should in general differ, cf. Section 4. Finally, in the case with no taxes on consumption of fuels, the total carbon tax per unit of carbon should also differ across all fossil fuels, see Appendix B.

It is straightforward to understand why total carbon taxes should always be equal when there are no policy restrictions: When a fuel is traded internationally, increased production or consumption of that fuel has impact on the international fuel prices, and thereby on emissions of CO_2 from the non-cooperating countries. However, as the international fuel prices depend on net demand from the cooperating countries, these prices do not change when production *and* consumption of the cooperating countries increases by one unit. Hence, the only effect is the increase in emissions from the cooperating countries by one unit of CO_2.

We have also given an empirical illustration of the theoretical analysis, assuming the cooperating countries consist of the OECD countries. In our reference scenario, global emissions of CO_2 from oil, coal and gas are reduced by 15 per cent relative to the actual level of emissions in 1990. With no constraints on the tax policy, the total carbon tax in OECD should equal about 100 USD per tonne carbon (1990 prices). The carbon tax comes in addition to all other (energy) taxes, which are constant in our model. Note that the optimal carbon taxes in OECD yield decreased non-OECD production (6 per cent in our reference scenario) and approximately unchanged consumption of CO_2 in non-OECD. It goes without saying that 100 USD is a very rough estimate. First, the total carbon tax is highly dependent on the OECD parameters. Secondly, we have not taken into consideration market failures. Finally, non-OECD countries may respond to the optimal carbon tax policy of the OECD; OPEC may e.g. impose measures to increase the international price of oil.

Throughout the paper we assumed competitive supply of oil, coal and gas in the cooperating countries. Our interpretation of the differences between consumer prices and marginal production costs as tax rates, cf. (8) and (9), was based on this assumption. Note, however, that the optimal quantities are unchanged when producers in the cooperating countries are not price takers. If only carbon taxes are used (in this case) to implement the optimal solution, then the carbon taxes should also correct for traditional market failures. Hence, the total carbon taxes should differ across fossil fuels. Alternatively, and more transparently, the optimal solution can be implemented by equal total carbon taxes and specific measures to correct for market failures. Hence, even with non-competitive market structures, equal carbon taxes can be part of the optimal policy.

Appendix A: Perfect equilibrium in all fuel markets

Assume that all fuels are traded on competitive international markets. In the non-cooperating countries, supply of each fuel depends only on its own price, $S_i(P_i)$, i=1,2,3, while demand for each fuel depends on all fuel prices, $D_i(P_1,P_2,P_3)$. The market equilibrium conditions for the international markets are:

$$
\text{(A.1)} \qquad \begin{aligned} x_1+S_1(P_1) &= y_1+D_1(P_1,P_2,P_3) \\ x_2+S_2(P_2) &= y_2+D_2(P_1,P_2,P_3) \\ x_3+S_3(P_3) &= y_3+D_3(P_1,P_2,P_3) \end{aligned}
$$

Each equation in (A.1) describes the market equilibrium for a fossil fuel; total supply from the two groups should be equal to total consumption. The three equations in (A.1) can be solved for the three fossil fuel prices, giving

$$
\text{(A.2)} \qquad P_i = f_i(y_1-x_1,y_2-x_2,y_3-x_3)
$$

The f_i-functions depend on all demand and supply functions in (A.1). We make the following assumptions about the demand and supply functions;

(A.i) $D'_{ii} < 0$

(A.ii) $D'_{ij} > 0$, $i \neq j$

(A.iii) $S'_i > 0$ and $s'_1 > 0$

(A.iv) The economy is well behaved in the sense that a positive shift in the demand for any fossil fuel increases the price of that fuel.

Lemma 1: Assumptions (A.ii), (A.iii) and (A.iv) imply that all the first order derivatives $f_{ij}'(y_1-x_1, y_2-x_2, y_3-x_3)$ are positive.

Proof: Consider the equations in (A.1). The partial market equilibrium condition for oil is given by

$$S_1(P_1) = D_1(P_1,P_2,P_3) + y_1 - x_1$$

An increase in y_1-x_1 will have the same effect on P_1 as a positive shift in $D_1(P_1,P_2,P_3)$. According to (A.iv) this implies that P_1 will rise, i.e. f'_{11} is positive. Next, consider the partial market equilibrium conditions for coal and gas:

$$S_2(P_2) = D_2(P_1,P_2,P_3) + y_2 - x_2$$
$$S_3(P_3) = D_2(P_1,P_2,P_3) + y_3 - x_3$$

A rise in P_1 (through a rise in y_1-x_1) implies according to (A.ii) a positive shift in both the demand for coal and gas. According to (A.iv) this implies that both P_2 and P_3 will rise. Hence, f'_{12} and f'_{13} are positive. Lemma 1 now follows directly from the symmetry in (A.1).

As an alternative to (A.iv) we may assume

(A.v) $$\frac{\partial(\sum_{i=1}^{3} D_i)}{\partial P_j} < 0 \quad \forall j$$

According to (A.v), an increase in the price of any fossil fuel reduces the sum of the demand - measured in carbon content - of all fossil fuels.

Lemma 2: Assumptions (A.i), (A.ii), (A.iii) and (A.v) imply that all $f_{ij}'(y_1-x_1, y_2-x_2, y_3-x_3)$ are positive

<u>Proof:</u> Let us first differentiate the equilibrium conditions in (A.1). In matrix form the result may be written as

(A.3) $\quad \mathbf{Hx} + \mathbf{c} = \mathbf{0}$

where

(A.4)

$$H=\begin{bmatrix} D'_{11}-S_1 & D'_{12} & D'_{13} \\ D'_{21} & D'_{22}-S'_2 & D'_{23} \\ D'_{31} & D'_{32} & D'_{33}-S'_3 \end{bmatrix}$$

$$x=\begin{bmatrix} dP_1 \\ dP_2 \\ dP_3 \end{bmatrix}$$

$$c=\begin{bmatrix} d(y_1-x_1) \\ d(y_2-x_2) \\ d(y_3-x_3) \end{bmatrix}$$

According to Lemma 2, we want to show that if $d(y_i - x_i) > 0$, then all $dP_j > 0$. Rewriting (A.3) gives

(A.5) $\quad (\mathbf{H} + k\mathbf{I})\mathbf{x} + \mathbf{c} = k\mathbf{x}$

where k is a positive scalar which is sufficiently large to ensure that **(H+kI)** only has positive elements. (A.5) can be written as a Leontief-system

(A.6) $\quad (k^{-1}\mathbf{H} + \mathbf{I})\mathbf{x} + k^{-1}\mathbf{c} = \mathbf{x}$

where

(A.7)

$$(k^{-1}H + I) = \begin{bmatrix} 1+k^{-1}(D'_{11}-S'_1) & k^{-1}(D'_{12}) & k^{-1}(D'_{13}) \\ k^{-1}(D'_{21}) & 1+k^{-1}(D'_{22}-S'_2) & k^{-1}(D'_{23}) \\ k^{-1}(D'_{31}) & k^{-1}(D'_{32}) & 1+k^{-1}(D'_{33}-S'_3) \end{bmatrix}$$

(A.7) is a matrix with only positive elements, and, according to (A.iii) and (A.v), the sum of each column is less than 1. It then follows from the characteristics of a Leontief-system that k^{-1} $\mathbf{c} \geq 0$ implies $\mathbf{x} \geq 0$.

We have thus shown that $dP_i \geq 0$. In fact, it follows from (A.3) that they must all be strictly positive. Consider first the case in which $d(y_1\text{-}x_1) > 0$ (i.e. $d(y_2\text{-}x_2) = d(y_3\text{-}x_3) = 0$). Assume first that $dP_1 = 0$. In this case if follows from the last two equations in (A.2) that $dP_2 = dP_3 = 0$. But $dP_1 = dP_2 = dP_3 = 0$ contradicts the first equation in (A.2), since $d(y_1\text{-}x_1) > 0$. Next, assume $dP_2 = 0$. Since $D_{ij} > 0$ and $dP_i \geq 0$, the second equation in (A.2) must imply $dP_1 = dP_3 = 0$, which again contradicts the first equation in (A.2). The same argument rules out the possibility of $dP_3 = 0$. Lemma 2 now follows directly from the symmetry in (A.3).

Finally, consider the g_i-functions introduced in Section 4. They are defined in a similar way as the f_i-functions, except that we have $s_i(P_i)$ instead of x_i in (A.1). All of the analysis above remains valid with $S_i(P_i)$ replaced by $s_i(P_i)+S_i(P_i)$, $y_i\text{-}x_i$ replaced by y_i, and $f_i(y_1\text{-}x_1, y_2\text{-}x_2, y_3\text{-}x_3)$ replaced by $g_i(y_1, y_2, y_3)$. Under reasonable assumptions, all g_{ij}' are thus positive under perfect competition.

Appendix B Optimal taxes on the production of carbon

Assume that there is no tax on the use of fossil fuels, i.e. $t_i^c=0$, i=1,2,3. From (9) this means that the marginal utility of fossil fuel consumption should be equal to the international price P_i. Hence

(B.1) $$U_{yi}'(y_1,y_2,y_3) = P_i$$

The relations in (B.1) define y_i as a function of P_1, P_2 and P_3,

(B.2) $$y_i = l_i(P_1,P_2,P_3)$$

Inserting (B.2) into the price functions (4) yields

(B.3) $$P_i = m_i(x_1,x_2,x_3)$$

Hence, all fossil fuel prices depend on the production level of oil, coal and gas.

The optimization problem of the cooperating countries is to maximize the objective function W, subject to the restriction of no taxes on the use of fossil fuels (defined implicitly by (B.2)) and the price functions (B.3). It is straight forward to derive the first order conditions

(B.4) $$c_i' = P_i - \sum_{j=1}^{3} m_{ji}' \cdot (y_i - x_i) - H'(\sum_{j=1}^{3} l_{ij}' m_{ji}' + \sum_{j=1}^{3}\sum_{h=1}^{3} D_{jh}' m_{hi}')$$

Using the definitions of the carbon taxes, the total tax on each fuel is

(B.5) $$t_i = \sum_{j=1}^{3} m'_{ji} \cdot (y_i - x_i) + H'(\sum_{j=1}^{3} l'_{ij}\, m'_{ji} + \sum_{j=1}^{3}\sum_{h=1}^{3} D'_{jh}\, m'_{hi})$$

As seen from (B.5), the carbon tax on each fuel consists of two terms; an optimal tariff (as the prices of fossil fuels depend on the production level of oil, coal and gas in the cooperating countries) and a pure carbon tax. As the magnitude of these terms differ, the total carbon taxes will in general differ.

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