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Optimal Energy Economics under Global Environmental Constraints

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OPTIMAL ENERGY ECONOMICS UNDER GLOBAL ENVIRONMENTAL CONSTRAINTS

Preface

Over the past decade there has been mounting concern that it will become increasingly difficult to reverse or adapt to climate changes resulting from human activities.

Owing to the accumulation for carbon dioxide and other greenhouse gases (GHG) the atmosphere is trapping more and more heat coming from the earth. The trapped heat is expected to cause increases in average yearly temperatures and other consequent climatic changes.

The major purpose of this book is to contribute to better policy making through improvements in models studying the economic impacts of carbon dioxide and other greenhouse gases, and to show ways in which economic instruments can effectively be put to use to alleviate such problems.

This approach differs in at least one major respect from common studies of the climatic change problem. We focus on the analysis, control and optimization of modelling forms rather than the collection and analysis of data. More concretely, we search for optimal fossil fuel use, research and technology policies rather than prediction of the future. Most studies of the problem exogenously specify technical developments and fossil fuel control policies and then predict future climatic changes. These prediction models incorporate a great deal of data and tend to be quite complex.

The advantage of an optimizing control model is increased flexibility in structural and dynamic assumptions on the economy allowing explicit 'what-if' questions to be asked about the possibility of controlling the growth in atmospheric CO_2 concentrations. We are able to use these models to draw some specific policy conclusions. By using and improving such models we intend to explore a wide range of substantial issues of CO_2 emission analysis of the subject. For example,

- are different import/export taxes on fossil fuels desirable in competitive and non-competitive world markets?
- do increasingly severe economic impacts of CO_2 reduce present optimal fossil fuel

use?

- do different relations between output and energy use have corresponding impacts on the long-run optimal level of atmospheric CO_2?
- in which cases can a paradoxical situation arise where a lack of cooperation among nations in controlling CO_2 leads to lower initial world fossil fuel use?
- what are the impacts of change in important parameters, such as the discount rate, risk aversion, energy productivity, and CO_2 retention on the present level of fossil fuel use and the long-run CO_2 level?
- what are the conditions under which the optimal use of fossil energy falls or rises during the transition to a future steady state?

Because present energy-economy-environmental (EEE) models do not optimize and do not treat such issues as cooperation, technical change and uncertainty in an integrated manner, these models may be too pessimistic about the possibility of controlling the growth in atmospheric CO_2 concentrations.

The omissions and the structure of present models place hidden, rigid and unnecessary constraints on the economy. We hope that this book helps to eliminate some of these problems and allow a more solid foundation from which policy conclusions can be drawn.

The global nature of potential CO_2 problems makes it very difficult to keep models simple and makes issues of uncertainty and international cooperation very important. The long time span lends significance to the modelling of technical change and also stresses the need to balance the welfare of present and future generations.

Previous and Related Studies

Doing research in economic models and applying them to the CO_2 problem requires that we take the physical aspects and findings of this problem as given inputs. We see that although controversy and uncertainty surround almost all the specific predictions of CO_2 effects, the basic conclusion that a doubling of atmospheric CO_2 will cause a significant temperature rise has remained constant. Variations in the physical parameters, based on different sources, can be confidently handled by sensitivity analysis in the numerical treatment of our models. An important more recent development is the emphasis on the additional temperature increases due to trace gases or so-called greenhouse gases (GHG), to which we will respond in our models.

Before we clearly identify the relevant work in this area we have some general remarks on some energy-economy studies used to examine CO_2 and other recent energy issues. We identify several limitations in these models which we will address in our models. We find that most major studies of the CO_2 problem use predictive models which do not include feedback effects. In the models we discuss, the process of accumulating and disposing of physical capital is handled by a variety of ad hoc methods and is not modelled explicitly. Technical progress is often considered to occur uniformly throughout the economy or is included by predicting specific technical developments. Though the treatment of technical progress is not deep in most studies of CO_2, the importance of predicting technical progress is recognized. Finally, the relationship between trade patterns and CO_2 accumulation is seldom considered in present energy - economy models.

Two major kinds of economic studies can be identified for dealing with the CO_2 problem. The first category treats economic and economic modelling issues in the context of an integrated framework of energy-economy and climate changes, the second category applies the theory of resource use and depletion to the management of CO_2 emissions.

In what follows we provide a selected overview of those studies which yield pertinent results comparable to our own or which are of methodological interest in modelling the energy-economy-climate interactions.

A few years ago the US National Research Council (1983) compiled a detailed investigation that up to now constitutes the most extensive, comprehensive and consistent examination of the climate change problem. It uses energy-economy, climate and agricultural models to predict future impacts of carbon dioxide and trace gas accumulation. The major conclusions are that no radical actions should be taken, that increases in carbon dioxide are likely, and that more research is necessary. In this report, the developers of the energy-economy model (W. Nordhaus and C. Yohe) note that the technology development and elasticity of substitution parameters critically affect the model's results.

It should be added that the method of modelling technical change and energy substitution possibilities is also critical and controversial. The economic modelling chapters of the NRC report have been updated by Yohe (1984). The conclusions of the report have not changed significantly.

Another collaborative study, the joint MIT-Stanford study (Rose, Miller, and Agnew, 1983), is of interest because it is one of few reports which search for alternatives to increasing

CO_2 and offer some positive choices. In the Edmonds and Reilly model (Edmonds and Reilly, 1983) S-shaped paths are exogenously specified for several new energy technologies. The MIT-Standard study modifies these paths and also looks at additional technologies. It finds that the adoption of realistic CO_2 reducing technologies, while not eliminating a significant CO_2 warming could increase the CO_2 doubling time to several centuries.

In attempting to discuss optimizing strategies W. Nordhaus (1980) made a seminal contribution by applying simple optimization models to the qualitative and quantitative analysis of the CO_2 problem. By letting the consumption equation depend on fossil energy use he determines the appropriate tax policy to control CO_2 and makes a quantitative estimate of this tax.

In most present studies of the CO_2 problem we find that technological change and technology substitution are specified exogenously or modelled in a very simple fashion.

In this regard, the logistic (S-curve) assumption on the diffusion of new technologies has become very popular, although it lacks sufficient economic explanatory power. For example, in a study by Perry et al (1982) an energy demand level and a fossil fuel use pattern is assumed. Fossil fuel use follows a logistic curve between the present and an assumed ultimate level of use. The rate of non-fossil energy growth needed to fill the gap, between fossil energy use and assumed total energy demand is then examined. This study emphasizes the importance of analysing the investment needed in non fossil energy to fill the gap but it does not present a model of the substitution process. In the model we suggest, the substitution process is a direct result of our maximization of welfare.

Modelling the impacts of changes in energy use on the economy is a major problem. A good starting point for such considerations would be the ETA-Macro model (Manne, 1977) though it has not been used for studying the CO_2 problem. This model can be described as a multisector, forward-looking model. It examines consumption and investment policies and their impact on national welfare. National welfare is measured by discounting utility from the present to a distant horizon. ETA-Macro consists of two models: a macro-model o∧the whole economy and a more detailed model of the energy sector. The model seems more sophisticated in its treatment of capital and the determination of the desirable level of energy use than those economic models presently used in CO_2 analysis. However, it is limited to the USA in geographic scope which makes it unsuitable for examining international problems such as CO_2. The model has no endogenous technical progress which we consider an

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important feature for the analysis of CO_2. On the other hand, the model has several features which would be desirable in models of energy, economy and the environment. It is optimizing and considers costs and benefits of capital investment. More recently, Manne (1990) has proposed Global 2100, a model adapted to the CO_2 problem but with similar structural features of ETA-Macro.

In setting up this approach we were influenced by applications of optimal control models to pollution problems (Fisher, 1981; Conrad and Clark, 1987) and relevant issues of resource use. Such models often show different structures, e.g. pollution affecting utility or production, the pollutant acting as a stock or a flow and the way abatement activities are available.

In this specific context, we find that many models and results on the depletion of a non-renewable resource could be applied with simple modifications to the CO_2 problem. Under two assumptions the problem of fossil fuel use in the face of increasing carbon dioxide is parallel to the problem of consumption of a limited resource. The first assumption is that the carbon dioxide rate is sufficiently small to be ignored. The second is that CO_2 impacts follow a "step" pattern: that is, CO_2 has no impact on productivity until a critical level, M_c, is reached. Then if the CO_2 level exceed M_c production falls to zero, or remains stagnant. One of the most serious effects in facing global climatic change is that of irreversibility, that is, given the accumulation of atmospheric CO_2 we will reach a critical level of the CO_2 budget where there is a point of no return (unless technologies are in place that effectively remove CO_2 from the budget). The interpretation of a critical level of atmospheric CO_2 accumulation where there is a precipitous drop of production means that we have reached the biophysical limits of growth.

An interesting treatment of endogenous neutral technical progress in a depletion model was suggested by Chiarella (1980). He proves the existence of a steady state growth path and a simple rule governing the rate of investment in research. Research investment along the optimal path should be carried out until the growth rate in the marginal accumulation of technology equals the difference between the marginal product due to an extra unit of research investment and the marginal product of capital.

A similar problem is the use of a limited non-renewable resource when the reserve of the resource is unknown. The model by Gilbert (1979) can be directly converted to a model of fossil fuel when the critical CO_2 level is uncertain. Under the above assumptions this problem is equivalent to determining the rate of fossil fuel use when the critical concentration of atmospheric carbon dioxide is unknown. The results show that the optimal use of fossil fuel is lower when uncertainty is properly considered when the expected values are assumed to be certain.

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Deshmukh and Pliska (1980, 1983) study more complex models of the same problem. The possibility of doing exploration to find new reserves is a significant addition in their models. The parallel in the CO_2 problem is research to increase the probability of finding a technology for the removal of CO_2 from the atmosphere. Their findings imply that in the periods between discoveries or research breakthroughs, fossil fuel use and consumption fall, but if research is very successful long-run fuel use may rise.

Conceptual Framework: The Reference Model

In exploring the shifts in energy, economy and CO_2 interactions we start with a basic 'simple' model assuming that energy is the only productive factor in the economy and that energy use causes CO_2 accumulation which lowers productivity. Consumption of goods in the economy causes a flow of utility. The decision-maker acts to maximise the integral of the discounted utility stream. The model has a single state variable, CO_2 concentration, and is autonomous. Unique features of the model are the very general production function and the assumption that the pollution does not affect utility directly.

Special interesting features of this model include:

- conditions which assure the existence of an optimal equilibrium,
- a phase plane analysis showing the conditions under which the shadow price of fossil fuels rises, and what conditions assure convergence to the equilibrium
- definition of an ideal tax to control CO_2 in a decentralized economy and factors which make estimating the tax difficult.

We develop models which can be solved numerically and applied to specific issues of interest. Two specific forms are presented. In the first, the negative impacts of CO_2 accumulation occur abruptly at specific levels of atmospheric CO_2; this is referred to as the step model of CO_2 impacts. In the second form, the negative impacts of CO_2 increase gradually and continuously with increases in atmospheric CO_2. We refer to this model as the modified Cobb-Douglas model. Furthermore, on the basis of this reference model, we will develop and assess several other useful economic models for CO_2 studies as well as supporting tools and models.

One model in particular deserves attention: a vintage model with expert knowledge and flexible opportunities for investment which will be helpful in examining issues related to technical progress.

Economic Policy Analysis

The models developed in this book are used to examine (i) technical progress, (ii) international trade and cooperation, and (iii) uncertainty and risk behaviour.

1. We show that and how, depending on the assumptions regarding technical progress, the optimal steady state of CO_2 concentration may rise or fall with increases in the steady state level of progress. Notably, an improved substitute for fossil fuels always reduces the long-run level of atmospheric CO_2; while an improvement in fossil fuel productivity may increase or decrease the level of atmospheric CO_2.

2. We show what the solutions are for a model with neutral, constant, and ongoing technical progress, and in which way higher levels of technical progress lead to lower long-run optimal levels of atmospheric CO_2.

3. Several trade issues regarding CO_2 pollution will be considered. We examine whether a country or group of countries concerned about increasing atmospheric CO_2 and trading in energy goods should increase its exports of fossil fuels in a competitive world market, and should tax exports of fossil fuels in a non-competitive market place. On which specific world supply and demand conditions should decisions be made to subsidize or not to subsidize exports of fossil fuel substitutes.

4. We examine several cases of international cooperation in controlling CO_2 accumulation. The base case is complete cooperation between two regions in maximizing consumption with complete awareness of the CO_2 problem. This case is compared with a situation in which no cooperation takes place until a critical CO_2 level is reached.

We explore why in the non-cooperative situation the critical level is reached sooner, even though the region concerned about CO_2 always emits less carbon than in the base case.

As we have so far established, a paradoxical situation can occur in which initial emissions of the two regions are lower in the non-cooperative case than in the cooperative case.

5. Substantial applications apply to uncertainty. We show that the results from studies of the optimal use of a resource which is in limited supply can be applied to the CO_2 problem. This similarity is important because the results regarding the use of limited resources are extensive and powerful. Then, using numerical examples, we show that an inappropriate treatment of risk can lead to significantly higher than optimal estimates of the desirable level of fossil fuel use.

6. Departures from the reference model involve multiple state models. The key difference between this model and the reference model is the possibility of improving the economy by investment in knowledge and physical capital. This allows the description of more realistic long term behaviour. We show that both a stationary equilibrium in which no growth occurs in the economy and a dynamic equilibrium in which the economy grows at a constant rate are possible. We expect interesting results that concern the impact of changes in the values of key parameters on the equilibrium growth rate. Some of the results which we derive from the model and which we consider more closely are:

- increases in the productivity of the energy sector increases the capital to knowledge ratio in equilibrium and the economy's rate of growth;
- an increase in either the social discount rate or the consumption elasticity of utility causes the economy to grow more slowly, and
- an increase in the depreciation rate of capital lowers the capital to knowledge ratio and the equilibrium growth rate.

7. We explore the properties of another model, allowing flexible technical progress, in which prices influence the pattern of technical progress and non-neutral technical progress is possible. This model has several interesting features. First, it is a vintage model, the fossil energy input required by a piece of capital is determined by the year in which it is purchased and cannot be changed after the capital is in place. Second, research and development can be performed to improve the productivity of new capital or to reduce its energy requirements,

the output required, the research budget, and cost of energy and capital are all exogenous. The objective is to minimize the cost of production.

8. In evaluating large-scale models of energy, economy, and CO_2 interactions we explore how CO_2 feedback effects, forms of optimization, uncertainty, technical change and Vin organizational forms explicitly treated/our models could be effectively used to greatly enhance the explanatory power of large-scale models.

Description of Chapters

The book is organized in nine chapters.

The first chapter deals with the scientific (e.g. geophysical, climatic, biochemical etc.) background of global environmental externalities and the major substantive issues involved in characterizing and identifying the CO_2 problem and trace gas accumulation. The structure of natural science models of such phenomena is discussed to gain an understanding of the physical complexities of the underlying issues. Chapter 2 presents the simplest, one state variable control model of an economy in which pollution of the prescribed kind occurs. We also analyze specific forms of this one state variable model which permits to trace the transition path to long-run performance. We also use the models to study major structural features such as technical progress, international cooperation and uncertainty. These features will be taken up separately each in later chapters.

In Chapter 3 a more complex model of the economy with endogenous, neutral technical progress is analyzed. This is an EEE model with several state variables in which the major difference from earlier models, in Chapter 2, is the possibility of improving the economy by investment in knowledge and physical capital. This allows a more realistic description of long-run behaviour and adjustment.

Furthermore, in Chapter 3 we construct a model which is very flexible in the pattern of technical development and examine the reaction of research to energy price changes and limits on energy use. A vintage model of technical progress is introduced in which fossil energy input required by a piece of capital is determined by the year in which it is purchased and cannot be changed after the capital is in place.

Following the simple aggregative models of long-run growth and environmental constraints developed in Chapter 2 and 3, we focus on a discussion of large-scale models of energy, economy and CO_2 interactions in Chapter 4. Our primary conclusions are that CO_2 feedback effects and simple forms of optimization could be added to conventional EEE models without much difficulty. They would greatly enhance the usefulness of these models.

Chapter 5 discusses and assesses several neo-classical growth models under global environmental constraints. It explores the interrelations between capital accumulation, global waste disposal and economic growth in emphasizing that environmental pollution reduces the production possibilities of the aggregate economy, but that global pollution might not retard economic growth.

Chapter 6, complementing Chapter 5, discusses a specific optimal economic growth model with a finite time horizon where fossil fuel resources constitute inputs to the aggregate production function bound by a critical cumulative CO_2 budget. The problem utilizes standard optimization procedures such as Pontryagin's maximum principle and dynamic programming. All of the approaches so far have adopted a social planning focus toward these issues.

Chapter 7 provides a rigorous treatment of uncertainty in view of optimal statistical decisions and stochastic dynamic programs. Using a model of optimal statistical decisions it is shown when it pays to 'act and learn' and when to 'learn and act'.

The value of information in reducing uncertainty can be shown to be sensitive to accuracy and likelihood of scientific research results.

The results are extended for the dynamic inter-temporal decision situation when the value of new information is an outcome of an optimal stochastic dynamic program.

Chapter 8 looks at an approach to optimal coordination in stochastic decentralized control systems.

Chapter 9 develops and assesses a class of EEE models for designing macro and sectoral policy choices under GHG emission constraints using a computable general equilibrium (CGE) approach.

Chapters 7-9 virtually address special issues relating to enhancing the complexity of modelling forms, enriching the core of the simple aggregate optimizing models of Chapters 2 and 3. They also serve to shed more light on the technical and substantive problems affecting EEE modelling: technology change, uncertainty, international trade and co-operation.

Acknowledgements

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CHAPTER 1

ENERGY, ECONOMY AND THE CO_2 PROBLEM

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1, 1 THE SCOPE OF THE PROBLEM

The focus of the first chapter is to address and analyze some specific issues at the interface of energy use and environmental management from a global perspective.

We start from the fact that carbon dioxide (CO_2) in the atmosphere affects the radiation balance of the earth, and that increasing (CO_2) concentrations are expected to cause a warmer climate.

Carbon dioxide is relatively transparent to energy as sunlight, but reflects or traps a large portion of this energy when radiated from the earth as heat. An important development in climatic studies is the identification of other gases with this same heat trapping property; these include: chlorofluoromethanes (major sources: spray cans and other commercial uses), nitrous oxide (major sources: jets, cars and unknown sources), and methane (major sources: anaerobic fermentation in rice fields, and natural gas leakages). These gases are often referred to as trace or greenhouse gases (GHG). Uncertainty about future increases in such gases significantly complicate the prediction of future temperatures. The prediction of future temperatures is further complicated by diverse factors. Significant feedback effects are expected to accompany any direct effects of CO_2 heat trapping. Warming may change cloud, snow, and ice cover and alter the earth's albedo or brightness. Consequently, the reflection and absorption of energy may increase or decrease. Also warming may cause the release of additional GHG which is trapped in frozen soils, accentuating the GHG problem. In addition, uncertain, major climatic disturbances are foreseen which are not connected with CO_2. Continental drift, fluctuations in the earth orbit, solar flux variations, and volcanic dust are all natural causes of climatic change. The release of particulates and changes in land-use are human activities which may impact local or regional climate.

We look into the suitability of energy resources, alone or in combinatiion, to satisfy these global environmental constraints, and identify reasonable scenarios for environmentally benign energy futures.

The last century has witnessed an unprecedented period of growth, in energy, economy, population and consumption. By the end of the sixties, in particular, anxiety was mounting about whether the world was beginning to collide with the ceiling of resource and environmental constraints, or whether it still had time to complete the gradual transition toward a "steady state" in which population and resources were in balance. The "limits to growth" debate culminated in several studies associated with the Club of Rome, predicting a catastrophic increase in mortality rates throughout the world beginning in the first decades of the 21st century, largely as a result of resource shortages. But even if resources were not limiting, global pollution would, a few years or decades later, lead to even greater catastrophe. The basic thesis was that the longer this catastrophe was postponed by "technological fixes" the more destructive would be the final collapse.

Consumption growth, as these studies argued, would so pollute the environment as to threaten human health and the life-supporting properties of the biosphere on which human existence depends.

There are three separate issues involved here:

(i) general chemical or radiation contamination of the environment (hazardous and nuclear wastes)

(ii) general deterioration of natural ecosystems (air and water pollution, soil erosion)

(iii) changes in the global climate. (CO_2 emission and trace gases, ozone depletion).

Changes in the global climate, if they occur - offer now the most challenging task for the control of global environmental pollution. We particularly focus on (iii) as a problem of energy production.

Most studies so far have singled out carbon dioxide (CO_2) as the most serious pollutant and with potentially irreversible consequences of the global climate.

So far, no economically feasible control technologies to capture any significant fraction of CO_2 emissions have been on the horizon. A prime characteristic of the CO_2 problem is the long time lags that may elapse between the cause and the identification of significant effects. Another characteristic is the high degree of uncertainty that usually attends predictions of future effects. Such uncertainty is compounded by the fact that CO_2 effects occur at various complex levels.

The first level of effect is the direct physical result of the activity, in this case the actual CO_2 concentration in the atmosphere. The principal evidence for a trend toward increasing CO_2 levels in the atmosphere comes from continuous observations over many (up to thirty) years, at various sites.

The second level addresses the partitioning of CO_2 among the atmosphere and other carbon reservoirs. If we look at the carbon cycle, with the exception of the atmosphere, the amount of carbon in each reservoir is somewhat uncertain. It seems there is a natural circulation of carbon among these different reservoirs, particularly through photosynthesis and oxidation.

This opens a biogeochemical perspective on the carbon cycle. We can take the view that various chemical reservoirs of the earth are comparable to the organs in a human body. [Ausubel (1980)]. Accordingly, the CO_2 problem emanates from human activities feeding carbon into circulation faster than the ability of the earth's organs to digest or metabolize it. This is largely due to the increasing consumption of fossil fuels and less so to the clearing of natural forests. Carbon dioxide emissions from fossil fuels combustion have been growing at the rate of 4.3 percent per annum since 1860 (with the exception of the periods of world disaster, World Wars I and II and the depression years of the 1930s [Rotty (1977), Flohn (1989)].

In 1860 the release rate was about 0.9 GT of carbon, in 1900 it was 10.5 GT, in 1940 it was 1.3 GT, in 1970 it was 4.2 GT, and in 1981 it was 5.5 GT/yr of carbon, with further increases since then.

About 48 to 56 percent of the CO_2 produced by the combustion of fossil fuels over the past two decades can account for the observed CO_2 increase in the atmosphere. Between 33 and 41 percent is thought to be taken by the oceans. This suggests that less than 15 percent must have gone into another sink. The role of the biosphere is currently uncertain and under research.

Educated guesses tend to assign a lower contribution to forest clearing, perhaps less than 20 percent of the fossil fuel emissions, and their potential contribution is much more limited than fossil fuel. But that only aggravates the search for the "missing" sink.

The next level of uncertainty is the direct climatic effect of increasing atmospheric CO_2 leading to an increase in the surface temperature - the so-called "greenhouse effect".

Calculations done with the "best" available, but still significantly superficial climate models, time-dependent general circulation models (GCM) indicate upon doubling of CO_2 concentrations (say from 300 ppm to 600 ppm) a possible increase in global temperature (ΔT_s) in the range 1.5 to 4.5 deg C. Most models also show how the global temperature increase will be unevenly distributed with probable amplification of up to a factor of 5 in the polar regions.

A major part of the variance in the several estimates of climate models of ΔT_s can be explained from the "built-in" assumptions of the relative importance of various interactive physical processes on climatic feedback mechanisms.

Figure 1 shows how these interactions occur in a very simplified network, used only for illustrative purposes. These processes could either amplify or damp by several times the calculations of ΔT_s.

Furthermore, there are other "greenhouse" trace gases being added to the atmosphere by human activities, such as chloroflouromethanes, methane, carbon monoxide, nitrous oxide, and ozone.

Estimates put the additional warming close to that of CO_2 at about 50 percent.

One major deficiency of all models is the inability to couple the ocean and the atmosphere in ways that describe real energy exchange and division of energy transport between the two. The ocean's heat transport, in the models, is held fixed within atmospheric circulation and changing wind stress, which is not realistic. The oceans exert tremendous influence on nearly all atmospheric processes and modelling of the fuel interaction is still in the development stage. Despite the uncertainties and probably systematic flaws in the climate models we should pay attention to a panel conclusion by the U.S. National Academy of Sciences more than ten years ago. [NAS(1979)].

> ".. We have tried but are unable to find any overlooked or underestimated physical effects that could reduce the currently estimated global warming due to doubling of atmospheric CO_2 to negligible proportions or reverse them altogether".

Can the CO_2 "signal" be filtered out from the "noise" during the past century?

Assuming $\Delta T_s = 4.04 \ln (M/M_0)$ where M is the carbon dioxide concentration, it can be calculated that the expected ΔT_s over the past century is in the order of 0.55 deg C which lies within the range of the natural interannual variations in T_s.

The next level of uncertainty concerns the timing and geographical distribution of changes due to the increased global surface temperature. The last and most important level of uncertainty is the translation of the effects of climatic change and increased CO_2 levels -

even if they were certain - into impacts on the environment and human activities in specific regions. This , in effect, closes the chain of causation in the man-climate interaction.

It is possible, however, to identify several sectors for major impacts with some confidence. Agriculture is the most prominent. There could be significant changes in the production potential of many areas as a result of temperature, precipitation and variability changes. Shift in climate zones and in loci of agricultural activity may create deoptimization of existing crops and farming activities. Other climate-sensitive areas comprising energy demand, water resources, fisheries, human health, population settlements, tourism and recreation appear of lesser significance than food production, but no serious estimates have been made of their magnitude. Second order effects such as the absence of frost and cold winters could imply proliferation of pests and pathogens adding to possible damage.

One difficulty in even a crude cost/benefit assessment of the previous impacts stems directly from speculation on the ability of mankind to anticipate the problems and adapt to them (Schelling, 1988).

Reduction of uncertainty and better, more reliable data on physical impacts and change will be of immense help in using models to calculate the optimal rate of CO_2 emissions. The objective, in energy and resource economics, is to determine the time paths of resource use that maximizes the discounted present value of the future stream of economic benefits.

In addition to the above discussed constellation of uncertainties in the physical climate modelling, carbon cycle models, ecological impacts and economic costs or benefits of the attendant change, there are uncertainties in the social and political domain as well as in mankind's future demand for fossil fuels.

But these latter uncertainties, political and social circumstances and fossil fuel demand, are of a necessarily different character: they are the outcome of human decisions and hence are more amenable to control -- not easy but not impossible. Thinking of CO_2 as a problem in the management of a scarce natural resource - the quality of the global atmosphere - during an eventual transition from fossil fuels to a non-fossil fuel economy focuses on efforts on developing appropriate transition strategies for the management of fossil fuel emissions: management *per se* since extractable fossil resources are sufficient to increase CO_2 concentrations by a factor of 5 to 8 over the pre-industrial level. If that happens, ΔT_s will increase by 6 to 9 Deg C within the next two to three centuries. This undoubtedly would be a disaster. Thus, the world's cumulative use of fossil fuels must be restricted to levels below the estimated recoverable resources.

We shall briefly describe the constraining factors that are the "givens" of our economic modelling. In Figure 2, curve A represents the present world energy use, unrestrained by CO_2 considerations. Curve B is a different world, where CO_2 concentrations satisfy two constraints:

(1) $M_{cl} \le M \le M_{cu}$

(2) $\frac{1\ M}{M \partial t} \le km,$ where m is a rate to be determined, and k being some fixed parameter.

The first constraint to be determined on M, represents a ceiling level that might be approached with "acceptable" cumulative consequences, but which probably should not be exceeded. In addition, this level should be below any "threshold" level which heralds catastrophic events.

The second constraint is a rate limitation which depends on the rate of climate change, reflecting in a general fashion the assumption that slow rates of change are more amenable to compensating adjustments in human institutions and in productive activities that depend on climate.

History shows that societies often have failed to recognize the importance of planning for climate changes and have responded differently to climatic perturbations and experienced quite disparate impacts [Rabb (1980)].

Assuming some plausible constraints on the climate system via CO_2 concentrations in the atmosphere we turn to the energy-economic system. Since the CO_2 problem is essentially a problem of high prospective energy demand, it is natural to concentrate our efforts there. There are four routes to reduce this growth in global energy use.

1 Reduction of population growth rate.
2 Reduction of per capita energy growth, with redistribution or without redistribution.
3 Reduction of world economic growth.

4. Decoupling of energy from economic growth through conservation, advances in energy conversion, more efficient technologies, and/or structural shifts in the economy.

The items 1. through 4. do not constitute free variables that we can adjust, they have a minimum or maximum.

Reasonable variations of those rates have to be discussed in view of the latest global energy demand projections.

We proceed as follows. We model mathematically the constrained trajectory of CO_2 increase versus time, and using a simple carbon cycle; we find global fossil fuel use.

The sensitivity of the fossil fuel scenarios to variations in input parameters is analyzed.

The change in the curvature of F_{fo}^B has important consequences on the growth of new energy technologies (the gap between F_{nfo}^B and F_{nfo}^A), see Figure 3.

A slow change in F_{fo} implies a fast change in this gap. This implied rate of change in the technological energy infrastructure is analysed and compared with the present and future industrial capabilities. Some of the discussed "rates of change" are:

- the market penetration rate,
- startup rates of new energy technologies
- the manufacturing capacity expansion rate

F_{nfo} in Figure 3 is a composite of various sources (e.g. solar, hydro, biomass, nuclear, etc.) with varied potential, sustained production, and diffusion rates.

Subsequently, we view the CO_2 problem in its proper international setting.

Recognizing that F_{fo}^B is the sum of fossil fuel use of various nations while M_{cu} and M_{cl} are globally fixed; and recognizing that global climate change will lead to various regional climatic impacts we will draw a composite hypothetical regional climate scenario based on those (sometimes contradictory) scenarios reported in the literature.

How rapidly could or should various groups of nations reduce their use of fossil fuel so that their CO_2 cumulative emissions shall not exceed those ceiling constraints?

This question and other similar questions posed in the global analysis are addressed with the help of a hypothetical multiregional model.

1.2 WORLD FOSSIL FUEL USE

Given the CO_2 constraints described in the previous section we will analyze plausible world fossil fuel paths with the aid of a simple carbon cycle model. The model will be used to probe the sensitivity of the results to variations in key parameters that would influence the energy system dynamics. From the economic point of view of fossil fuel use the argument is often advanced that market forces should (and will) play a dominant role in allocating resources, and thus no other mechanism should intervene in this "optimum" process. According to this view, the "cost effectiveness" process is automatically operative, hence there is no reason for constraining exogenously the growth of fossil fuel use.

To explore these ideas further, note that the resource base for fossil fuels is indeed very large if it includes resources that will cost much more than they do now. About half of the conventional oil and natural gas is recovered inexpensively. The rest comes from a variety of sources that are associated with higher production costs, poorer fields that require drilling more holes, production from continental shelves, deeper basins and polar regions etc. All these activities are, moreover, associated with significantly larger environmental impacts. If we assume that cleaning up spills, reclaiming mined-out lands to useful purposes, and hazardous waste management are all operations whose cost must be internalized, then these "dirty" fuels become even more expensive.

The standard observation of resource economists e.g. that one never runs out of a resource but simply reaches a point where it is cheaper to use a substitute, is to some extent correct in the case of fossil fuels. Unfortunately, very few reliable estimates of fossil fuel resources versus cost curves are available.

It is estimated that remaining and recoverable resources of fossil fuels (Table 1) contain nearly 3900 Gt (of carbon), enough to increase the concentration of CO_2 in the earth's atmosphere by a factor of four.

This means a CO_2 limit of even 1000 ppm would restrict the release of fossil carbon well below the 3900 Gt available in recoverable ore, gas and coal. A limit of 700 ppm might restrict the release to 1400-2200 Gt, depending on the airborne fraction. Table 2 summarizes the recoverable resources by the type and cost category.

Given an economically ultimate resource base of 5000 TWyr, an obvious question is what do plausible production paths look like?

Figure 4 shows four such curves each within an integrated area of 5000 TWyr.

Curves 1 and 2 represent high energy consumption, while Curves 3 and 4 represent low consumption. The difference between Curves 1 versus 2 or 3 versus 4 is whether the difficulties associated with nuclear power will be overcome (Curves 2 and 4), or not, in which case coal will play a major role (Curves 1 and 3). An appropriately weighted integral of the curves in Figure 4 represents the projected increase to CO_2 from anthropogenic sources. Figure 5 displays the outcome. A CO_2 concentration of 500 ppm(v) will be exceeded in the period 2035-2050, and a 700 ppm(v) will be crossed in 2070-2120 whatever the fossil fuel path will be.

Path 1 implies a 2.6 pc/yr growth in fossil fuel while Path 4 shows only about 1.1 pc/yr. In comparison, the historical fossil fuel growth rate has been over decades around 4.3 pc/yr. From this simple exercise and the previous observations we see that CO_2 concentrations will not be restricted by physical depletion or economic factors below the ceiling limits indicated in Sec. 1.

Moreover, although lower fossil fuel growth rates give us more time they would not eliminate the problem. They will satisfy the rate constraint limitation, but not the ceiling. Therefore, the latter will need to be buried into the system response.

1.2.1 A Simplified Modelling Approach

Plans for increased energy use yield various scenarios for increased CO_2 concentrations with time. To capture in simple terms the nature of the problem we assume:

1 A minimum realistic level M_{cl} of CO_2 exists, determined by total integrated demand for fossil fuels added up over all future time, before the world has changed over to energy sources that do not increase CO_2 after that (principally solar and/or nuclear in various forms).

We assume that if less total fossil fuels are used during all future time, there will be considerable social unrest due to unmet demand.

2 A maximum allowable level M_{cu} of CO_2 exists, determined by unacceptable climatic and/or biological and/or botanical effects. If it is exceeded, bad things happen (See Sec. 1).

3 We can plan our global fossil energy use so that the CO_2 Curve F increases smoothly to some value that is acceptable.

The curvature of the CO_2 buildup with time implies that society thought ahead, planned accordingly and avoided the trouble. Do we really have to plan ahead, even in the face of uncertainties, or can we let matters take their normal course, and later take crash measures when scientific research has finally shown with "certainty" that we should not exceed a given maximum level of CO_2. That is, can we behave so that the concentration increases, then stop in time?

It appears that we cannot do that because a discontinuous and sudden stop in the use of fossil fuel is practically impossible, it would imply a complete shift in technological infrastructure away from fossil fuel.

Technological breakthroughs notwithstanding, this is impossible for the world as a whole.

In order to describe the dynamics of carbon accumulation meeting the CO_2 ceiling constraints we assume the rate of buildup of CO_2 in the atmosphere to be represented by the following equations.

$$\dot{M}(t) = kF_{fo}(t) + l\dot{A}(t) - \dot{R}(t)$$

where the dots denote time derivatives

$M(t)$	is the accumulated mass of carbon in the atmosphere (in tons) at time t,
$F_{fo}(t)$	is the rate of fossil fuel use (TWyr/yr)
k	is the average emission rate of carbon per unit of fuel energy
$l\dot{A}(t)$	is the net release rate of carbon from clearing tropical forests
$\dot{R}(t)$	is the net transfer rate of carbon from the atmosphere to other reservoirs (sinks).

The objective is to find $F_{fo}(t)$ that conforms to a prescribed asymptotic M_α.

Unfortunately, most terms in the equation are only vaguely known.

$l\dot{A}(t)$ has been estimated, in the past few years with significant variations. The estimates are based on forest inventory statistics with fluxes calculated from a forest clearing rate times estimated carbon stored in particular forests.

R(t) represents the sinks of carbon dioxide. Currently, only the deep ocean is regarded as the ultimate sink of carbon.

Comparing fossil fuel use and the reconstructed and actual atmospheric record suggests an "airborne fraction" (AF) that according to most calculations with cycle models (Rosenberg et al., 1988), is between 0.55 and 0.65 depending on the oceanic vertical mixing rate and the behaviour of the biosphere.

One can establish an approximate relation between the atmospheric mass of carbon M(t) and the fossil fuel burning rate, $F_{fo}(t)$:

$$F_{fo}(t) \approx \frac{1}{k \cdot AF} \cdot \quad \dot{M}(t) \tag{1}$$

This relation is the starting point for the development of the model.

We shall assume that the carbon in the atmosphere (added since Δt_o) can be modelled as a logistic function in the form:

$$M(t) = \frac{\beta}{1 + \gamma e^{-\alpha(t-t_0)}} + \lambda \tag{2}$$

where $\beta, \gamma, \lambda, \alpha$ = constants

t = time

t_o = action initiation time (AIT)

Since $M(t_o) = M_o$ it follows that:

$$\lambda \equiv M_0 - \frac{\beta}{1 + \gamma}$$

A ceiling constraint on CO_2 is imposed:

$$M_{cl} \le M_c \le M_{cu}$$

The model developed above attempts to describe a plausible world response to the impending CO_2 problem. The final world trajectory or response function could be thought of as a composite of three stages: during the first describing the period up to AIT, the response function follows closely the reference scenarios, hence it represents a period in which fossil fuel use is unconstrained by consideration of the CO_2 buildup. The AIT is however not the time when discussions or negotiations to limit CO_2 begin, but rather the time when the resulting policies begin to be effective. The second phase starting at the AIT signals the retardation of fossil fuel growth (because of the CO_2 build-up) and ends with fossil fuel use at its peak. The third phase of the response describes a "decay" and general "phase out" of the fossil fuel consumption.

1.2.2 Projections of CO_2 Concentrations

Most previous predictions of CO_2 concentration have been based on the assumption of exponential growth in the CO_2 production rate. Table 3 shows the CO_2 concentration for exponential growth of 4.3 pc/yr (the historical rate of carbon release) 2 pc/yr (the 1970-1985 growth rate) and 2.6 pc/yr, representing a carbon rich high energy scenario.

On comparing these three projections the following observations are in order:

(i) Doubling of CO_2 (ie 600 ppm) would have been reached by the year 2030 *if* historical growth rates were continued.

(ii) Moderating the fuel consumption rate to 2.6 pc/yr would have delayed the doubling time by 13 years; even moderating the growth to 2 pc/yr will buy only an extra five years. Thus, at most, moderating exponential growth will buy us two decades.

(iii) Constraining the trajectory to a 1000 ppm asymptote will delay the doubling by three decades (from the 4.3 pc/yr date) and 15 years from the corresponding exponential path.

(iv) For an ultimate constraint of 700 ppm, the delay is 45 years and 35 years, respectively.

Indeed, there is a marked difference between the constrained and the exponential paths, although there is some overlap in the first few decades.

1.2.3 Energy System Constraints and Opportunities

In order to meet global environmental constraints we can infer from the previous analysis that non-fossil energy sources will have to make a major contribution to world energy supply within the next few decades. This is not surprising to anyone anticipating the decline of importance of oil and gas (which presently supply more than 60 percent of the world's energy). However, the recognition that remaining recoverable fossil fuel supplies are really very large, -- 5000 TW years or more, more than 20 times the amount used in the world so far and enough, at the present rate of consumption to last another five centuries -- makes it less than self-evident that the expected transition to long-term renewable energy resources is in fact considered an urgent necessity among all nations in the world.

It is the anticipated further growth in world energy requirements (as reflected in high and low scenarios), coupled with prospective limitations on fossil fuel use well below the physically available resources, that gives rise to the early need for a major growth in non-fossil energy sources.

The question arises whether the transition from fossil to non-fossil fuels is likely to be a difficult or easy one to manage. The details of the transition are complex, and possible answers still appear to be open-ended.

But here we analyse a few constraints that better illuminate the transition. The constraints embedded in this transition are many: technical, political, strategic and economic. No sharp distinctions exist among the various kinds of constraints, but we find it helpful to group them under various headings.

1.2.4 Technological Patterns

Historical data on world energy consumption, when plotted versus time as a fractional share of different primary energy sources, follows a very regular pattern. This observation has given rise to the hypothesis that primary energy sources are competing for market shares, in producing intermediate and final products. Substitution of an existing technology or industrial processes to satisfy a given need by a new emerging technology has been the subject of a large number of theoretical and empirical studies evidenced in the journal Technological Forecasting and Social Change.

One general finding is that almost all binary substitution processes (e.g. steam engine versus electric power), expressed in fractional terms, follow characteristic S-shaped curves which have been used for forecasting future competition between the two alternative technologies.

(This way to look at technological substitution processes has never been in much favour among economists because they feel that such processes lack a choice-theoretic or explanatory basis, see Stoneman, 1981.)

However, if we can find an endogenous explanation for the way technologies emerge and proliferate we will be able to parametrize such processes along the line of some logistic substitution model.

Most of the studies of technological substitution are based on the use of the logistic function. The logistic function, however, is not the only S-shaped function, but it is perhaps the most suitable one for empirical studies. Another S-shaped function, the Gompertz curve, has also been frequently used, especially to describe population, plant, and animal growth.

One of the first such studies that showed how technological substitution can be described by a S-shaped curve was the pioneering work of Griliches (1957) on the diffusion of hybrid corn seed in the USA. Griliches showed that hybrid corn replaced traditional corn seed in different states in a very similar way: the S-shaped substitution was only displaced in time by a few years and lasted differing lengths of time from one to another.

Following the work of Griliches, Mansfield (1961) developed a model to explain the penetration of an innovation. He suggested that the penetration is directly proportional to the difference between the expected profit and expected investment associated with the innovation.

However, the first systematic attempt at forecasting technological change was made by Fisher and Pry (1970). This work extended Mansfield's findings, but considered only fractional shares of a market controlled by two competing technologies.

Let f be the total market share.

Most of the modelling approaches can be summarized by considering the following fundamental model

$$\frac{df}{dt} = (\beta + \alpha f)(f_m - f) \tag{3}$$

$$f(t=0) = f_i$$

where α, β = constants

f_m = upper limit on market share

The constant α can be interpreted as the index of influenced adoption so that $\alpha f(f_m - f)$ can be thought of as the imitation component. Similarly, β can be interpreted as the index of uninfluenced adoption, so that $\beta(f_m - f)$ can be explained as the innovation component.

The Fisher-Pry model can be obtained from (3) by setting $\beta = 0.0$ and $f_m = 1.0$; that is

$$\frac{df}{dt} = \alpha f(1 - f) \tag{4}$$

The differential equation has the solution:

$$\ln\left(\frac{f}{1-f}\right) = \alpha t + \gamma, \text{ or} \tag{5}$$

$$f = \frac{1}{1 + \exp(-\alpha t - \gamma)} \tag{6}$$

This process may be thought of as pure imitation diffusion process.

In (4), if f = 0, so is $\frac{df}{dt}$. Thus its validity requires that the process has risen above some initial threshold by other means. This implies that $f(t_0)>0$. That is, in order for diffusion to occur according to the model, there must be a finite proportion of the population that has already adopted the innovation.

(3) with $\beta=0$ can be solved for f(t):

$$f(t) = \frac{f_m}{1+\left(\frac{f_m-f_0}{f_0}\right)e^{-\alpha f_m(t-t_0)}} \tag{7}$$

where $f_0 = f(t_0)$

To find the maximum diffusion rate, set df/dt = 0, which gives for t*,

$$t^* = t_0 - \frac{\ln f_0 - \ln (f_m - f_0)}{\alpha f_m}$$

hence $f(t^*) = f_m/2$. Also evaluating the change in t*, as α varies yields,

$$\frac{dt^*}{d\alpha} = \frac{\ln f_0 - \ln(f_m - f_0)}{\alpha^2} \tag{8}$$

Since $0 \leq f_0 \leq 1$ it follows that $\frac{dt^*}{d\alpha} < 0$ so that any increase in the imitation index α, will increase f(t) for all $t > t_0$; that is, it will shift the logistic curve to the left.

(3) is also a first order differential equation. Its unique solution can be found by specifying an initial condition,

$$f(t) = \frac{f_m - \left[\frac{\beta(f_m-f_0)}{\beta+\alpha f_0}\right] e^{-(\beta+\alpha f_m)(t-t_0)}}{1+\left[\frac{\alpha(f_m-f_0)}{\beta+\alpha f_0}\right] e^{-(\beta+\alpha f_m)(t-t_0)}} \tag{9}$$

Note that $f(t)$ is not symmetrical about t^*, since $f(t^*) = f_\infty/2 - \beta/2\alpha$, the majority of the adoptions in the diffusion process given by (3) will occur after the rate of diffusion achieves a maximum.

To handle more than two competing technologies, Marchetti and Nakicenovic (1979) generalized the Fisher-Pry model since in such cases logistic substitution cannot be preserved in all substitution processes.

Every given technology undergoes three distinct substitution phases: growth, saturation and decline. They assumed that only one technology is in the saturation phase at any given time; that declining technologies fade away steadily at logistic rates; and that new technologies enter the market and grow at logistic rates. The current saturating technology is then left with the residual market share. After the current saturating technology has reached a logistic rate of decline, the next newer technology enters its saturation phase and the process is repeated until all but the most recent technology are in decline.

In order to test this hypothesis, Marchetti and Nakicenovic have applied their model to three different levels of energy system aggregation:

- Primary energy inputs for the world as a whole
- Primary energy inputs for single nations or regions
- Energy subsystems, such as electric utilities.

In total they used 60 data bases to generate 300 examples of 30 different spatial and structural subsets of the world energy system. The goodness of fit was found consistently high in all examples.

The first fact to be observed about the curves is the extreme regularity and slowness of the substitution. It takes about 100 years to grow from 1 percent to 50 percent of the market. This length of time, analogous to the time constant of the system, is called the market penetration time. The regularity refers not only to the fact that the rate of penetration remains constant over such very long periods, when so many perturbing processes seem to take place, but also to the fact that all perturbations are reabsorbed elastically without influencing the trend. Another observation to be made on the curves is that the penetration trends have almost the same "time constant" for all energy sources except nuclear energy. Nuclear energy achieved only 1 percent share of primary energy in the early 1970s. Thus its future penetration rate cannot be distilled from the historical data since the initial phases of the market penetration do not stabilize to long-term substitution trends for some time.

Table 4 presents the market penetration rates as well as market penetration times (t_m) of the various energy supply technologies.

Starting with (5), t_m can be calculated as follows:

$$t_m = [\ln(\frac{0.5}{1-0.5}) - \ln(\frac{0.01}{1-0.01})]\alpha^{-1}$$

It is clear from Table 4 that characteristic global penetration times are of the order of 100 years. Also, the smaller the region or the country, the shorter are those times.

From the preceding discussion it is prudent to assume that new technologies that will be necessary as future complements to and replacements for fossil fuel will not be introduced faster than technologies were introduced in the past. Also note that the dynamics of technological substitution is more complex than depicted here. Since the penetration rate tends to decrease with decreasing energy growth rate it can be assumed that future penetration rates will be slowed correspondingly. In particular, we need to explain penetration rates endogenously, a part of an interaction of economic factors within an economic model. This will be pursued later.

1.2.5 Penetration of Non-Fossil Energy Technology

The principal non-fossil energy sources available are nuclear power and the renewable energy sources: direct and indirect solar and geothermal energy, though the possible contribution of such non-fossil technologies at any particular time is not at all certain. In addition, biomass very likely represents a major energy source, although the data on its use are sparse. The various solar technologies are in markedly different phases of development, with some, such as solar hot water heating, passive space conditioning and wind power already beginning to compete with conventional sources, while others are still the focus of R&D. (The Appendix contains an example of market penetration of non-fossil fuels based on previously described techniques.) We now turn to a discussion of the potential of each of these sources.

Solar Market Penetration

The penetration rates of solar energy technologies will depend to a certain extent on the competitive environment of all energy supplies, and the uncertain cost estimate for most potentially significant solar options limit the procedures that can be used to determine solar market penetration rates.

There are currently numerous market penetration analyses of the solar energy market, but some economists (Warkov and Meyer, 1982; Feldman and Wirtshafter, 1980) are sceptical of their validity. The criticisms levelled at those models are also applicable to any market penetration models. These economists contend that such models are grounded on only very simple behavioural assumptions. Therefore the structure of the models cannot be tested. Also, because of this lack of basis in behavioural theory, these analyses are limited to explaining behaviour, not predicting it. Finally, the one claim to legitimacy of these analyses is a foundation based on well-developed models of diffusion processes. However, as pointed out previously, the applicability of diffusion models may be limited by the implicit assumption of the pure imitation diffusion process.

Assuming a 1 percent share for solar in the year 2000 (in IIASA scenarios (Häfele (1981)) the actual share of solar was only 0.6 percent (High), or 0.15 percent (Low)), and taking a market penetration rate for solar consistent with the upper end of those of other energy sources in the past; will lead to f_s (market share of solar in total primary energy) of 6 percent by 2030 and 18 percent by 2050. For comparison, note that IIASA predicts for f_s only 1.4 percent by 2030 (High Scenario), or 1.3 percent (Low Scenario). Even if we add to solar sources the "other" category in IIASA scenarios (which include bio-gas, geothermal and commercial wood use), then their share by 2030 is on the order of 3.65 percent for both scenarios.

However, many analysts argue that with adequate government incentives, the rate of market penetration of solar techniques can be far greater than that suggested by the application of the historical penetration rates of other energy sources.

First, proper incentives, including adequate support for research and development, can substantially shorten the time required to bring a technology to commercial readiness. Second, the initial stages of commercialization can be shortened through such incentives. This is actually what is observed in the market penetration dynamics of technologies in their early growing phase.

However, once the penetration has reached several percent, the dynamics of the system take over. In other words, during the developmental phase, subsidies and external capital can be used to support new technologies even if the direct costs of these technologies are somewhat higher than the competition. However, after achieving commercial maturity, the penetration rates depend on the actual competitive situation. Non-profitable technologies are rarely supported for very long time periods. Government support can no longer sustain extremely rapid growth; market and system effects become dominant, and the penetration behaviour becomes smooth and regular.

The Nuclear Option

The nuclear share is very scenario-specific, given the fixed solar prediction in either low or high scenarios. But assuming solar would meet the challenge, what are the residual burdens nuclear should bear? There are many variants analysed, but we shall discuss a representative sample.

Taking the year 2000 as possible Action Initiation Time, nuclear power could contribute between 21 and 53 percent by the year 2030 for the High Scenario and CO2 limits ranging from 500 to 700 ppm. The Low Scenario, on the other hand, would require 4-38 percent under the same conditions. In terms of reactors (with a capacity of 1 GWe) the entire scenario range comes to 500 to 11,000 reactors. This range is very wide for useful policy discussions. Note that the worldwide installed nuclear capacity is presently 350 GWe.

In all cases studied, only after the year 2025 is the nuclear share markedly affected by the assumed solar penetration. One particular argument seems to speak for the nuclear option: this is electricity growth. Counter arguments include cost considerations (Barnes, 1991) and other nuclear related issues (Williams, 1990). Currently the end markets open for nuclear power are mainly in electricity production, so let us look for possible opportunities there. Table 5 shows the projected electricity generation and installed capacity in IIASA scenarios (Häfele, 1981).

Nuclear power represented in 1975 worldwide about 5 percent of both generated and installed electric capacity rising to 17 per cent in 1989.

Inter alia, the scenarios embody a transition to electricity as the reference energy system and the associated secondary energy carried could be hydrogen. Moreover, since most of the renewable energy sources discussed earlier tend to use electricity as secondary energy carrier, they compete directly with nuclear for the same market. But, if the reference energy system is electricity, the "oversupply" could be directed to generate hydrogen.

Hydrogen would be a partner for electricity because it can be stored and transported. But the uses of hydrogen as a final energy require a significant adaptation of the existing infrastructure and thus require time and capital investment.

The quoted projected electricity generation in IIASA scenarios are only constraints in the near term and involve a variety of assumptions about electrification in various parts of the world. Over time the trend of increasing penetration of electricity into end use markets slows down significantly, especially in industrialized countries. Electrification rates, however, could be increased from IIASA assumptions if electricity would enter more rapidly into the heat market in buildings for space heating, water heating and cooling. Also, opportunities exist in the industrial heat market - process and space heat - and if heat pumps penetrate the market more rapidly than hitherto assumed. Another way to accelerate the trend is by increasing the electrification of railways and urban mass transit systems throughout the world and introducing electric cars more rapidly into urban travel. However, if the whole transportation sector - which represents about 25 percent of final energy use - is electrified, then secondary electricity would increase to 11.1 and 7.2 TW yr/yr in high and low scenarios respectively. This new demand for electricity will eliminate the "oversupply" of electricity created by restrictions on fossil fuel use.

We should not take lightly the implications of such a policy; it heralds new structural changes in the way societies around the world are being organised. It would involve transformation of our view of electricity. Electricity could come to be seen as a primary energy source in the sense it would become the source of gas and liquid fuels. It could become the "standard" or reference against which all substitutions would be measured. In IIASA's words (Háfele (1981)), it would be the "currency of the future world".

The other half of the future energy structure would be hydrogen. Depending on the extent to which natural gas networks will be built over the next few decades, new investment both in installations and materials and equipment would be needed to put in place the infrastructure of long distance transmission and local distribution networks needed for a smooth transition to a hydrogen economy.

1.2.6 Conclusion

From the tentative analysis we see the following steps as essential for coping effectively with the CO_2 problem:

- A strong global program to use energy more rationally and efficiently (commonly called "conservation")
- Development and utilization of more non-fossil sources in the institutional interim
- Accelerated efforts to shorten the time lag between effective action time on problem resolution.

The envisioned transition comes just at a time of considerable social and political dissent about how to solve the energy problem, a time when conventional oil and gas are becoming very much more costly and at a time of rising expectations on the part of the LDC and former East-Bloc countries who are in mid-transition to a different economic system.

1.3 INTERNATIONAL ASPECTS

The discussion in previous sections has been confined to the global response to impending CO_2-induced climatic change. However, the effects will result in extremely varied regional impact. For example, some areas of the globe might experience longer growing seasons or more favourable rainfall patterns. At the same time, certain highly productive agricultural lands could be subjected to prolonged or permanent drought.

It is thus likely that as more is learned about the problem and as more refinements are introduced into the General Circulation Models (GCMs) capability, some nations will begin to perceive themselves as, on balance, "winners", while others will come to see themselves as "losers". At that point, diverse regional objectives might prove divisive and diminish the prospects for establishing the international cooperation needed to cope best with the CO_2 problem.

1.3.1 Climate Scenarios

There are many researchers around the world studying the spatial distribution of the future changes in climate, some using numerical modelling like GCMs, others using primarily past warm periods as analogues of the future.

With the help of these models it is possible to show, temperature changes associated with a doubling of atmospheric CO_2. It produces estimates of average changes according to latitude. The changes are relatively small at the equator and dramatically large at the pole.

GCMs incorporate a hydrological cycle that permit a study of precipitation, evaporation, and soil moisture over the continent.

From previous considerations, we can classify the seven regions used in this study into "wetter" (W) or "drier" (D) categories based on agreement of at least two sources:

Region I	(NA)	US(D); Canada (W)
Region II	(SU/EE)	Soviet Union, Eastern Europe
Region III	(WE/AUS)	W. Europe (D); Australia (W)
Region IV	(LA)	Mexico (W); Rest/Mixed
Region V	(AF/SEA)	Northern & eastern Africa & India (W)
Region VI	(ME/NAF)	(W) except the fertile crescent may be mixed
Region VII	(C/CPA)	Southern China (W), Northern (D)

It should be stressed that this exercise - translating global warming into regional conditions - is at most a highly tentative speculation and based often on conflicting sources, but these three points need to be borne in mind:

(a) Regional Climatic Scenarios are based on sparse data and often in disagreement.

(b) These studies are only "scenarios". A scenario can only provide a guide to the patterns of climate change which could occur as the result of global warming, but that does not guarantee that the same conditions will recur in the future. There are going to be changes in oceanic and cryospheric boundary conditions. Those changes will influence the large scale circulation patterns that in turn affect the regional temperature and precipitation.

(c) Even with this level of uncertainty there seems to be a basis for assigning a higher probability to the direction of the change in some regions, such as region I, V, VI and VII.

1.3.2 Distributive Climatic Impacts

As the casual speculations begin to suggest, there is no well-defined process for translating the global expected warming into spatial climatic changes. The second logical translation of these anticipated changes into effects on agriculture and economy, and thirdly into societal responses is even more uncertain. We are essentially multiplying uncertainties.

The point of the previous analysis is largely that climatic change is likely to have important distributive implications. Some regions will find themselves with more favourable climates, others with less favourable climates. Indeed, distributive issues could well be of great importance, both among the seven regions and among groups within each region and among the nations of each group.

In the context of these general comments we shall explore the impacts on one sector, agriculture, whose effects are both serious and less uncertain.

1.3.3 World Food Problem

Attention to the world food problem, under present trends, is long overdue. Anticipating probable disruption and loss of production in major crop-producing areas, because of projected climatic changes, gives the problem the dimensions of a major world crisis.

In food and agriculture, the rate of growth in production in the developing world averaged about 3 percent a year in the 1970s, but the increase in food production failed to keep up with the growth of population in more than half of the developing countries, particularly the poorer ones. The undernourished in the developing market economies are at least 420 million, and continue to increase in number. The immediate situation and outlook have become more precarious than in the past.

The rising costs of imported agricultural imports severely thwart the efforts of many developing countries to increase their food and agricultural production. Especially noticeable is the rapid acceleration of fertilizer prices, one of the most critical inputs. World carry-over stocks of cereals at the end of the season will represent only about 18 percent of consumption, which is the minimum proportion required for world food security. Inefficient distribution, handling and transportation of food presents logistic difficulties adding to the gravity of the problem.

1.3.4 Framework for World-Regions Analysis

Even in its rudimentary form, the foregoing analysis of climatic impacts on regional cereal production suggests the potential conflict between would-be "winners" and "losers" in the climate game. In the cases studied, the probable major losers might just be the United States, the USSR and Europe, and major winners China, India, parts of Africa and Latin America. The fate of other regions is less clear. But even that grouping is unsatisfactorily vague.

The negative impacts of such cumulative change on the USA and the USSR is likely to create severe tensions in the global socio-political network leading to future confrontation within existing socio-political subsystems as resources become inadequate.

This selective decline of productivity among the major economic and food producing nations *will not* be necessarily offset by the increase in developing countries because of lack of industrial infrastructure, transportation and distribution systems. In addition, questions will inevitably arise concerning responsibility for the costs of the adverse climatic impacts. In practice, this means that nations would need to agree on measures to determine the proportional indemnity to be charged to each region or country. But countries, ultimately, are free to comply with or ignore international agreements.

In the opinion of many noted global energy analysts a solution should be sought based on global international cooperation and interdependence and which satisfies conditions of equity and fairness.

As an illustration of one possible approach, the following regional scenarios were constructed. The reason for such an analysis is to illustrate the varied regional responses to a global decision to restrict CO_2 levels to certain limits.

Global analyses tend to imagine everyone doing the same thing at the same time. They do not, and the spread in time has important consequences. For example, history tells us that countries that proceed from a state of relatively little industrialization to a state of fairly advanced industrialization take typically 30 to 40 years or more (Japan, Germany, for instance). Thus, we can easily imagine a transition to new technological styles that might take a century to implement, even if all major less-industrialized countries started now and following the same track toward new technologies, we foresee almost half a century of transition, and that kind of lockstep will not happen.

These qualitative arguments have guided our construction of the regional scenarios. This was translated into the scenarios by assuming that the high-energy-using industrialized countries would shift first from fossil to non-fossil energy technologies, with the less industrialized countries allowed to use fossil fuels for a longer time.

A fundamental assumption in the following analysis is the creation of an international regime entrusted with setting up CO_2 standards and a mechanism or framework for carbon quota allowances for each region. The stringency built into the energy scenarios was that world population should be constrained to 10 billion by 2100, only 2.2 times the present population. Also, per capita energy consumption was gradually reduced even to negative values. Observe that on a per capita basis, the less industrialized countries <u>never</u> enjoy a level comparable to any point of time in the industrialized countries.

A crucial question arises on setting up the "carbon quota" system. This study explores only two bases for a solution: per capita basis or total prospective cumulative energy consumption. No penalties were incurred for pre-1980 CO_2 emissions. The former basis posed another problem: which population levels should be used--present levels, stabilized levels or some cumulative measure? However, on average, setting up standards on total cumulative energy consumption allows the industrialized countries (Region I-III) 56 per cent more in terms of carbon shares.

The figures in Table 6 reveal very explicitly the political explosiveness of the CO_2 problem. If global action is warranted to curtail fossil fuel consumption, it would be near to impossible to persuade any of the countries in Region IV through VII to reduce or even to hold their fossil growth at present rates. There should be enough delay to catch up with the almost eleven-fold difference between the developed and the developing regions (on per capita basis).

Thus, in our model, the AIT for Regions I-III is 1990, while for Regions IV-VII it is 2010-2020. It should be clear that this lag, an "equity lag", is the very political minimum.

Once the regional carbon shares are allotted, then each region decides independently how best to utilize it. This in turn is reflected in the regional choice of fossil fuel mix. We assume for the period up to the AIT that fossil fuel consumption will follow regional scenarios.

The need for an international regime is fairly demonstrated in Table 7. The North controls 78 to 83 per cent of the world's coal endowment. The United States and the former Soviet Union alone have some 74 per cent and with China, that comes to 88 per cent. Such an uneven distribution of coal resources looks very suspicious. Perhaps the South has not been explored as well yet. On the other hand, the remaining oil and gas resources are more evenly distributed (in terms of the North-South axis). Thus, regions IV to VII contain between 53 and 61 per cent of the world's oil and gas resources.

From these comments we can anticipate problematic regions: Region II (SU/EE) and Region VII (C/CPA), because of their coal wealth, might not be as cooperative, though SU/EE will be negatively affected, which may counterbalance its response. Regions IV and V would not have much bargaining power. WE/JANZ (Region III), too, will be in a weak position despite its technological capability. ME/NAF (Region VI), though having no known coal resources, still will be able to exert a somewhat strong position because of its oil and gas assets. Appreciating the potential complexities and impediments to a global consensus on the CO_2 question, we proceed to discuss next the results of one possible regional scenario construction.

1.3.5 Adjustment Process

Adaptation is perhaps the path of least resistance with respect to expected changes in climate (Schelling, 1988/1992). Qualitatively, it can easily be shown that migration and industrialization will be the basic implication of any climate change - especially a slow one - as it impacts upon agriculture. Change in agricultural productivity means that the ratio of population density per unit of agricultural productivity is changed, and such changes can be compensated by corresponding changes in the population density (migration) or by increasing agriculture or other economic activities (reeducation and industrialization).

Explicit government intervention to change behaviour could also assist in the adaptation. For example, zoning laws could be planned to restrict new buildings from being constructed in areas of probable flooding because of projected sea level rise. However, recent efforts to prevent construction in flood plains have been notably unsuccessful (Lave, 1981). Historically, people do not behave the way "foreseen" by advocates of adaptation. A good example of how myopic the adjustments might turn out to be was presented by Ronald Ridker (1981). Ridker described how coastal populations are likely to adjust to a slow but inevitable rise in sea level. In view of the slowness of the change (on the order of a few feet each century), and man's tendency to use a positive discount rate, it is more likely that sea walls and dikes will be built than that people will evacuate. And once such sea walls are built, it will appear cheaper to make them a bit thicker and higher than to evacuate the area. Eventually, much of the human race could find itself living below sea level, with the probability of a catastrophic breach in the dikes growing over the centuries. This is an example of a situation in which man's normal response to adaptation could eventually become self-destructive.

There are considerable costs involved in this adaptation process as well. It might include major new investment, significant change in production methods, development of new business relations, and disruption and dislocation of present human settlements. Adaptation is inherently redistributive, and hence could be inequitable especially for the developing countries. Most of the developing countries, lacking strong technological infrastructures, are highly vulnerable especially to changes in agriculture and water supply. Thus, adaptation could very well accentuate the North-South cleavage. Adaptation seems to be working well and is well founded by economic theory (see Chapter 2) when climate change is only moderate, but is much more difficult to justify if changes are significant because of second-order more severe consequences (Nordhaus, 1991).

1.3.6 Economic Cost Analysis of Various Strategies

Nordhaus (1977) pioneered investigations into the costs of constraining CO_2 concentrations to pre-determined levels via taxation. Using a market allocation model linked to a simplified carbon cycle model, Nordhaus considered CO_2 emissions as a resource in short supply, allocating them between sectors in such a way as to maximise national income. Losses are discounted at 10%/yr and by minimizing economic losses one obtains "emission paths" that are "allowable".

However, this model fails to allow for deployment rates of new non-fossil technologies and probably assumes new capacities well in place when needed. These omissions permit Nordhaus to conclude that it does not pay to curtail carbon dioxide emission until nearly the time when the limit is reached. For example, in the uncontrolled scenario, doubling of the pre-industrial concentration is reached by 2040 but abatement measures become necessary only in the period 2010 to 2020.

Nordhaus further calculates shadow prices of carbon dioxide. The shadow price indicates how much the objective function would increase if the constraint were relaxed one unit. The objective function is here, the real income of consumers. For the case of doubling constraint, the shadow price starts in 1980 at $0.14/ton carbon, rises to $68/ton by 2040, and reaches a plateau of on the order of $94/ton throughout the rest of the century.

Later, Nordhaus (1980) has used control theory to find "optimal" emission paths. Thus, he attempts to maximize the discounted expected utility of consumption:

$$MaxW = \int_0^\infty e^{-rt}\, U[C(t)]\, dt$$

where:

W(t) = welfare functions

r = pure rate of social time preference

C(t) = real consumption

U[C(t)] = utility of real consumption

subject to constraints:

$$C(t) = f[F(t)] - h[M(t)]$$

$$\dot{M} = \beta F(t) - \delta M(t)$$

where:

$F(t)$ = emissions of CO_2
$M(t)$ = increase of atmospheric concentration of CO_2 from pre-industrial level
β, = carbon cycle constraints
$f[F(t)]$ = consumption
$h[M(t)]$ = loss in consumption due to CO_2 buildup.

Nordhaus estimates, $h[2 \times M_0]$ (doubling), from various sources including his own estimates and finds the economic value of climate change to range from $301 billion ($1975) as benefit to pessimistic loss of $691 billion. In his calculation he takes the "best guess" to be a loss of *$180 billion.*

Nordhaus decomposes the goods discount rates into a pure social rate of time preference (r) and growth discount (α g). The choice of a redistributive parameter (α) which is the elasticity of marginal utility of income, is crucial to the results.

Starting from Nordhaus's "best guess" we can test how sensitive the results are to his basic assumptions. If the costs of climate change are closer to his "pessimistic" assumptions, then the current shadow price increase to $38/ton and the steady state, to $323/ton. Also, the control rate becomes 43 percent in 1980 and reaches 100 percent in the steady state. If the discount rate is reduced from the assumed 13 percent to about 5 percent, full control of CO_2 emissions will have to be imposed. Obviously, such an analysis is highly simplified and questionable in terms of basic assumptions and even of approach.

Apart from discounting, which we discuss elsewhere, Chapter 2 ~~Gottinger (1990~~), Nordhaus assumes the cost of CO_2 buildup to be linear with concentration neglecting the non-linearity of the temperature response as well as possible catastrophic discontinuities. Further he assumes, based on his earlier study (Nordhaus, 1977) a parameterized abatement function ignoring possible backstop technology.

1.3.7 Equity Issues

All the previous studies discussed were concerned with the changes from global mean. However, in many cases the redistributive regional costs might be very different and not accounted for on the average.

The natural global climatic losses estimated (Kates, 1979) at $30 billion per year originate from floods (53 per cent), tropical cyclones (27 per cent) and droughts (15 per cent). What is striking from Kates' study is the great inequity in the spatial distribution of deaths and losses. For example, of the 250,000 people who die each year from natural hazards, 95 per cent are citizens of poorer nations. Also, the annual drought losses in Tanzania are 1.8 per cent of GNP, while the comparable figure in Australia is only 0.1 per cent. Other examples show similar results. On the average, in terms of per cent of GNP, Climatic hazards are about 25 times more severe in developing countries than in developed countries.

If the equity issue is at the heart of the CO_2 problem, then the optimal control or other allocative efficiency theories has to be relativized. These theories, emphasizing "efficiency", tend to net out the losses from the gains with no consideration of distributive impacts.

In the meantime, issues of distributional justice have assumed priority in arguments about international order (Spash and d'Arge, 1989).

Assuming we can calculate for each strategy the time paths of annual costs and benefits, we still are faced with the difficulty of expressing the results into a measure to reflect their relative present day seriousness. This is often done by discounting. But the long term span involved here puts us into issues of intertemporal fairness: a problem of tradeoffs between succeeding human generations. When the present generation evaluates alternative uses of the atmosphere it is making judgments about the welfare of future generations relative to the welfare of the current generation. This judgment, often, centers on the choice of a "correct" social rate of discount, which is an extensively debated question in economics.

Another difficulty arises because the redistribution of costs and benefits, a theoretical underpinning of cost/benefit analysis, can only be effected in one direction--from present to future generations. Page (1977) argues that compensation is likely to be only hypothetical and not real, making the whole discounting procedure meaningless on ethical grounds since actual compensation is not likely to be paid.

It is the intertemporal ethical rule that in balancing risks to human life in the present and in the future one is inclined to feel that equal numbers of lives should receive equal weight, making the "present value" of future human life independent of the time at which it is lived, in contrast to the present value of a bundle of consumption goods, that specially complicates the evaluation process.

Such considerations lead some economists, such as d'Arge, Schultze and Brookshire (1982), to discard traditional benefit/cost analysis in favour of one based on ethical beliefs for evaluation of the CO_2 issue. They find that differing ethical systems imply differing discount rates. They further suggest a way to postpone a decision to establish an "optimal" environmental ethic for future generations by estimating the present generation's willingness to pay to avoid environmental risks to future generations. Hopefully, such an estimate might embody the present generation's ethical beliefs.

Clearly, then, many argue that resource depletion and the future long-term quality of the environment are not merely problems of market failure, but distributional problems. They are even more difficult than internalizing cost, because people and nations will not agree on the distributional criteria for societies. Should the criteria emphasize maximization of returns to the international community? Conservation of natural resources? Passing on a stock of environmental wealth per capita at least equal in value to the one which was inherited? Or, in contrast, passing on a legacy of infrastructure from development and resiliency which will minimize the "loss" from climate change? (Ausubel, 1980b).

1.3.8 Uncertainty Analysis

The high levels of uncertainty have prompted some policymakers scientists to adopt the "wait and see" approach, since no agreement or even a process towards "optimal" strategy is on the horizon. Thus, they caution against any major quick actions until better data and information is available (~~Gottinger, 1991~~). It is, however, quite possible that none of these substantial uncertainties will be reduced in the next decade (Ausubel, 1980b).

hapter 7

In other words, the conventional assumptions of learning over time may be irrelevant in this issue. It is possible that we will face virtually the same decision in a decade that we face now, with only slightly more reliable information. This is so partly because the climate change is plagued with the problem of "indivisabilities" and "scant sets" (Olson, 1982). These are areas where our knowledge is generally so meager and the stakes are so high, but also that we cannot expect to get reliable answers either cheaply or quickly because a decisive experiment entails a policy change and, because historical experience with scant sets is so slowly informative. It is clear that decision-makers must attempt to live with and accommodate to the high degree of uncertainty permeating the CO_2 issue. Therefore, policymakers must realize that decisions will have to be made without certain cost/benefit information and that waiting until the uncertainty is resolved will be probably too late to make an effective decision.

More attention should be directed at resolving or reducing the effects of "malign uncertainties" (uncertainties which make it impossible to determine whether the outcome of a particular policy will be reasonably good or very bad), as opposed to resolving "benign uncertainties" (those which make it impossible to tell which of two good strategies will be better) (Burgess, 1980). This means that uncertainties and therefore losses become asymmetric around the optimal decision and have to be treated accordingly (Morgan and Henrion, 1990).

1.3.9 Institutional Barriers

The "slow" cumulative buildup of CO_2 in the atmosphere poses a series of challenges to the international community and its institutions. The unique blend of political, economic, ethical, legal and scientific issues that the CO_2 problem raises propel existing institutions to break new ground.

Indeed, the CO_2-induced climate change is a virtual prototype of a problem poorly matched to existing human institutions: its time span is longer than any political leader's career, and the potential effects are enormous, conceivably dwarfing those of normal, man-made technical and social change. This kind of problem presents an almost insurmountable challenge to institutions designed for times when societies were less complex and man's abilities for doing "good" and "bad" much more limited and thinking much more restricted in time and space.

1.3.10 Design of an International Regime

Undoubtedly, for the CO_2 issue and many other "global commons", some form of international regime is not only desirable but necessary. Design of operative mechanisms for a CO_2 regime can be partially extracted from studying other mechanisms developed at the regional or international level to deal with "similar" situations.

The envisioned international CO_2 regime requires much more substantial global agreement, control, management, and allocation of resources. Most of the international agreements discussed have failed to reach substantial agreement (beyond agreeing to study the matter further, monitor, and so forth). Even if substantive agreements were reached, there is real grounds for doubting that paper agreements are being seriously enforced.

The relative lack of enforcement machinery internationally necessarily conditions the character of international negotiations. Thus, it is often less costly and easier to hold meetings, conferences and other forums and even draft "statements of principles" than to actually do something substantial. We already are experiencing this in the context of the proliferation of meetings and papers on the CO_2 issue.

The difficulties arising from investing a CO_2 international regime with requisite responsibilities and authority and even legitimate command of coercion stems directly from the reluctance of nations to yield sovereignty to international bodies, for it means some loss of control over decisions that directly affect important national interests and domestic constituencies. However, in practice, some delegation of responsibility cannot be avoided and a variety of means are attempted by each nation to make it more acceptable.

A serious investigation of "optimal" strategies should be initiated under which we can organize reasonably effective coordinated efforts among sovereign states to reach substantive agreements on the CO_2 issue. Policies should attempt to find powerful self-interest and incentives which could be a mixture of selfish and social motivation, to foster an acceptable agreement. The fact that diverse opinions and high uncertainties surrounding the relative redistribution of the climate on the regional level might be helpful for achieving early consensus on action.

The actual tactical steps required to initiate an early international regime for CO_2 should be explored very carefully with an eye to the requirements of political legitimacy and the necessity for efficiency and internal consistency in decision-making on highly complex issues.

The above conclusions lead to the following policy-oriented recommendations:

1. International cooperation is needed in many areas in order to ensure timely and orderly "transition" to inevitable non-fossil energy systems. Areas of cooperation include:

   (a) More efficient and rational worldwide energy use and perhaps subsidies of certain end-use-efficient technologies.
   (b) Arresting the increasing fuelwood crisis in the LDCs.
   (c) International R&D in non-fossil energy systems, development, and commercialization. Attention should be given to avoid "monism" in energy policy. All feasible options should be pursued.
   (d) Establishing a world energy financing centre with low interest loans to aid the LDCs in technology acquisition, adaptation and training.

2. It should be of the highest priority to incorporate the CO_2 issue into the world energy policy-making process and CO_2 should serve as the discriminating factor in the choice of energy options.
3. A serious investigation to establish an international institution to deal with the CO_2 issue should be promptly initiated. Consideration should be directed to the following:

   (a) How best to optimize effective coordinated efforts among sovereign states.
   (b) Setting of standards or "acceptable" CO_2 limits that will serve as policy targets.
   (c) A system of carbon "quotas" or allocation schemes.
   (d) Coordinated scientific cooperation in data gathering, monitoring and emission regulation.
   (e) International agreement on enforcement machinery.
   (f) The needs of the LDCs, their ability to substitute, and their developmental objectives.

4. The US, EC, and the OECD countries should plan in the next decade to take early action at limiting their fossil fuel use. This is required regardless of the CO_2 problem.
5. There is a need to develop guidelines for world coal trade and reach early agreements toward "internationalization" of the trade, especially with the signs of the second coming of the "coal age".

In concluding, it is appropriate to recognize the limitations of the analysis; partly due to the paucity of relevant data, the constellation of uncertainties pervading all aspects of the CO_2 problem, the lack of an integrative framework, and above all the "social engineering" ethical, value-laden implications of the issues raised.

Table 1. Energy and Carbon Content of Recoverable Fossil Fuels [1,2]

	Energy Content (10^{21}J)	**Carbon[3] Content** (10^{15}J)
Oil		
Reserves 640 x 10^9 bbl	3.9	74
Resources 2000 x 10^9 bbl	12.2	232
Gas		
Reserves 2500 Tcf	2.7	38
Resources 9000 Tcf	9.7	138
Coal		
Reserves 636 Gtce	18.8	440
Resources 5000 Gtce	147.0	3500
Total	169.0	3870

1) Includes only conventional deposits of oil, gas, and coal. Excludes heavy crudes, oil shares, tar sands, methane, hydrates etc.

2) Resources are cumulative, they include reserves. These are estimated recoverable resources, not resources in place.
Units 1 bbl = 42 gal. = 0.159 m^3, 1Tcf = 10^{12} ft^3 = 2.83 x $10^{10}m^3$; 1 Gtce = 10^{15} grams of standard coal equivalent at 7 cal/g.

3) Carbon content/energy content is assumed to be approximately as follows: oil 1.9, gas 1.4, coal 2.4 grams carbon/MJ.

Sources: Adapted from Háfele (1981) and Rosenberg (1989)

Table 2. Summary of Economically Recoverable Additional Resources by Cost Category

Resource	Coal		Oil			Natural Gas		
Cost Category[1]	1	2	1	2	3	1	2	3
World	560	3065	264	200	373	267	141	130

1) Cost categories represent estimate of cost either at or below the stated volume of recoverable resources (in constant $ 1980)

For oil and natural gas: Cat. 1 : $15/boe
Cat. 2 : $15-20 boe
Cat. 3 : $20-25 boe

For coal: Cat. 1 : $25/tce
Cat. 2 : $25-50 tce.

Sources: Adapted from Háfele (1981)

Table 3. Dates by which various CO_2 levels are reached in the constrained and unconstrained scenarios.

CO_2 Level (ppmv)	Annual Exponential Growth (pc/yr)			Constrained Trajectories Asymptotes (ppm)		
	2.0	2.6	4.3	600	700	1000
500	2023	2023	2018	2050	2043	2037
600	2047	2042	2029		2075	2057
700	2063	2054	2036			2074

Table 4. Energy Technologies Market Penetration Rates & Times

	Technology	Penetration Rates (α) (%/yr)	t_m (Yrs)
World primary energy supply	Oil	4.9	94
	Natural Gas	4.8	95
	Coal	2.7	170
U.S. primary energy supply	Oil	5.3	86
	Natural Gas	4.5	102
	Coal	7.0/4.6*	66/99*
OECD - Europe primary energy supply	Oil	10.0	46
primary energy supply	Natural Gas	15.7	29
	Nuclear	6.9	66

*The first figure refers to the growth stage, while the second refers to the decline stage.

Table 5. Global Electricity Generation

	Base Year	**High Scenario**		**Low Scenario**	
	1975	2000	2030	2000	2030
Electricity, secondary (TW yr/yr)	0.75	2.1	4.7	1.7	3.0
Installed capacity (GWe)	1600	4390	9845	3550	6320

Table 6. Regional Share of CO_2 Emissions in 1980

Region	Share (%)	World Population (%)
I	28.7	6.0
II	24.7	9.2
III	28.1	14.2
IV	4.1	8.1
V	4.6	36.0
VI	1.8	3.4
VII	8.0	23.1
Developed countries	81.1	29.4
Developing countries	18.5	70.6

Table 7. Share of Carbon Wealth by Region

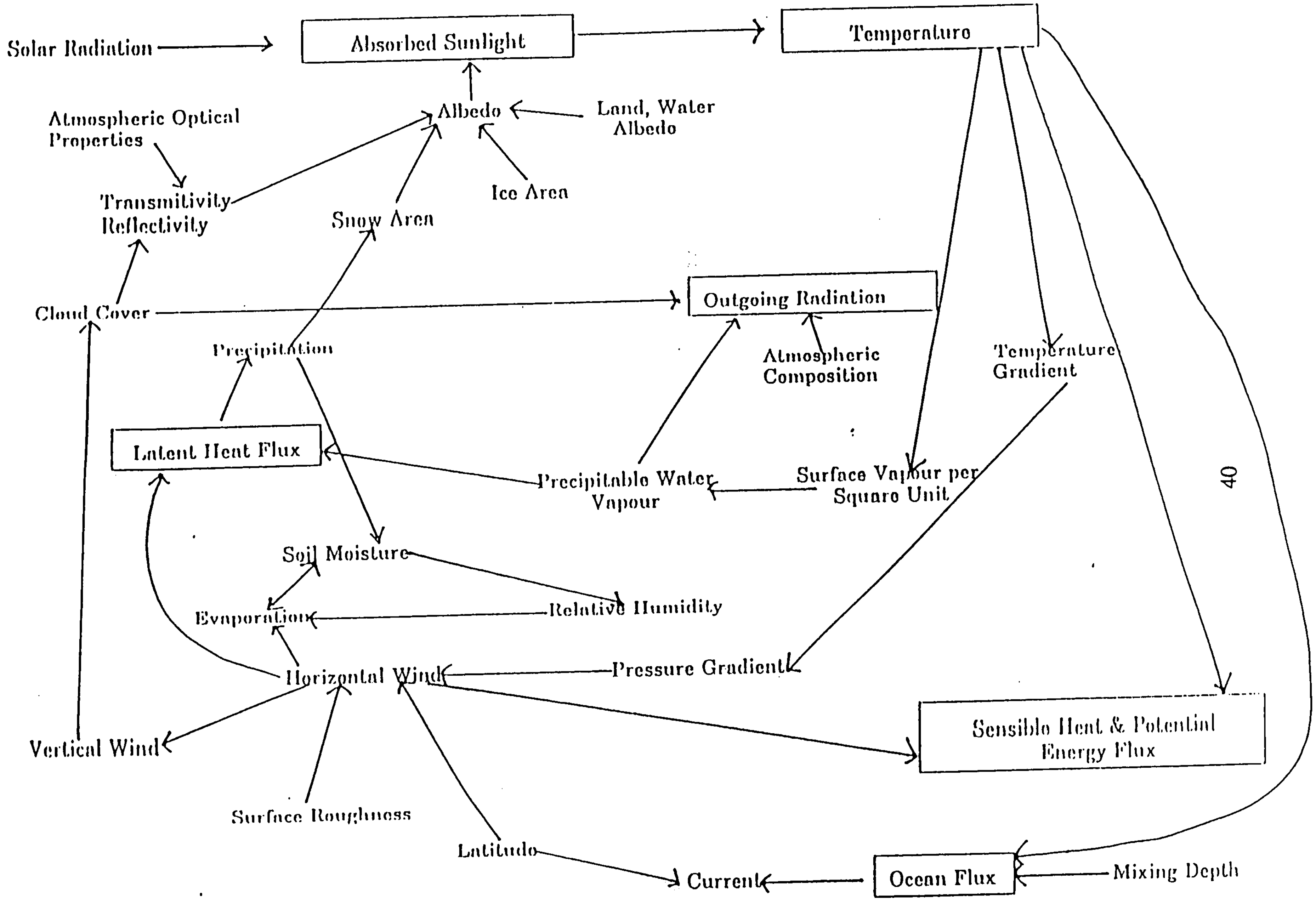
Region	Coal[a]	Oil & Gas[b]
I	25.7/25.7	20.1/12.4
II	37.0/49.1	21.7/20.6
III	15.5/8.4	5.7/6.2
IV	1.3/0.4	18.4/7.6
V	6.8/2.1	7.5/6.9
VI	0/0	21.2/42.6
VII	13.7/14.6	5.31/3.9

(a) First number is based on estimated economically recoverable coal resources; the second is based on total geological resources.

(b) Both numbers are based on total oil and gas economically recoverable resources but the first includes contributions from unconventional resources, the second does not.

Source: Based on data from Häfele (1981), Ausubel (1980b).

Figure 1 The Interactions, Linkages and Feedback Loops in the Climate System (after Kellog (1974)).



CLIMATIC CAUSE-AND-EFFECT (FEEDBACK) LINKAGES

Figure 2 Schematic Illustration of Two CO_2 Paths: "A" and "B"

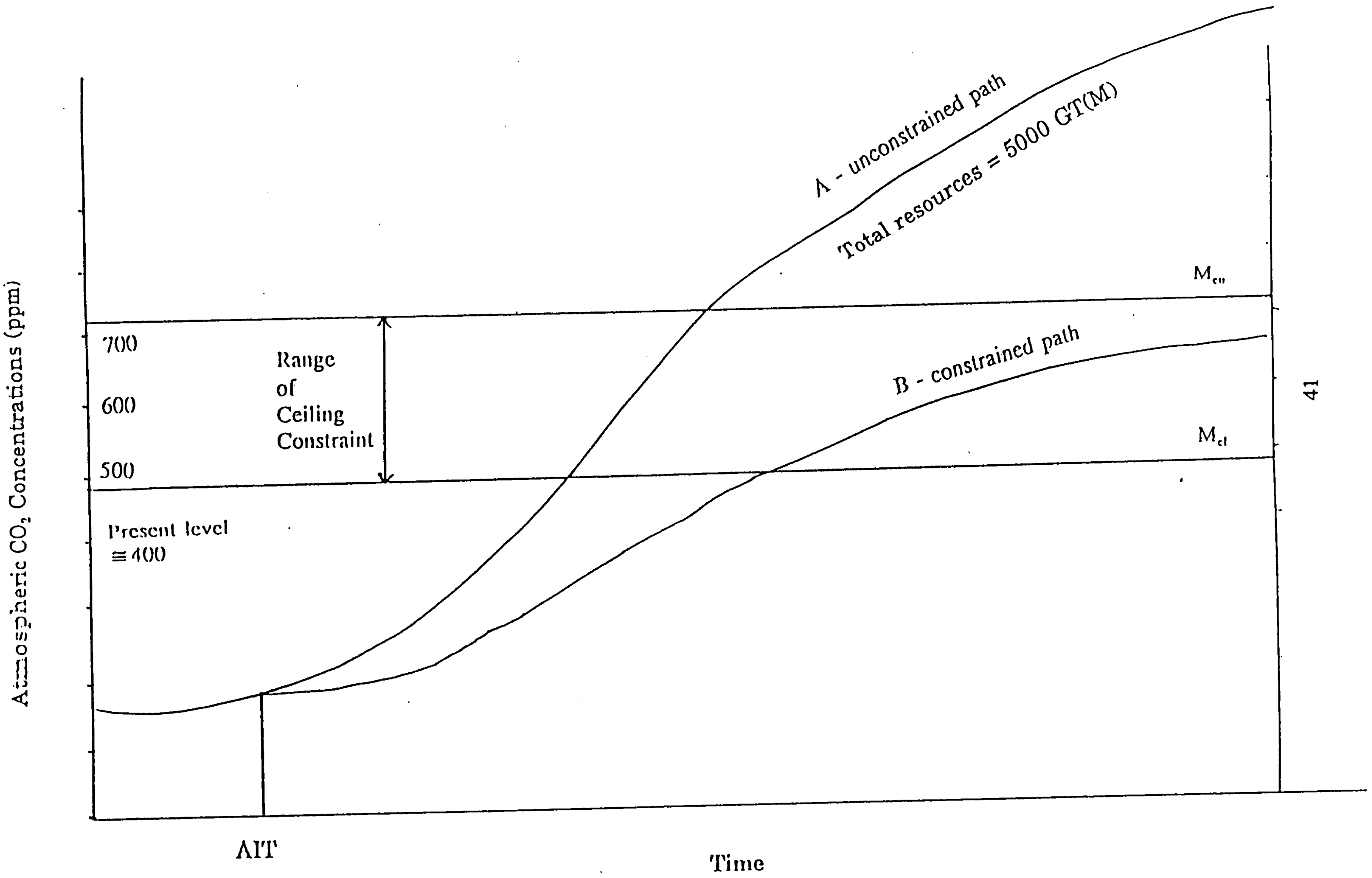


Figure 3 Global Fossil Fuel Use Scenarios Corresponding to CO_2 Paths: "A" and "B"

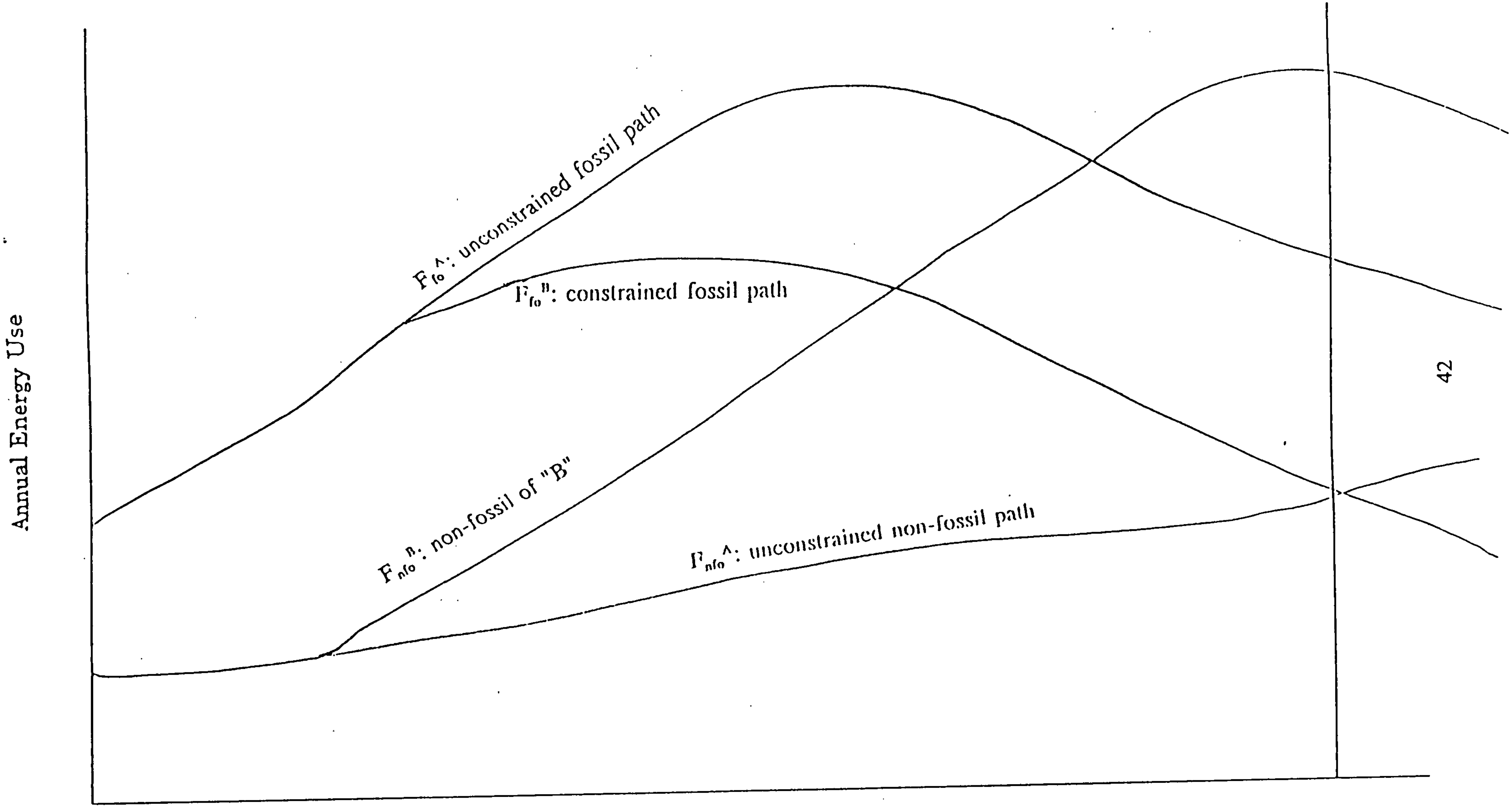


Figure 4 Feasible fossil fuel production paths for a total ultimate cumulative consumption of 5000 TWyr

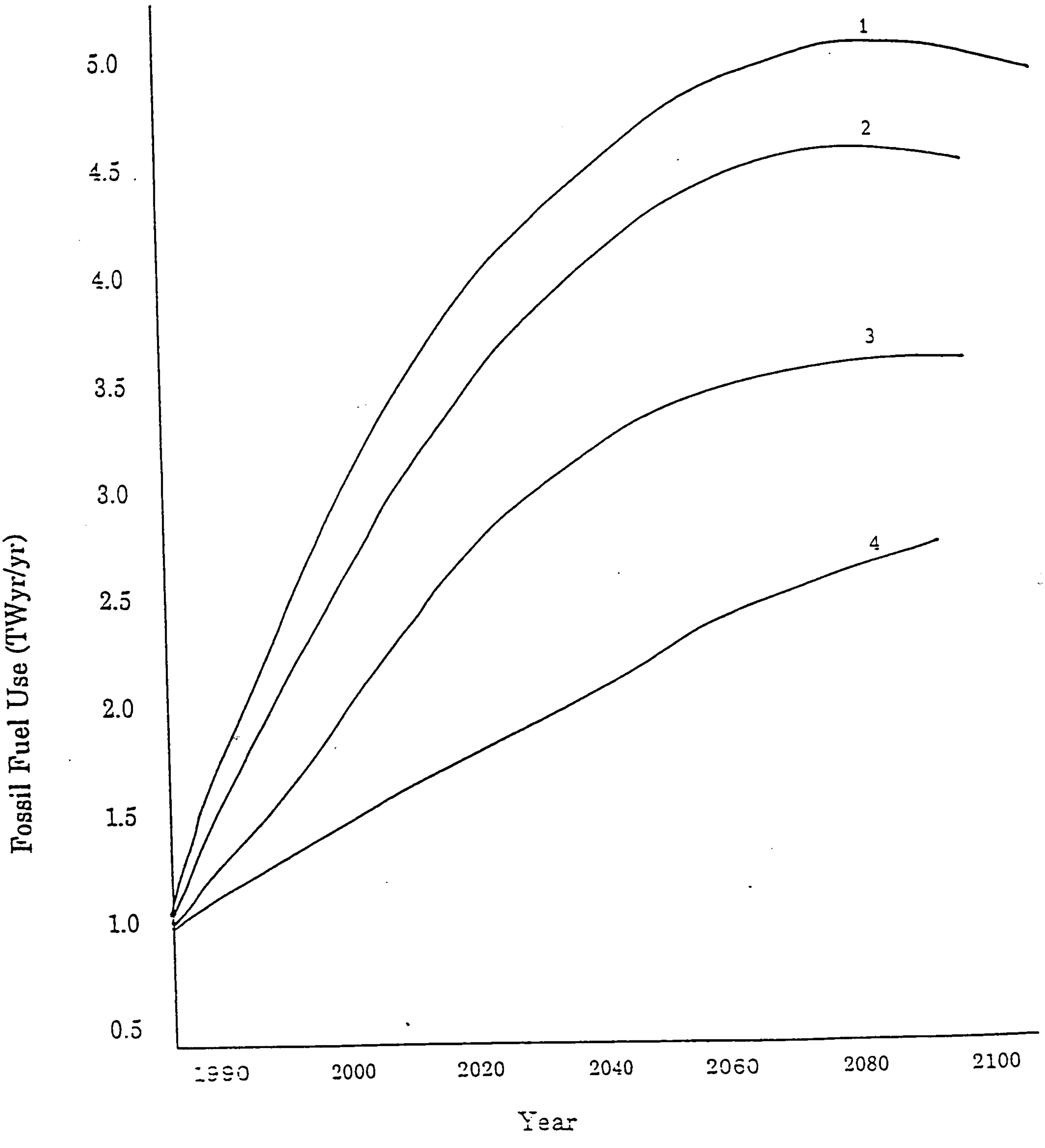
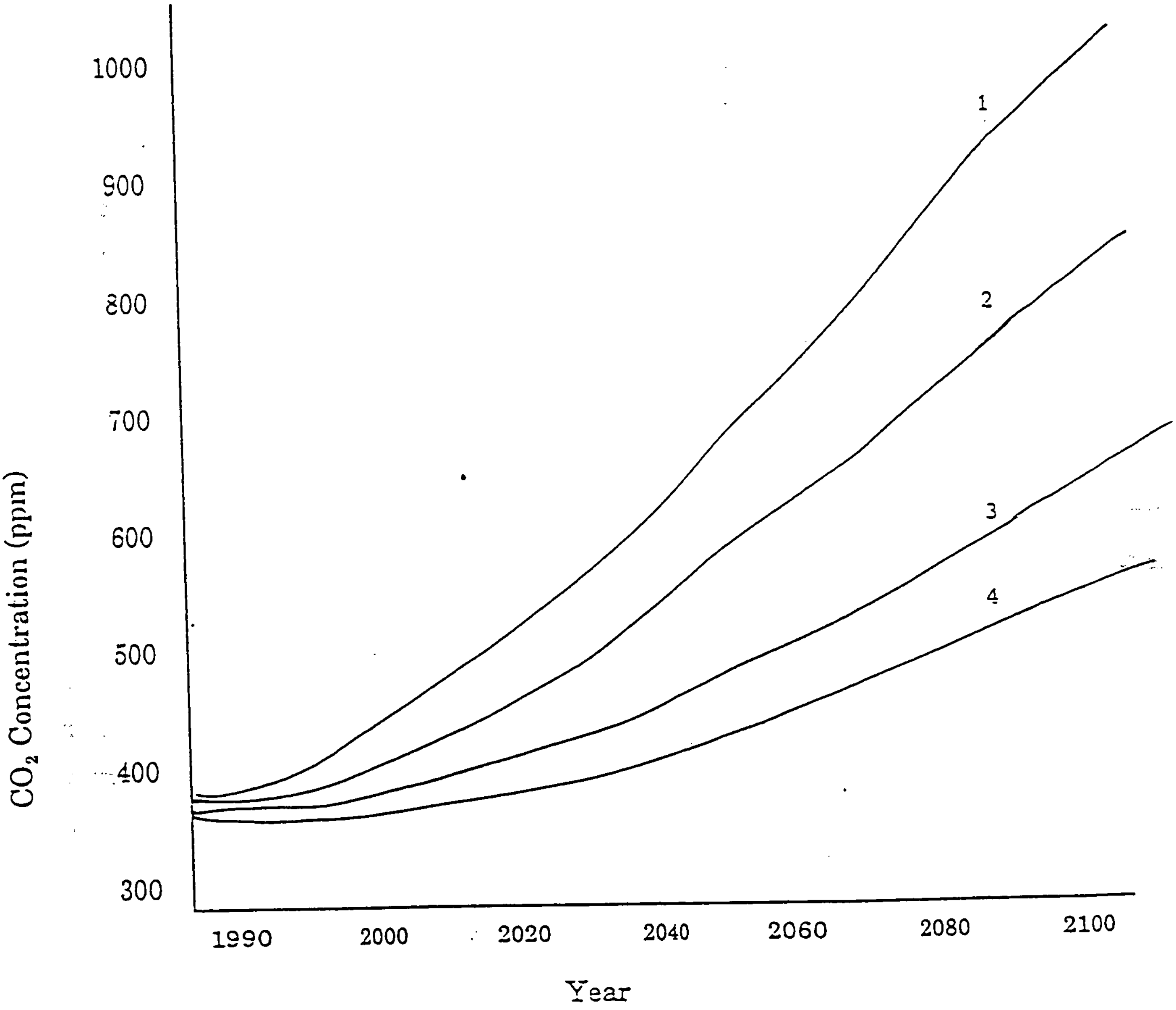


Figure 5 Range of projected CO_2 concentration for an ultimate consumption of 5000 TWyr and production functions in Figure 5. (Airborne fraction ≡ 0.5)



Appendix

Market Penetration of Non-Fossil Fuels

Under normal conditions replacement time for industrial equipment could run from 30 to 50 years.

We have the non-fossil component of our scenarios plotted as log y = log [f(1-f)] where f is the market share of the non-fossil component. Three logistic curves are computed and the scenarios are of the form $\log y = \alpha(t - t_0) + \gamma$

α is the penetration rate

$t_0 = 1980$

γ is a constant.

γ was calculated on the assumption that the new non-fossil component could be treated as one single aggregate growing from an initial market share of 8.6 percent (1990). It is clear that such an assumption is hardly valid, and most probably will yield to an overestimation of the actual potential of non-fossil supplies. For one thing, the hydro fraction already has saturated and is on the decline. Thus, the actual growing component of the non-fossil component is mainly nuclear, which contributes now in the order of 3 percent of total primary input.

In fig. A.1, for the High Scenario, we display two curves from our scenarios, namely for AIT of 1990 and the year 2000 along with logistic curves of constant penetration rates of 6.2, 5.2 and 4.2%/yr, representing an accelerated penetration, a historical trend and moderate to slow penetration, respectively (see Table 4).

The AIT = 1990 curve follows closely a logistic trend with 6.2%/yr until the year 2030, at which time the non-fossil share is on the order of 66 percent. Afterwards, it departs to a slower trend of about 4.6%/yr. On the other hand, the later AIT curve displays non-logistic behaviour in the early decades before AIT, averaging a penetration rate of about 1.7%/yr. Subsequently, three distinct phases of growth can be discerned: "catch up" phase, ending about 2035, characterized by a high unprecedented penetration of about 8.6%/yr; a transition and readjustment phase ending by 2050; a saturation phase marked by slower penetration of 6.5%/yr, but still faster than the accelerated penetration curve.

The Low Scenario, as opposed to the High one, is expected to represent a reduced penetration rate. But is this the case? Fig. A.2 shows corresponding curves as shown in

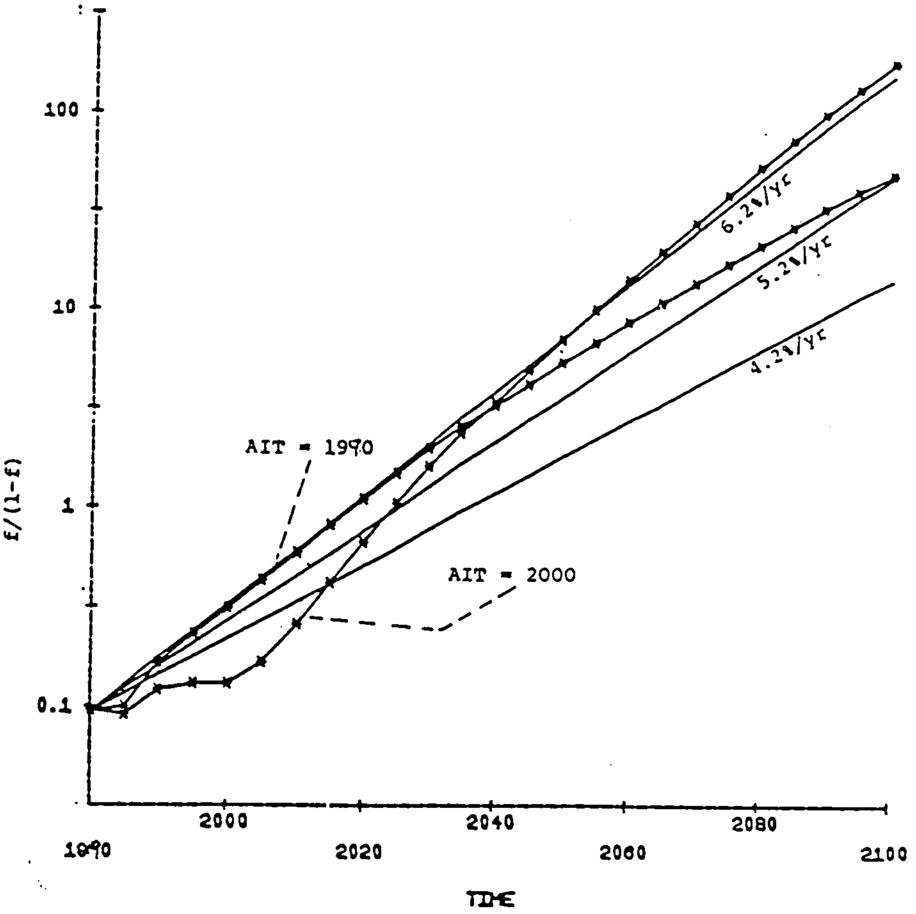


Figure A.1: Market penetration analysis: Logarithmic plot of the transformation f/(1-f) where f is the non-fossil market share in total primary energy, here the High Scenario, HC/LN with a CO_2 limit of 500 ppm. Smooth straight lines are logistic paths with constant penetration rate.

Fig. A.1, but for the Low Scenario. The AIT = 1990 curve follows very closely the historical substitution trend up to 2040 at which the non-fossil fractional share is 63 percent. Afterwards it follows a saturation rate of about 3.8%/yr.

In the AIT = 2000 substitution process four trends are evident. The first, lasting until around the year 2005, represents the unconstrained penetration (i.e., no. CO_2 constraint) and shows slow penetration of about 1.4%/yr; the second extends to the year 2035 with a mean penetration of 8.1%/yr - a very high rate under all circumstances; the third stage is a transition step before long-term substitution processes prevail. This stage end by the year 2050; the long-term trend defines the fourth phase and follows closely the historical substitution trend.

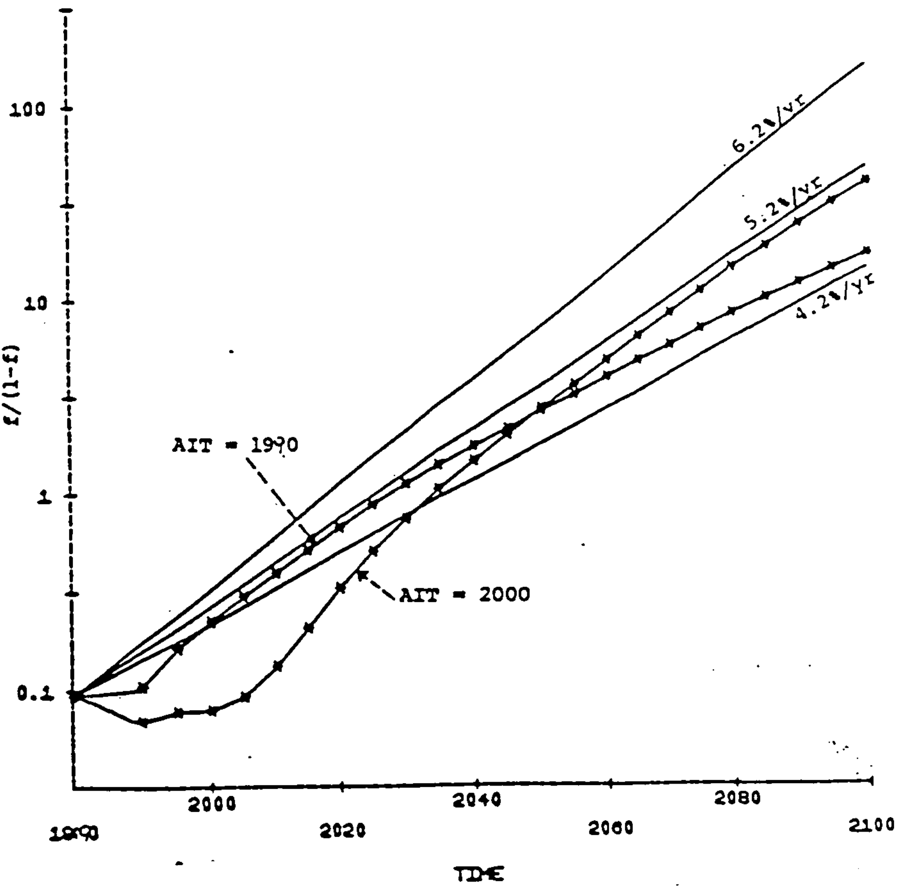


Figure A.2: Market Penetration analysis: Logarithmic plot of the transformation f/(l-f) where f is the non-fossil market share in total primary energy, here the Low Scenario, HC/LN with a CO_2 limit of 500 ppm. Smooth straight lines are logistic paths with constant penetration rate.

The previous discussion has been limited to a CO_2 limit of 500 ppm; but how about other limits, say, for the 600 ppm asymptote. The AIT = 1990 manifests two trends: 4.6%/yr extending to the year 2035 and 3.6%/yr after that. As before, the AIT = 2000 reveals four phases of penetration with annual rates: 1.6 percent ending by 2000; 5.8 percent ending by 2035; transition phase ending 2055; and 4.6 percent. These penetration rates are still high compared either to historical or IIASA projected trends.

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CHAPTER 2

ECONOMIC MODELS OF OPTIMAL ENERGY USE UNDER ENVIRONMENTAL CONSTRAINTS

2.1 INTRODUCTION

The major purpose of this chapter is to contribute to better policy making through improvements in models studying the economic impacts of the carbon dioxide problem, and to show ways in which economic instruments can effectively be put to use to alleviate such a problem.

This approach differs in at least one major aspect from common studies of the climatic change problem. We focus on the analysis, control and optimization of modelling forms rather than the collection and analysis of data. More concretely, we search for optimal fossil fuel use, research and technology policies rather than predicting the future. Most studies of the problem exogenously specify technical developments and fossil fuel control policies and then predict future climatic changes. These prediction models incorporate a great deal of data and tend to be quite complex.

The advantage of an optimizing control model is increased flexibility in structural and dynamic assumptions on the economy allowing explicit 'what-if' questions to be asked about the possibility of controlling the growth in atmospheric CO_2 concentrations.

Let us start by looking at a class of single state aggregate optimal control models. They allow consideration of static production but also technical change. In the latter case we take care of the fact that very small rates of ongoing technical change can have an enormous impact because of the very long time-span associated with the CO_2 problem, that is the 100-150 years until major effects occur.

In this class of single state models, the level of atmospheric CO_2 is the only state variable. The only policy or control variable chosen is fossil fuel use. (In a follow-up paper we will explore a class of multiple state models including additional state variables, stocks of physical capital and levels of knowledge).

The simplest type of technical change is a finite or limited improvement in a technology. Because such a change is not ongoing, the model remains static and relatively easy to examine. Ongoing but uncontrolled technical change is also examined with a single state model.

Some major policy conclusions can be derived from this class of model:

(1) One can show that depending on the assumptions regarding technical progress, the

optimal steady state CO_2 concentration may rise or fall with increases in the steady state level of progress. Notably, an improved substitute for fossil fuels always reduces the long-run level of atmospheric CO_2, while an improvement in fossil fuel productivity may increase or decrease the level of atmospheric CO_2.

(2) Solutions of a model with neutral, constant and ongoing technical progress and the basic static model are very similar. In the model with technical progress, higher levels of technical progress lead to lower long-run optimal levels of atmospheric CO_2.

(3) We examine two cases of international co-operation in controlling CO_2 accumulation. The base case is complete co-operation between two regions in maximizing consumption with complete awareness of the CO_2 problem. This case is compared with a situation in which no co-operation takes place until a critical CO_2 level is reached. Our most important finding is that in the non-co-operative situation the critical level is reached sooner, even though the region concerned about CO_2 always emits less carbon than in the base case.

(4) The last applications are on uncertainty. We first show that the results from studies of the optimal use of a resource which is in limited supply can be applied to the CO_2 problem. This similarity is important because the results regarding the use of limited resources are extensive and powerful. Then, using numerical examples, we show that an inappropriate treatment of risk can lead to significantly higher than optimal estimates of the desirable level of fossil fuel use.

2.2 ECONOMIC STUDIES ON THE CO_2 PROBLEM

Two major kinds of economic studies can be identified for dealing with the CO_2 problem. The first category treats economic and economic modelling issues in the context of an integrated framework of energy-economy and climate changes, the second category applies the theory of resource use and depletion to the management of CO_2 emissions.

In what follows we provide a selected overview of those studies which yield pertinent results comparable to our own or which are of methodological interest in modelling the energy-economy-climate interactions.

A few years ago the US National Research Council (1983) compiled a detailed investigation that up to now constitutes the most extensive, comprehensive and consistent examination of the climate change problem. It uses energy-economy, climate and agricultural models to predict future impacts of carbon dioxide and trace gas accumulation. The major conclusions are that no radical actions should be taken, that increases in carbon dioxide are likely, and that more research is necessary. In this report, the developers of the energy-economy model (W. Nordhaus and C. Yohe) note that the technology development and elasticity of substitution parameters critically affect the model's results.

It should be added that the method of modelling technical change and energy substitution possibilities is also critical and controversial. The economic modelling chapters of the NRC report have been updated by Yohe (1984). The conclusions of the report have not changed significantly.

Another collaborative study, the joint MIT-Stanford study (Rose, Miller, and Agnew, 1983), is of interest because it is one of few reports which search for alternatives to increasing CO_2 and offer some positive choices. In the Edmonds and Reilly model (Edmonds and Reilly, 1983) S-shaped paths are exogenously specified for several new energy technologies. The MIT-Stanford study modifies these paths and also looks at additional technologies. It finds that the adoption of realistic CO_2 reducing technologies, while not eliminating a significant CO_2 warming could increase the CO_2 doubling time to several centuries.

In attempting to discuss optimizing strategies W. Nordhaus (1980) made a seminal contribution by applying simple optimization models to the qualitative and quantitative analysis of the CO_2 problem. By letting the consumption equation depend on fossil energy use he determines the appropriate tax policy to control CO_2 and makes a quantitative estimate of this tax.

In most present studies of the CO_2 problem we find that technological change and technology substitution are specified exogenously or modelled in a very simple fashion.

In this regard, the logistic (S-curve) assumption on the diffusion of new technologies has become very popular, although it lacks sufficient economic explanatory power. For example, in a study by Perry et al (1982) an energy demand level and a fossil fuel use pattern is assumed. Fossil fuel use follows a logistic curve between the present and an assumed ultimate level of use. The rate of non-fossil energy growth needed to fill the gap, between fossil energy use and assumed total energy demand is then examined. This study emphasizes the importance of analysing the investment needed in non fossil energy to fill this gap but it does not present a model of the substitution process. In the model we suggest, the substitution process is a direct result of our maximization of welfare.

Modelling the impacts of changes in energy use on the economy is a major problem. A good starting point for such considerations would be the ETA-Macro model (Manne, 1977) though it has not been used for studying the CO_2 problem. This model can be described as a multisector, forward-looking model. It examines consumption and investment policies and their impact on national welfare. National welfare is measured by discounting utility from the present to a distant horizon. ETA-Macro consists of two models: a macro-model of the whole economy and a more detailed model of the energy sector. The model seems more sophisticated in its treatment of capital and the determination of the desirable level of energy use than those economic models presently used in CO_2 analysis. However, it is limited to the USA in geographic scope which makes it unsuitable for examining international problems such as CO_2. The model has no endogenous technical progress which we consider an important feature for the analysis of CO_2. On the other hand, the model has several features which would be desirable in models of energy, economy and the environment. It is optimizing and considers costs and benefits of capital investment.

Very recently, there has been a flurry of economic modelling and assessment studies of CO_2 effects, in particular by A Manne and R G Richels, that have appeared in the 1990 issues of the Energy Journal.

In setting up this approach we were influenced by applications of optimal control models to pollution problems (Fisher, 1981; Conrad and Clark, 1987). Such models often show different structures, e.g. pollution affecting utility or production, the pollutant acting as a stock or a flow and the way abatement activities are available.

In this specific context, we find that many models and results on the depletion of a non-renewable resource could be applied with simple modifications to the CO_2 problem. Under two assumptions the problem of fossil fuel use in the face of increasing carbon dioxide is parallel to the problem of consumption of a limited resource. The first assumption is that the carbon dioxide absorption rate is sufficiently small to be ignored. The second is that CO_2 impacts follow a "step" pattern: that is, CO_2 has no impact on productivity until a critical level, M_c, is reached. Then if the CO_2 level exceeds M_c production falls to zero, or remains stagnant. One of the most serious effects in facing global warming is that of irreversibility, that is, given the accumulation of atmospheric CO_2 we will reach a critical level of the CO_2 budget where there is a point of no return (unless technologies are in place that effectively remove CO_2 from the budget). The interpretation of a critical level of atmospheric CO_2 accumulation where there is a precipitous drop of production means that we have reached the biophysical limits of growth.

An interesting treatment of endogenous neutral technical progress in a depletion model was suggested by Chiarella (1980). He proves the existence of a steady state growth path and a simple rule governing the rate of investment in research. Research investment along the optimal path should be carried out until the growth rate in the marginal accumulation of technology equals the difference between the marginal product due to an extra unit of research investment and the marginal product of capital.

A similar problem is the use of a limited non-renewable resource when the reserve of the resource is unknown. The model by Gilbert (1979) can be directly converted to a model of fossil fuel use when the critical CO_2 level is uncertain. Under the above assumptions this problem is equivalent to determining the rate of fossil fuel use when the critical concentration of atmospheric carbon dioxide is unknown. The results show that the optimal use of fossil fuel is lower when uncertainty is properly considered than when the expected values are assumed to be certain.

Deshmukh and Pliska (1980, 1983) study more complex models of the same problem. The possibility of doing exploration to find new reserves is a significant addition in their models. The parallel in the CO_2 problem is research to increase the probability of finding a technology for the removal of CO_2 from the atmosphere. Their findings imply that in the periods between discoveries or research breakthroughs, fossil fuel use and consumption fall, but if research is very successful long-run fuel use may rise.

2.3 PRELIMINARY DEFINITIONS AND THE GENERAL MODEL

Our interest in model structure and policy options for energy policies leads us to the use of forward-looking, optimizing aggregate models. The class of models we are analysing here uses mathematical techniques of optimal control (Kamien and Schwartz, 1981). This requires: a measure of welfare or benefit as an objective function, and a definition of how policies, the control variables, affect specific aspects of the world, the state variables. A complete optimal control model of the CO_2 problem is an ideal rather than a reality, although outlining a detailed model provides a reference when examining simple models focusing on specific issues.

The basic ingredient of our model is an uncomplicated measure of welfare denoted by J. J is the sum over all time of the discounted flow of welfare. U is the utility or the flow of welfare at any instant. The social discount rate is r. Utility depends only on consumption, C. Consumption, production, and investment all have the same, single measure. Production is designated Y. There are numerous investment possibilities represented by the vector $\underline{I}$. At least two world regions are imagined and trade can occur between them. The vector of traded goods is $\underline{X}$, and the prices of the goods are in the vector p. Consumption equals production minus the sum of investments in capital goods plus the returns from trade. Production depends on current inputs, capital inputs, atmospheric CO_2, and the use of imports. Current imports are not stored and affect production as a flow. Capital inputs can be accumulated and affect production as a stock, $\underline{K}$. We distinguish two types of current inputs, fossil fuels, $\underline{F}$, and all other current inputs, $\underline{E}$. Atmospheric CO_2 is a single number denoted by M. The welfare measure, J, can be expressed as:

$$\max_{\underline{F},\, \underline{E},\, \underline{I},\, \underline{X}} \qquad J = \int_{0}^{\infty} e^{-rt}\, U(C(t))\, dt \tag{1}$$

Production and consumption are determined by

$$C = Y - \underline{1}\,\underline{I} + \underline{p}\,\underline{X} \tag{2}$$

$$Y = f(\underline{F}, M, \underline{E}, \underline{K}, \underline{X}) \tag{3}$$

and $\underline{1}$ is the unit vector.

We assume that one region has the leadership in world trade, and its decisions on trade influence world prices through a vector function

$$p = g(\underline{X}) \tag{4}$$

The regional use of fossil fuels, $\underline{F}_i$ also reacts to the trade policy, this reaction is determined by the vector function:

$$\underline{F}_i = h(\underline{X}) \tag{5}$$

The capital stocks and the level of atmospheric CO_2 are the state variables. Knowledge is treated as a special type of capital stock which does not depreciate. The current capital stocks and the CO_2 level may affect their own rate of change through depreciation and reabsorption respectively.

The dynamics of their change are expressed as follows:

$$[d\underline{K}/dt] \equiv \dot{\underline{K}} = j(\underline{I}, \underline{K}) \tag{6}$$

$$[dM/dt] \equiv \dot{M} = k(\underline{F}, \underline{F}_i, M) \tag{7}$$

The general model introduces many of the concepts to be elaborated: the importance of a welfare measure, feedback to production from increasing CO_2, changes in physical capital and knowledge, the ability to control these changes, and the impact of trade on fossil fuel and production. In its general form the model is too complex to solve for relevant results; therefore, we develop a set of simple models to examine these concepts. The simplification is achieved in two ways: by reducing the number of variables and restricting the functional forms.

2.4 A SIMPLIFIED MODEL

This model contains only the most important elements of the general optimal control model introduced in the previous section. As in the general model, the economy considered has only one consumption good, C, and a flow of utility. U(C), results from consuming this good. The objective is to maximize, J, which equals the utility flow discounted at the rate r and integrated from the present time, t=0, to infinity.

We assume that production of the good, C, depends only on the use of fossil fuels, F, and the level of carbon dioxide accumulated in the atmosphere, M. Control of fossil fuel use is the sole means of managing the economy. No decisions on investment and trade, as in the general model, are made. Thus, the assumptions are:

$$J^* = \underset{F}{Max} J = \underset{F}{Max} \int_0^\infty e^{-rt}\, U(C)\ dt \tag{1}$$

$$C = f(F, M) \tag{2}$$

We assume that production is finite at any finite level of fossil fuel use and that the production function is continuous in F.

The equation for change in the atmospheric carbon dioxide level is more specific than in the general model. We assume that each unit of fossil fuel use emits a fixed amount of CO_2 into the atmosphere; therefore, fossil fuel use and CO_2 accumulation can be measured in similar units. Finally, CO_2 leaves the atmosphere naturally at a rate proportional to the CO_2 concentration. The proportionality factor or "reabsorption rate" is α. We specify the relations which determine M:

$$dM/dt = F - \alpha M \tag{3}$$

$$M(0) = M_o \tag{4}$$

This model already contains three specific (economic) assumptions. First, neither emissions nor carbon dioxide accumulation impact utility (comfort or health) directly; the impacts occur through productivity changes. Second, the accumulated CO_2 is the pollutant not the rate of CO_2 emissions, differentiating CO_2 from such pollutants as SO_2 or particulates. Third we assume a very simple equation for CO_2 accumulation. Both the retention of CO_2 in the atmosphere and its reabsorption may in fact be nonlinear and change over time. The model differs from the earlier general model in two fundamental ways: there are no opportunities for investment and there is only one world region.

Several additional assumptions pertain to utility and production. Utility increases with consumption, but at a decreasing rate:

$$U' > 0 \quad \text{and} \quad U'' < 0 \tag{5}$$

where primes stand for derivatives.

Whatever the level of CO_2 accumulation, we assume that production increases with increases of fossil fuel use (close to zero) and that the rate of increase in production slows with additional fossil fuel inputs. But production declines with the accumulation of CO_2. Thus, the conditions hold with fossil energy being burnt.

$$\partial f(0,M)/\partial F > 0 \tag{6}$$

$$\partial^2 f/\partial F^2 \leq 0 \tag{7}$$

$$\partial f/\partial M \leq 0 \tag{8}$$

$$F \geq 0 \tag{9}$$

2.4.1 Necessary Conditions

First establish the Hamiltonian H:

$$H(F, M, \phi) = U(C) + \phi(F-\alpha M) \tag{10}$$

The Hamiltonian equals the utility flow plus the current value of increasing carbon dioxide concentrations. The adjoint value ϕ represents the marginal value of increasing CO_2 concentrations. The term λ F is added to form the Lagrangian L, and assures that $F \geq O$:

$$L = H + \lambda F \tag{11}$$

From (11) we derive the necessary conditions

$$\partial L/\partial F = U'(\partial f/\partial F) + \phi + \lambda = 0 \tag{12}$$

$$d\phi/dt = r\phi - \partial L/\partial M = (r+\alpha)\phi - U'(\partial f/\partial M) \tag{13}$$

$$\lambda F = 0, \lambda \geq 0 \tag{14}$$

Since (12) implies that ϕ is less than zero, this agrees with our intuition that the value of increasing CO_2 should be negative. In order to deal with a CO_2 induced cost, we define q equal to -ϕ, and q can be referred to as the shadow price of CO_2 emissions. We can now restate (12) and (13) as

$$U'(\partial f/\partial F) - q + \lambda = 0 \tag{15}$$

$$dq/dt = (r+\alpha)q + U'(\partial f/\partial M) \tag{16}$$

or

$$q = \int_t^{\infty} -e^{-(r+\alpha)(\tau-t)} U'(\partial f/\partial M)\, d\tau$$

(15) and (16) are crucial conditions that lend themselves immediately to economic interpretations. (15) implies that fossil fuels are used up to the point at which the marginal contribution to utility equals the shadow price, unless fuel use is forced to zero. Unless the marginal utility is increasing, shadow price increases will drive down the use of fossil fuels. The condition in (16) is easily understood in its integral form. Increases in atmospheric CO_2 lower productivity and thus cause a disutility. The cost of CO_2 is the discounted sum of the marginal harm or disutility due to an increase in CO_2. We discount at the rate α because a

unit of CO_2 emitted presently disappears at the rate of α, and we discount at r to put future losses on a present value basis.

A further necessary condition determines maxima or minima.

By defining $U_{ff}=U'(\partial^2 f/\partial F^2)+U''(\partial f/\partial F)^2$ (17)

the necessary condition for a maximum can be stated as

$$\partial^2 H/\partial F^2=U_{ff}\leq 0 \tag{18}$$

2.4.2 Sufficient Conditions

We assume that the first and second derivatives of the production function are defined. (In problems where the derivatives are not defined sufficiency must be proved by other methods)[1]. We use sufficiency conditions that require that the necessary conditions are met and that the utility function is jointly concave in M and F. The proof only applies when the state equation (3) is linear in F and M.

The definition in (17) and the two following makes the statement of concavity more concise:

$$U_{mm}=U'(\partial^2 f/\partial M^2)+U''(\partial f/\partial M)^2 \tag{19}$$

$$U_{fm}=U'(\partial^2 f/\partial F\partial dM)+U''(\partial f/\partial F)(\partial f/\partial M) \tag{20}$$

The concavity conditions can be stated as follows:

$$U_{ff}\leq 0 \tag{21}$$

and

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[1] In finite horizon problems with a structure similar to (1) - (9) an additional necessary condition determines the value of the adjoint variable, ρ or ϕ, at the terminal time. Halkin (1974) showed that the simple condition on the adjoint variable in the finite horizon case does not extend to the infinite horizon case. However, there exist conditions on the shape of the function to be maximized and the state equation which in combination with (14), (15) and (16) are sufficient to determine an optimum.

$$U_{ff}U_{mm}-(U_{fm})^2 \geq 0 \tag{22}$$

From (22) to assume concavity of the utility function the second partial derivative of f with respect to M must be less than or equal to zero, this assumes that U_{mm} is less than or equal to zero. This requirement matches the assumption of many scientists that CO_2 impacts will accelerate at higher levels of CO_2. Another sufficiency condition is simply that q(t) does not get too big. This condition can be stated as:

$$e^{-rt}\, q(t) \rightarrow 0 \quad \text{as } t \rightarrow \infty \tag{23}$$

The condition is satisfied if q has a finite equilibrium value or if q grows at a rate less than r. The conditions in (15), (16), (21), (22) and (23) are sufficient to assume that an optimal path for the control variable has been found.

2.4.3 Definition and Optimality of Equilibrium

In the context of controlling fossil fuel use, the notion of equilibrium is interesting because it predicts the distant future, indicates the general direction of movement from the present to the long run.

The equilibrium is defined in terms of q and M. To specify the equilibrium conditions we assume that for F greater than zero that F can be found (as a function of q and M from (15) and (16)). This function is specified as $\psi(q,M)$. F is constant when q and M are constant. From (3) if M is constant

$$\psi(q,M)-\alpha M=0 \tag{24}$$

Equation (24) defines combinations of shadow price, q, and CO_2 concentration M which keep the concentration of CO_2 constant. M is greater than zero, therefore, from (24), F is greater than zero in equilibrium.

Setting dq/dt equal to zero over time in (16) and substituting for F gives a second condition on the equilibrium

$$(r+\alpha)\,q+U'\left(\partial f(\psi(q,M),M)/\partial M\right)=0 \tag{25}$$

(25) constitutes something like a price/damage equation. It assures that in equilibrium the higher the incremental damage of CO_2 the higher the price of fossil fuels.

If the equilibrium satisfies (15), (16), (21), (22) and (23) it is optimal. q is finite in the equilibrium satisfying the sufficiency condition in (23). If we assume that the production function has a curvature there satisfying (21) and (22) we know the equilibrium is optimal.

Equilibrium condition (25) can be restated in terms of the more familiar rate of substitution

$$-(\partial f/\partial F)\,/\,(\partial f/\partial M)=dM/dF=1/(r+\alpha) \tag{25'}$$

Along an isoquant dM/dF, the slope of the isoquant equals the negative of the ratio in (25'). In Figure 1, the curve ab is the locus of all points such that $dM/dF = 1/(r + \alpha)$ along an isoquant. These points represent an efficient balance between the marginal gains from increased emissions and losses from increased atmospheric CO_2. $1/(r + \alpha)$ can be thought of as the price ratio of the value of increased fossil fuel use to decreased CO_2 levels. OC is the line along which F equals α M, the set of stationary points. The equilibrium is at the intersection of curves ab and OC in the figure. The line OC always has a steeper slope than dM/dF because r is greater than zero. Higher values of r lower dM/dF and move the equilibrium along OC. Because of the relative slopes of ab and OC, this means we move to higher levels of atmospheric CO_2 and lower levels of long-run production as r increases. Not surprisingly, a high discount rate causes us to value long-run consumption less.

2.4.4 Illustration by a Phase Plane Diagram

For conventional production functions, the equilibrium can be represented in a phase plane diagram as in Fig 1a. The phase plane diagram is a valuable tool not only because it shows the equilibrium but because it shows the changes in variables over time. In Appendix A, we precisely define the stability and existence conditions which assure that Fig 1a represents the equilibrium, that a path to the equilibrium exists, that the path is unique, and that therefore an optimal equilibrium is a long-run optimum.

Equations (24) and (25) are plotted in Fig 1a as curves AB and CD respectively. On the left-hand side of Fig 1a the curve AB lies above CD; the curves slope toward each other and intersect at equilibrium, 0. The relation between the curves results from our previous assumptions regarding the slopes of the production function, f(F, M). If the shadow price is above AB, fossil fuels are relatively expensive, fossil fuel use is reduced, and the CO_2 level increases for the opposite reasons.

These movements are indicated by the small arrows in the diagram. To maintain the steady state, that is to satisfy (24), fossil fuel use must be low when the CO_2 level, M, is low and high when M is high.

Therefore, along AB when M is near zero the steady state shadow price, q, is large lowering fossil fuel use, and, at higher values of M, q is lower. Curve CD traces the "price vs harm" equation, (25). Along this line the shadow price of CO_2 emissions equals the long-run harm due to a marginal increase in CO_2 concentration. To the left of this curve, dq/dt is positive and to the right dq/dt is negative, again illustrated by the arrows. If the CO_2 concentration is low, M to the left of CD, the harm due to CO_2 is low relative to the future impacts; therefore, the shadow price of emissions is increasing. The opposite effects occur to the right of CD. With low historic levels of atmospheric CO_2, CO_2 increases have caused little harm. This implies that, at low levels of M, the marginal harm due to CO_2 is low; and to satisfy (25), the shadow price, q, must also be low. As M increases along the "price vs harm" curve, CD, q also increases. The curved arrows in the phase plane diagram describe the change in variables over time. When the equilibrium meets the sufficiency conditions noted earlier and the phase plane can be illustrated as in Fig 1a, it is optimal to choose the unique q so that the level of atmospheric CO_2 increases monotonically towards the equilibrium from levels of CO_2 less than the equilibrium. FO in Fig 1a represents such an optimal path. This means that both the shadow price, q, and atmospheric CO_2, M, increase with time. The use of fossil fuels, F, decreases monotonically, as can be seen by taking the total differential of (15).

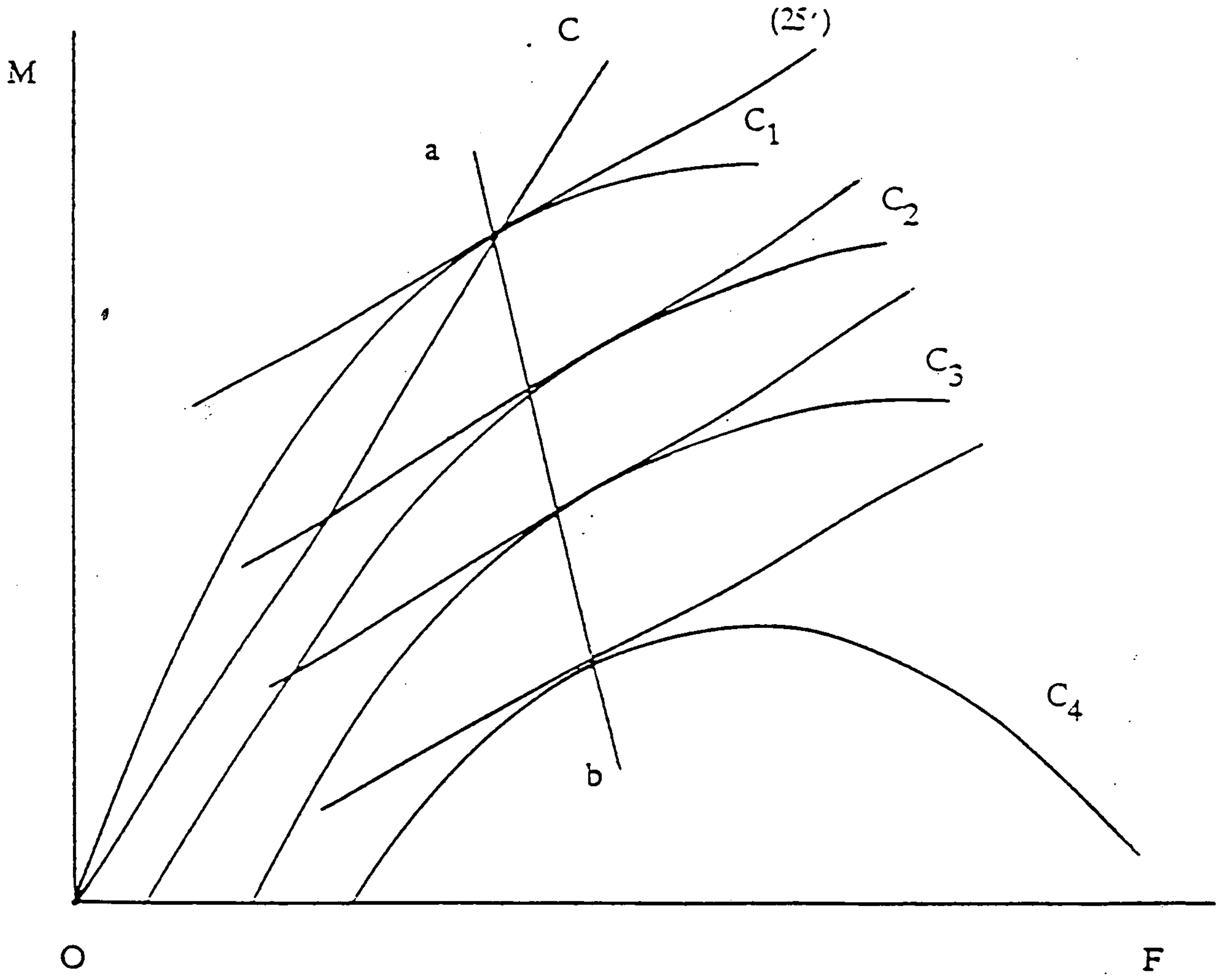

Figure 1 Solution of Model

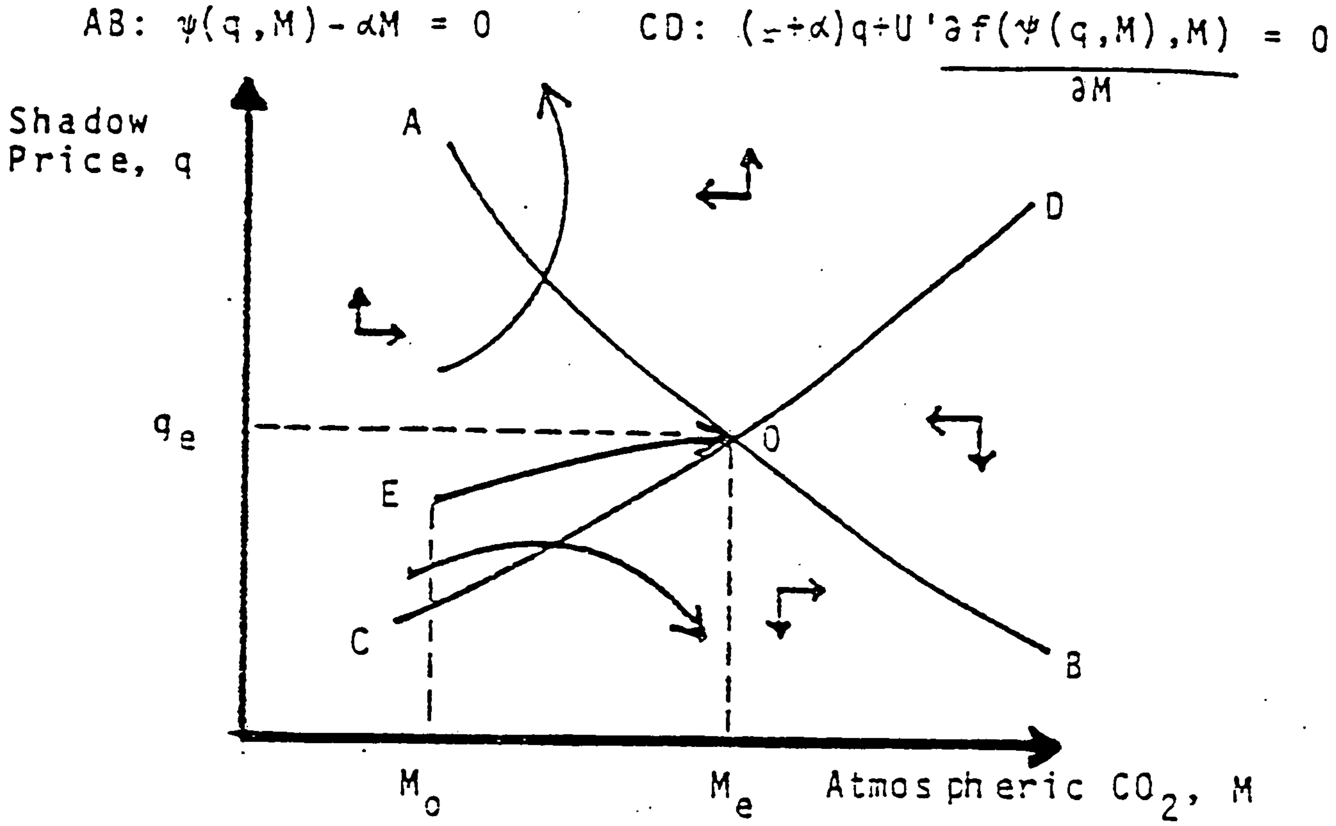

Figure 1a Phase Plane Diagram

2.5 A DISCRETE TYPE IMPACT OF CO_2 EMISSIONS

In this section we analyse, as a more specific form of the model, the negative impact of CO_2 emissions that occur abruptly at specific levels of atmospheric CO_2.

In the simplest way we describe how consumption C is affected by CO_2 accumulation:

$$C=\begin{cases} f(F)\,, M \leq M_c \\ 0, \quad M > M_c \end{cases} \tag{1}$$

f being a production function with $f'(0)>0, f''<0$ which means that, at zero fossil fuel use, fossil fuels are always productive, although they may be unproductive at higher levels of use. The basic assumption is that carbon dioxide has no impact on production until it reaches a critical level M_c. First assume that if CO_2 levels exceed M_c, production falls to zero and the step function's simplicity has very drastic consequences. Damages from CO_2 accumulation could rise rapidly at a critical level.

The necessary conditions for the basic problem when C is given by (1) are:

$$q=\begin{cases} U'f', M \leq M_c \\ 0, \quad M_c < M \end{cases} \tag{2}$$

$$\frac{dq}{dt}=(r+\alpha)\,q, M \neq M_c \tag{3}$$

Because the optimization is over an infinite horizon, there is no simple necessary transversality condition on the adjoining variable, q.

The problem is further complicated because the derivative of the Hamiltonian with respect to M does not exist at M_c. However, a careful analysis of the problem can determine q and F at M_c. If the function f gives a maximum at F_m and F_m is less than or equal to α M_c, there is a simple answer. Producing at the maximum for all time creates the highest possible utility. If producing at the maximum never raises M above M_c, we set q equal to zero and F at F_m. Both (2) and (3) are then satisfied. If F at the maximum of f is greater than αM_c or if f has no maximum, the solution is more complex. But it can be shown that it is always optimal to use the entire CO_2 capacity, that is burn fossil fuels in a manner which raises atmospheric CO_2 to the critical level M_c.

The analysis first determines that F equals αM_c when M equals M_c. Second, q is determined by (2), (3) and the additional necessary condition that q(t) is a continuous function (except where M is constrained).

The calculation of q is simplified by finding T in the time horizon at which M equals M_c. The existence, uniqueness and finiteness of T is then proved which directly leads to the conclusion that the optimal solution exists and is unique.

The following heuristic considerations lead to this result. If M equals M_c, F must be less than or equal to αM_c; if not, M becomes greater than M_c and production drops to zero. The properties of U and f assure that (2) can be inverted and a function g, with g(q) equal to F exists, which is monotonically decreasing in q. (2) further assures that q is greater than or equal to zero. (2) and (3) together assure that dF/dt is less than or equal to zero for M less than or equal to M_c. If F is strictly less than αM_c, then αM_c, or equivalently, q is strictly greater than $U'f'(\alpha M_c)$, F would remain less than αM_c for all times. Such a path is dominated by many alternative paths including F equals αM_c. Therefore, F equals αM_c when M equals M_c. This is pretty much in the spirit of Krelle's (1987) description of an ecological equilibrium though obtained from different model reasoning. These observations can be summarized in a proposition whose statement and proof is left for the Appendix B.

T is unique and exists. Assuming an optimum exists, it follows that the q_o, F and M which satisfy the necessary conditions are all unique, exist and are optimal. Figure 2 illustrates typical paths of fossil fuel use and CO_2 accumulation in the step model. M rises and F falls over time, both reach their equilibrium values at T.

The assumption that production falls to zero when the critical CO_2 level is reached is very extreme. We describe a model of slightly greater complexity that avoids this extreme assumption:

$$C=\begin{cases} f(F)\ , & M \le M_c \\ \beta f(F)\ , & M > M_c \end{cases} \tag{4}$$

where $\beta \le 1$.

In the previous model we did not require that f have a maximum. However, in this model we must assume a maximum of f exists or the integral in the Appendix B equation

(B3) is unbounded. We assume that F_m greater than αM_c maximizes f.

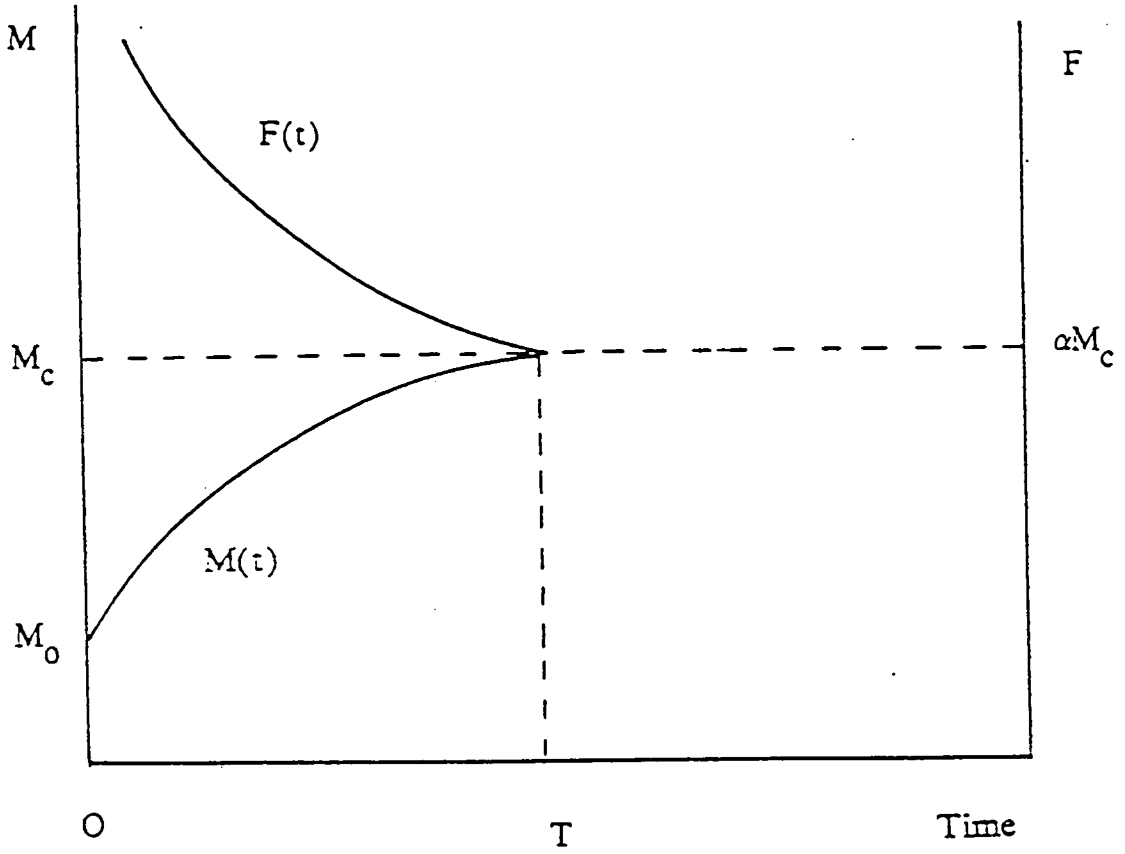

Figure 2 Illustration of Paths of F and M in Step Model

The necessary conditions are

$$q=\begin{cases} U'f', M \leq M_c \\ \beta U'f', M > M_c \end{cases} \tag{5}$$

$$\frac{dq}{dt}=(r+\alpha)\,q, \qquad M \neq Mc \tag{6}$$

In addition q(t) must be continuous.

Again the discontinuity of production at M_c complicates the problem. However, by examining paths of q which satisfy the three necessary conditions the possible solutions can be reduced to two.

If an optimum exists (5) assures that q is greater than or equal to zero; further, q is less than or equal to $U'f'(\alpha M_c)$ as shown in the previous case. If the value of q at time zero, q_o, is zero, then (6) and continuity assure that q equals zero for all t. F equals F_m for all t, M equals M_c at some finite time, and M goes to F_m/α as t goes to infinity. If q is greater than zero, q rises exponentially until q equals $U'f'(\alpha M_c)$. q rising further is non-optimal; therefore, M must equal M_c when q equals $U'f'(\alpha M_c)$. The problem is identical to that presented in the previous case.

Consider the optimal action when M_o equals M_c. F equals either αM_c or F_m for all time; therefore utility flow will be constant for all time at either $U(f(\alpha M_c))$ or $U(\beta f(F_m))$. The value of F which maximizes output will be optimal. If $f(\alpha M_c)$ is greater than $\beta f(F_m)$, F equals αM_c is optimal. If $f(\alpha M_c)$ equals $\beta f(F_m)$, the actions are equivalent. If $f(\alpha M_c)$ is less than $\beta f(F_m)$, F equals F_m is optimal.

Figure 3 illustrates both the production function and the optimal solution. We consider two step sizes β_1 and β_2 . As illustrated by the dotted line (Curve A) and along the left axis, $\beta_1 f(F_m)$ is greater than $f(\alpha M_c)$; therefore it is optimal to use fossil fuels at F_m. As illustrated by the dashed line (Curve B and along the left axis), $\beta_2 f(F_m)$ is less than $f(\alpha M_c)$, therefore, in this second case long-run fossil fuel use is αM_c.

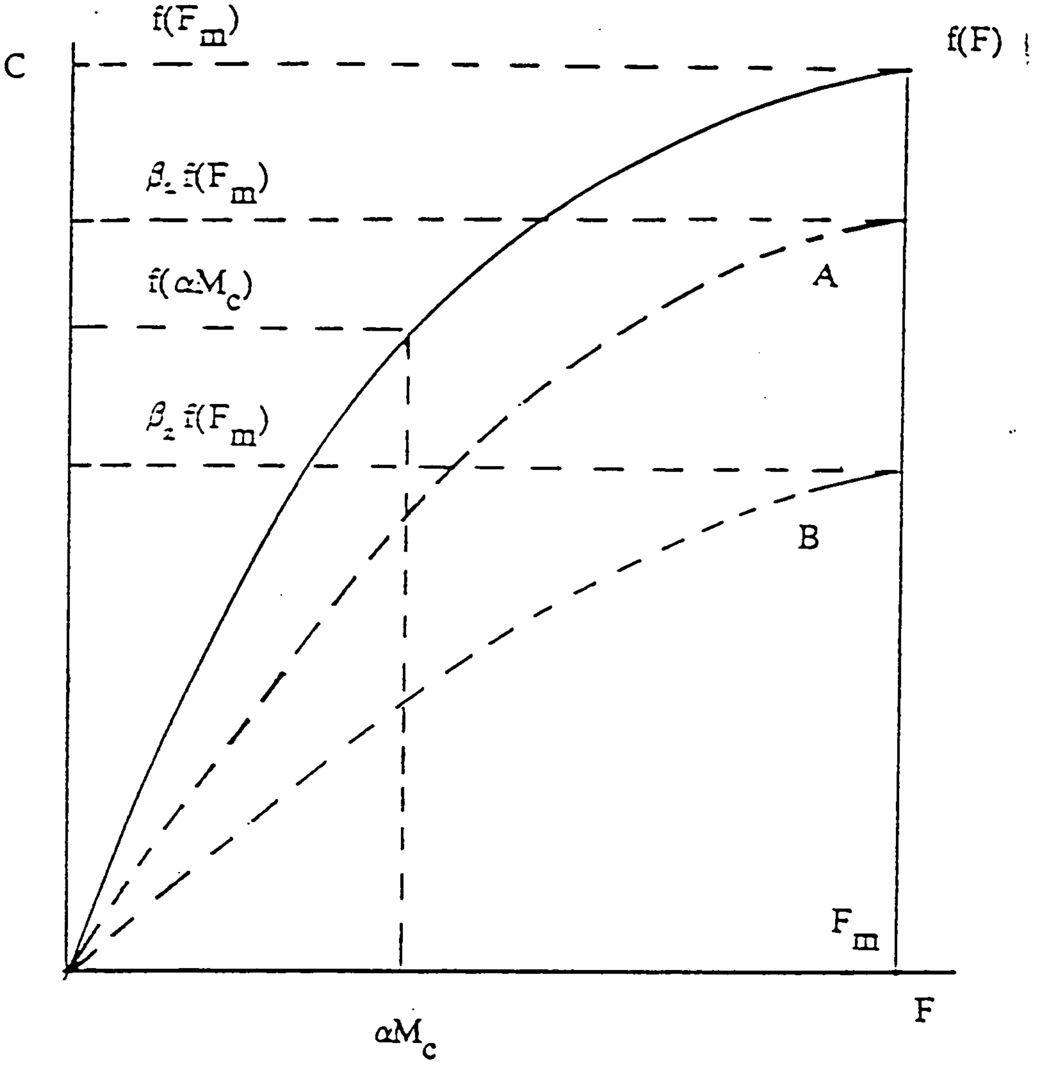

Figure 3 Two Levels of CO_2 Impact

2.6 FURTHER SPECIFICATION OF THE MODEL

Further specification of the model allows the sensitivity analysis of particular variables. Two major results emerge: an approximation of F as a function of M and a demonstration of the importance of the productivity and efficiency in the fossil fuel sector in determining the present optimal fuel use. We specify U(C) and f(F) by:

$$U(C)=\begin{cases}\frac{C^{1-\gamma}}{1-\gamma}, \gamma\neq 1\\ \ln C, \gamma=1\end{cases} \tag{1}$$

$$f(F)=\lambda F^{\varepsilon} \tag{2}$$

These functional forms have all the properties formerly assumed for U(C) and f(F), lead to a simple solution, and are familiar in the economics literature. The elasticity of consumption and production with respect to fossil fuel use (i.e. the per cent change in consumption and production over the per cent change in fossil fuel use) is constant at ε, referred to as the fossil fuel productivity factor.

The utility function is sometimes referred to as the constant relative risk aversion utility function. γ is the consumption elasticity of utility or the relative risk aversion. Equation 5 (2) takes the form:

$$\left.\begin{matrix} M\leq M_c, \varepsilon\lambda F^{-(1-\varepsilon)(1-\gamma)} \\ M>M_c, \quad 0 \end{matrix}\right\}=q \tag{3}$$

Then using the other necessary conditions, F(t), as a function of αM_c is found:

$$F(t)=\alpha M_c e^{A(T-t)}=F(0)e^{-At} \tag{4}$$

where

$$A=\frac{r+\alpha}{1-\varepsilon(1-\gamma)} \tag{5}$$

Note that F(t) is independent of the scaling factor λ .

Equation (4) for F(t) can be substituted into equation (B1) (Appendix B) and a simple expression in T derived:

$$\frac{\alpha M_c}{(A-\alpha)M_o}\left(e^{(A-\alpha)T}-1\right)+e^{-\alpha T}(M_c/M_o)=0 \tag{6}$$

A comparative statics analysis of (6) shows that: T decreases when the social discount rate or the fossil fuel productivity, r and ε respectively, increase: T increases when the consumption elasticity of utility, γ increases; and the change in T with changes in the rate of CO_2 absorption α, is indefinite.

The key equation is the simple approximation for F, the emissions policy, derived from (4).

$$F=A(M_c-M)+\alpha M \tag{7}$$

This shows that unless A is much greater than α. the present policy is sensitive to all the model's parameters. A comparative statics analysis of (7) shows how the fossil fuel use pattern reacts to parameter changes within the range of interest. Highlights of the main impacts can be summed up:

- Because of the lower value of future consumption when the discount rate is high, initial fossil fuel use will increase with increases in the social discount rate.

- A higher consumption elasticity of demand, γ , tends to reduce present fossil fuel use. In general, a higher γ moves the economy to a more uniform consumption pattern. In this case, since consumption is falling, a higher consumption elasticity of demand reduces present fossil fuel use and increases future fossil fuel use.

- An increase in reabsorption, α, increases present fossil fuel use. When reabsorption is high, present emissions of CO_2 have less impact on future atmospheric CO_2 levels, encouraging higher present fossil fuel use.

- The initial use of fossil fuels increases with increases in the fossil fuel productivity factor, ε. This can be best understood by relating the fossil fuel productivity factor to the price elasticity of demand for the fossil fuel input which we call σ. σ equals $1/(1-\varepsilon)$, thus in this model high productivity implies high price elasticity of demand. From 5(6) the shadow price of fuels increases at a rate dependent only on the social discount rate and CO$_2$ reabsorption; therefore, the higher the price elasticity the more rapidly the fossil fuel use declines. A high rate of decline implies high initial use and a rapid approach to the equilibrium.

Values for T and $F(0)/M_c$ for a variety of parameter values are given in Table 1. The table shows the very significant impact which the fossil fuel productivity factor has on the optimal present level of fossil fuel use and the time to equilibrium. A high productivity factor results in very high fossil fuel use and a very short time to equilibrium. This suggests both that if the productivity is high it is not optimal to stringently restrict present fossil fuel use and that if fossil fuel productivity is low, it is important to conserve fossil fuel and delay the time at which very low levels of fossil fuel use are necessary.

Table 1: T and F(0) for a Variety of Parameter Levels

$\varepsilon(1-\gamma)$	α	r	M_c/M_0	T	$F(0)/M_c$
0.5	0.001	0.02	2	77.44	0.0205
0.95	0.001	0.02	2	13.80	0.1915
0.5	0.002	0.02	2	69.87	0.0210
0.95	0.002	0.02	2	12.48	0.1830
0.5	0.001	0.06	2	34.48	0.0605
0.95	0.001	0.06	2	5.40	0.0592
0.5	0.002	0.06	2	28.96	0.0610
0.95	0.002	0.06	2	4.88	0.5830
0.5	0.001	0.02	8	91.25	0.0351
0.95	0.001	0.02	8	15.26	0.3344
0.5	0.002	0.02	8	75.51	0.3525
0.95	0.002	0.02	8	14.01	0.3188
0.5	0.001	0.06	8	39.12	0.1051
0.95	0.001	0.06	8	5.88	1.0344
0.5	0.002	0.06	8	33.59	0.1053
0.95	0.002	0.06	8	5.36	1.0188

2.7 AN EXTENDED COBB-DOUGLAS MODEL

In the discrete type model the impacts of CO_2 manifest themselves suddenly, in contrast, an extended Cobb-Douglas type model shows the negative effects occurring gradually. We use the dfinition of utility presented in 6(1). Again, the model assumes that a critical level of CO_2 accumulation, M_c, exists at which production drops to zero.

M_c is used to define two variables, F_s and M_s: $M_s = M_c - M$ and $F_s = F - \alpha M_c$.

M_s gives the distance to the critical level of CO_2 concentration, and F_s is the distance from the level of emissions which maintains the critical CO_2 level. Consumption increases with increases in either of these variables. The production/consumption function is:

$$C = F_s^{\epsilon} M_s^{1-\epsilon} \tag{1}$$

We refer to this model as an extended Cobb-Douglas model because equation (1) is a Cobb-Douglas form. From (1) it is clear that it will never be optimal to exceed the criticl level of atmospheric CO_2 and that the CO_2 level will always be increasing because F will be greater than αM_c.

An assumption inherent in (1) is that the economy produces less and less over time.

M_c and ε are the significant parameters of the model. For F much greater than αM_c, ε is approximately the fossil fuel elasticity of production, however, as F approaches αM_c the elasticity of production, however, as F approaches αM_c the elasticity of production with respect to fossil fuel use approaches infinity. The model is limited in that the elasticity of production with respect to CO_2 concentration is completely determined by ε. Using the variables M_s and F_s, the equation that describes the dynamic changes in M can be restated as:

$$d\,M_s/dt = -(F_s + \alpha M_s) \tag{2}$$

The necessary conditions for the problem are:

$$\epsilon C^{-\beta}\ [M_s/F_s]^{1-\epsilon} = q \tag{3}$$

with β as the consumption elasticity of utility.

$$\frac{dq}{dt} = (r + \alpha)\, q - (1-\varepsilon)\; C^{-\beta}\; [F_s/M_s]^{\varepsilon} \tag{4}$$

In the model, F_s and M_s equal zero at the equilibrium or steady state. At the equilibrium, there is no production or consumption. Marginal utility is infinite while total utility is minus infinity (for $\beta<1$). If the initial CO_2 level is less than the critical level, it is not optimal to reach the equilibrium. Fortunately, because of the familiar form chosen, we can guess a solution for the transition path. We assume that M_s changes at an exponential rate, g: Therefore,

$$M_s = M_s(0)\, e^{gt} \tag{5}$$

$$F_s = F_s(0)\, e^{gt} = -(g + \alpha)\; M_s(0)\, e^{gt} \tag{6}$$

$$C = C(0)\, e^{gt} = (-(g + \alpha))^{\varepsilon}\, M_s(0)\, e^{gt} \tag{7}$$

From (6) and (7) it is obvious that this solution can only be optimal if g plus α is less than zero.

Examining (3) we see that q changes at a rate of $-\beta g$. Using (3), (4) can be rewritten as

$$\frac{dq}{dt} = [r + \alpha\; \frac{(1 - \varepsilon)}{\varepsilon}\; \frac{F_s}{M_s}]\, q \tag{8}$$

By dividing q through (8), the left hand side becomes the rate of change in q, and the following expression is derived

$$-\beta g = r + \alpha\; \frac{(1 - \varepsilon)}{\varepsilon}\; \frac{F_s}{M_s} \tag{9}$$

By substituting for F_s/M_s and rearranging terms, we derive

$$-g = \frac{r\varepsilon + \alpha}{1 - \varepsilon(1 - \beta)} \tag{10}$$

Because the parameters are positive the numerator is always positive in (10), and because ε and $1-\beta$ are less than one the denominator is always positive. g is therefore always negative. This assures that consumption is always decreasing and that the integral of discounted consumption is limited. If g satisfies (10) the assumed solution is optimal. The similarity between the equation for g and the equation for A, 6(5), is obvious. If we rewrite (6) as

$$F_s(t) = F_s(0)\, eg^{t} \tag{11}$$

the similarity between the path of emissions in this model and that in the discrete-type model (expressed in 6(4)) is evident. The fossil fuel policy can also be expressed in a manner parallel to equation 6(7).

$$F = -g(M_c - M) + \alpha M \tag{12}$$

In this model, unlike the step model, the equilibrium is not reached in finite time. The restrictions of $\int$ and F to changes in the parameters are the same as the restrictions of A and F in the previous model. Increases in r, ε and α all increase the present level of fossil fuel use. An increase in β decreases the present use of fossil fuels.

2.8 DISCUSSION AND PERSPECTIVE

Simple step and Cobb-Douglas type models of CO_2 - economy interactions have been analyzed. Consumption equals F^ϵ in the step model and $AF_g^\epsilon M_g^{1-\epsilon}$ in the Cobb-Douglas model. For the Cobb-Douglas model, the consumption-fossil fuel use relation changes at different CO_2 levels.

In both models there is a critical carbon dioxide level at which consumption and production drop to zero, and this critical level is the equilibrium level of CO_2 accumulation. Furthermore, the optimal fossil fuel use path is a declining exponential. The level of present fossil fuel use responds to parameters similarly in the models: present fossil fuel use increases with increases in the discount rate, the rate of CO_2 absorption, and the elasticity of production with respect to the fossil fuel measure. The fossil fuel use reaches equilibrium in a finite time.

Small and simple models of this kind are useful for generating ideas, supplementing large-scale models, and improving modelling approaches.

Several areas of economic research will be particularly important to CO_2. One important area is the effectiveness of research support in changing the mix of energy technologies. Another area of research is the use of market controls such as excise taxes, export limits, subsidies and cartels; to reduce or increase the use of a commodity.

The CO_2 problem should be considered in determining the type of research in the energy area which government supports. From a CO_2 perspective, economic incentives for the development of coal and oil shale are questionable. It appears that improvements in all non-fossil fuels do not necessarily lower future levels of CO_2. To displace fossil fuels, alternative non-fossil fuels must be highly substitutable. This study suggests two other factors which policy makers should be aware of when considering the economics of CO_2. As with many other environmental problems, any policies which increase the perceived discount rate will exacerbate the CO_2 problem by reducing our concern about the future. The shadow price attributed to fossil fuels by expected value economic models will be lower than the true shadow price for a risk averse society.

In the remainder of this chapter we pursue policy-directed applications of the simple models proposed so far for a range of three major issues: technical progress, international co-operation, and uncertainty. Each of these issues will be treated in more detail in separate chapters.

Because of technical progress affecting energy production or finding mitigating strategies against CO_2 emissions (climatic engineering) the CO_2 problem may be highly sensitive to changes of technical progress parameters.

The worldwide nature of climate change, the dispersion of major fossil fuel resources, and the variation in possible effects of climate change all suggest the importance and problems of international co-operation in developing a CO_2 control policy.

Enormous uncertainty surrounds the future levels and effects of atmospheric CO_2 (see The Energy Journal, 1991). A specific type of uncertainty, structural uncertainty, associated with the modelling process has impacts on present optimal policies which have not been previously considered.

2.9 A TAXONOMY OF TECHNICAL CHANGE

The long-run level of atmospheric CO_2 depends on both the degree of technical change and its form. In the following cases, technical change varies in its impacts on the productivity of inputs and the reabsorption of CO_2. We show that these differences create very different incentives for the long-run use of fossil fuels.

In an ongoing system of technical progress, the system of state equations has an additional parameter S, e.g. knowledge. Let a function of three variables represent consumption.

$$C = f(F, M, S) \tag{1}$$

In this system of equations, the existence of equilibrium, its optimality, and stability are governed by the same conditions as in the previously presented simple models.

We examine several kinds of technical change. In all cases the equilibrium is examined under the previously stated conditions.

1. Neutral technical progress

$$f(F, M, S) = S\, h(F, M) \tag{2}$$

We refer to this as a neutral technical progress because the ratio of this partial derivative of f with respect to F and the partial derivative of f with respect to M is independent of the level of technical progress.

Examining condition 4(25') the constancy of this ratio under different levels of neutral technical progress assures that the long-run carbon dioxide concentration is invariant.

2. Development of non-fossil, substitutable energy

Along this line the production function represents technical progress in the development of a completely substitutable non-fossil fuel:

$$f(F, M, S) = h(F + S, M) \tag{3}$$

The equilibrium condition is the same condition as in 4(25') except that it is evaluated at F + S and M:

$$-[\partial h(F+S,M)/\partial F]/[\partial h(F+S,M)/\partial M] = dM/dF = 1/(r+\alpha) \tag{4}$$

dM/dF is defined along an isoquant of h.

The immediate impact of the level of technical progress can be found by comparative statics or by a graphic examination of the solution. By neglecting details it shows that more technical progress reduces the equilibrium level of CO_2 and fossil fuel use.

3. Removal of atmospheric CO_2

Removal of CO_2 changes the dynamics of CO_2 accumulation. Let S be the rate of CO_2 removal, the equation for the change in atmospheric CO_2 is:

$$dM/dt = F - (\alpha M + S) \tag{5}$$

The equilibrium condition is then $F = \alpha M + S$, and this case parallels the previous case with a similar interpretation.

4. Fossil fuel enhancing technical progress

Changes in fossil fuel productivity would result in a future production function:

$$f(F, M, S) = h(SF, M) \tag{6}$$

In equilibrium the following equation must hold along an isoquant of h

$$-S[\partial h(SF,M)/\partial F]/[\partial h(SF,M)/\partial M] = S(dM/dF) = 1/(r+\alpha) \quad (7)$$

Because the slopes of the isoquants may increase or decrease at any given value of F, the locus of points satisfying (7) may move to the right or left. The change in the equilibrium level of atmospheric CO_2 is, therefore, uncertain.

5. Emissions Purification

Scrubbing CO_2 from emissions changes the dynamics of CO_2 accumulation as

$$dM/dt = F/S - \alpha M \quad (8)$$

The equilibrium condition is then $F = S\alpha M$. This case corresponds to Case 4, and the same conclusion holds. The equivalence can be seen if the effects of substituting a variable F_o equal to F/S into the production function and (8) are considered.

6. Amelioration of CO_2 impacts (generated by forms of climatic engineering)

One representation of relieving CO_2 impacts would change the production function as

$$f(F, M, S) = h(F, M - S) \quad (9)$$

This case is similar to 2. Consequently, the long-run equilibrium level of CO_2 is raised with higher levels of technical progress.

A more general taxonomy is possible for production functions that are invariant under some general transformations to be called "neutral technical progress" (Sato, 1981). Such a taxonomy could translate itself into specific technology induced policies like reforestation, recovery of CO_2 from power plants, storage in the oceans, disposal in depleted gas reservoirs, energy technology substitution etc (Okken et al, 1989).

In the context of a narrower focus, e.g. energy technology substitution, very recent studies by Manne and Richels (1990), on the basis of their Global 2100 model, Have given rise to assessment of a broad spectrum of CO_2 benign technologies.

2.10 IMPACTS OF TECHNOLOGICAL CHANGE IN CO_2 EMISSION CONTROL

It has been shown that the long-run level of atmospheric CO_2 depends on both the degree of technical change and its form. A taxonomy of technologies has been given that vary in their impacts on productivity of inputs and the reabsorption of CO_2. Here we present a diagrammatic analysis on the impacts of technological change in CO_2 emission control.

As a classification we look at the following major effects of finite technical progress:

1. non-fossil substitute: $f(F,M,S) = h(F+S,M)$ (Sec. 9.2)
2. fossil fuel enhancement: $f(F,M,S) = h(FS,M)$ (Sec. 9.4)
3. impact amelioration: $f(F,M,S) = h(F,M-S)$ (Sec. 9.6)
4. exogeneous neutral technical progress: $f(F,M,T) = h(F,M)S(t)$ (Sec. 9.1)

<u>Case 1: Non-fossil, substitutable energy:</u>

The development of a completely substitutable non-fossil fuel can be characterized by

$$f(F,M,S) = h(F+S,M)$$

The equilibrium condition of the basic reference model is evaluated at F+S and M:

$$-\frac{\frac{\partial h(F+S,M)}{\partial F}}{\frac{\partial h(F+S,M)}{\partial M}} = \frac{dM}{dF} = \frac{1}{(r+\alpha)} \tag{1}$$

dM/dF is defined along an isoquant of h. In this formulation the impact of the level of technical progress can be found by comparative statics or by a graphic examination of the solution. Figure 4 shows the isoquants for the technology $h(X,M)$ where $X = F+S$. This type of technical progress changes the value of X at any point F along the horizontal axis, it essentially moves the vertical axis to the right at higher levels of productivity. In Figure 4, for technical pogress equal to S_1, O_1C_1 is the stationary CO_2 line, $M/F = 1/\alpha$. At the higher level of technical progress S_2,O_2C_2 is the constant CO_2 line. Figure 4 shows that O_2C_2 cuts the balanced input line, ab, at a lower level, and thus that more technical progress reduces the equilibrium level of CO_2 and fossil fuel use.

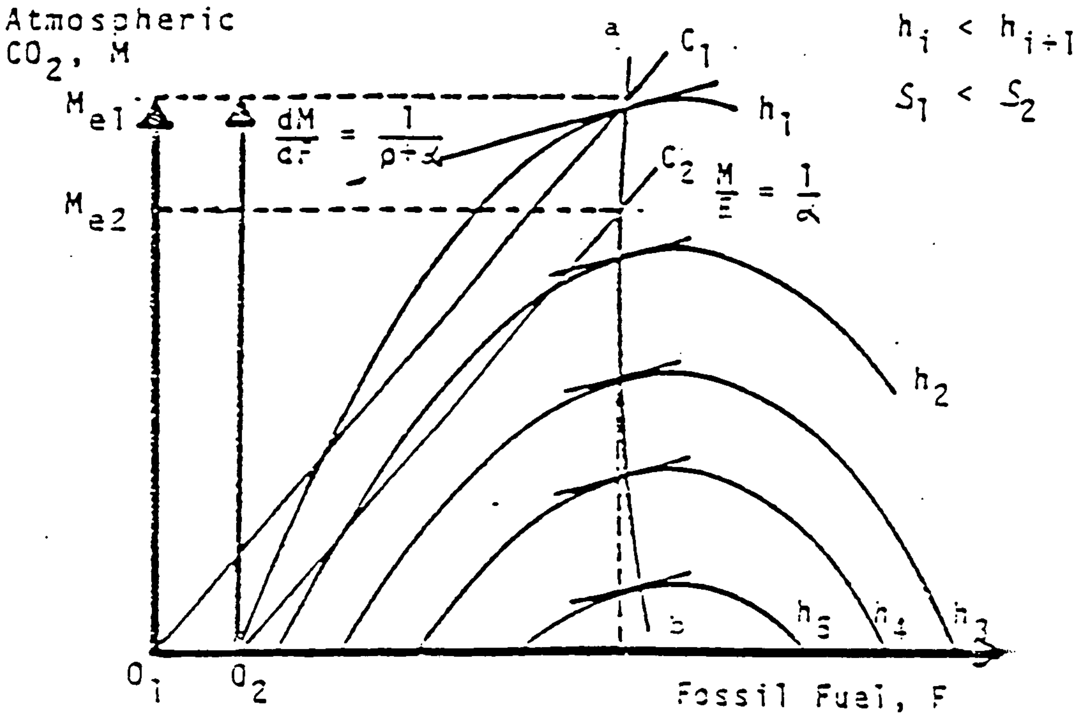

Figure 4: Case 1: A Non-fossil Substitute

The removal of CO_2 parallels this case, it changes the dynamics of CO_2 accumulation. Let S be the rate of CO_2 removal, then the state equation for the change in atmospheric CO_2 is:

$$\dot{M} = F - \alpha M - S$$

The equilibrium condition is then $F = \alpha M + S$. The change can again be represented as a shift of the vertical axis in our solution diagram to the right, a shown in Figure 4.

Case 2: Fossil Fuel Enhancement

Changes in fossil fuel productivity would result in a future production function:

$$f(F,M,S) = h(FS,M)$$

In equilibrium the following equation must hold along an isoquant of h:

$$-S\frac{\frac{\partial h(FS,M)}{\partial F}}{\frac{\partial h(FS,M)}{\partial M}} = S\frac{dM}{dF} = \frac{1}{r+\alpha} \tag{2}$$

This equation again can be analyzed graphically. an examination of a single isoquant in the F-M plane at two values of S, S_1 and S_2 helps explain the result. In Figure 5 the curve OC_1 represents the equation $h(FS_1,M)$. When S equals S_2, production along the curve is $h(FS_2,M)$. A higher level of technical progress compresses the isoquant toward the axis.

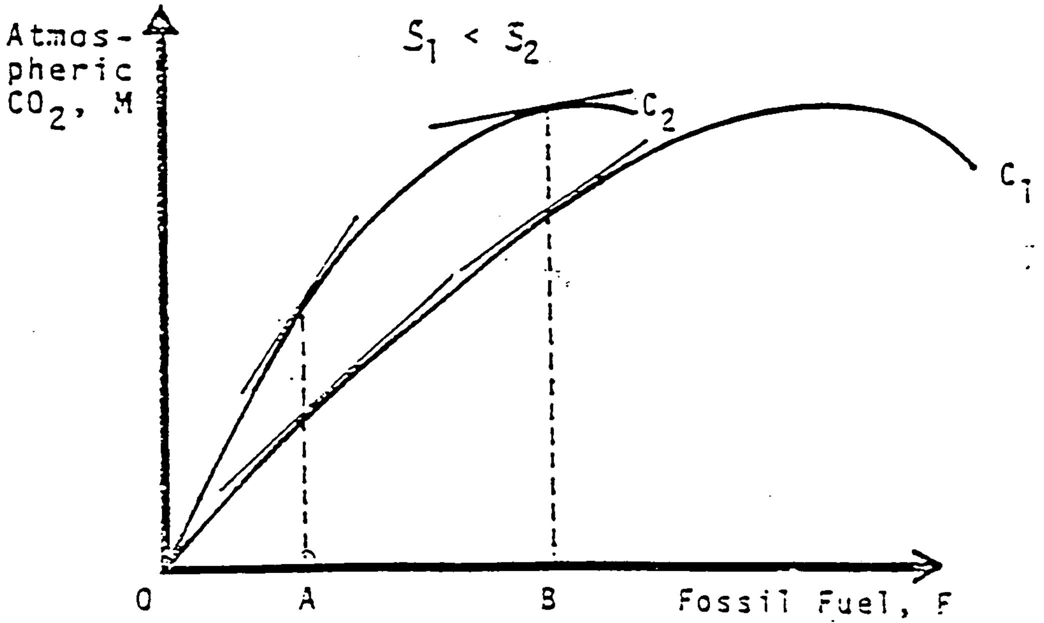

Figure 5: Case 2: Impact of Fossil Fuel Enhancement

In Figure 5 technical progress increases or decreases the slope of dM/dF along the isoquant depending on the value of F.

At A the slope increases and at B the slope decreases. Because the slopes of the isoquants may increase or decrease at any given value of F, the locus of points satisfying the rate of substitution equation (2) may move to the right or left. The change in the equilibrium level of atmospheric CO_2 is, therefore, uncertain.

An equivalent type of impact, as fossil fuel enhancement, is given by emissions scrubbing. Scrubbing CO_2 from emissions changes the dynamics of CO_2 accumulation as

$$\dot{M} = (F/S)\alpha M$$

The equilibrium condition is then $F = S\alpha M$. The same conclusion as in Figure 5 holds. The equivalence can easily be seen if the effects of substituting a variable F_s equal to F/S into the state equation and the production function are considered.

Case 3. Amelioration of CO_2 Impacts

Furthermore, one representation of ameliorating CO_2 impacts would change the production function as:

$$f(F,M,S) = h(F,M - S)$$

This case bears similarity to Case 1. However, in this instance the horizontal axis is raised as illustrated in Figure 6.

In Figure 6 only the isoquants for S equal to S_1 are shown, the isoquants when S equals S_2 would be moved vertically by S_2 - S_1 while not changing the constant CO_2 line. Consequently, the long run equilibrium level of CO_2 is raised with higher levels of technical progress.

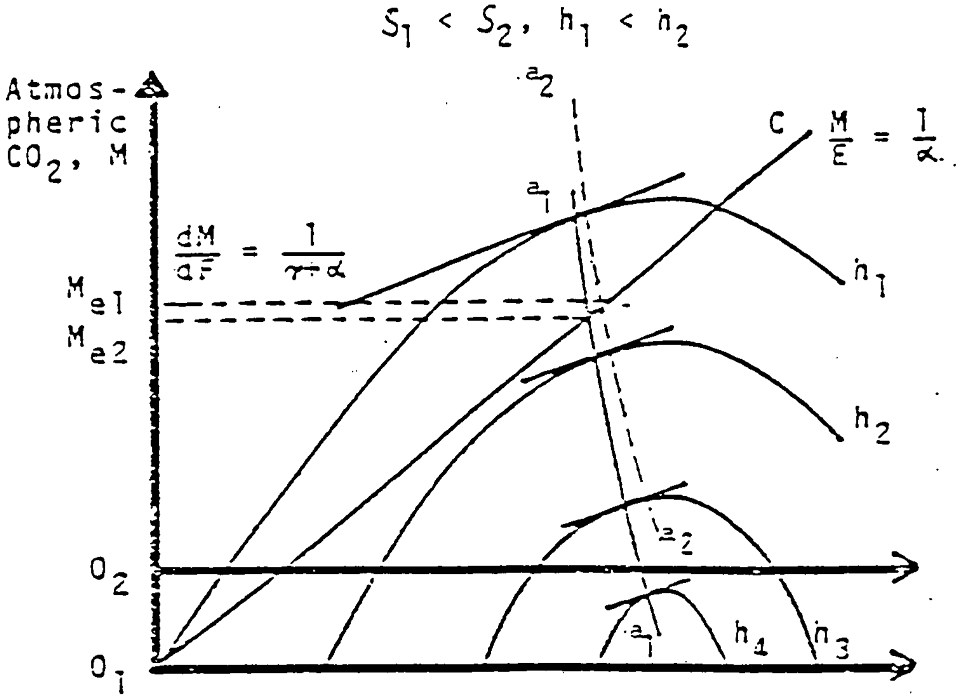

Figure 6: Case 3: Amelioration of CO_2 Impacts

Case 4. Exogenous Neutral Technical Progress

To examine ongoing technical progress we assume the production function to be separable as below:

$f(F,M,T) = h(F,M)S(t)$

Given this production function, we restate the equilibrium condition of the price damage equation, equation 4(25),

$$\frac{\partial q}{\partial F}\frac{dF}{dt}+\frac{\partial q}{\partial M}\frac{dM}{dt}+\frac{\partial q}{\partial S}\frac{dS}{dt}+\frac{\partial q}{\partial U'}\frac{dU'}{dt}=(r+\alpha)\,U'\frac{\partial h}{\partial F}S+U'\frac{\partial h}{\partial M}S \tag{3}$$

In general, no equilibrium solution exists, however, when S increases exponentially with time and the consumption elasticity of utility is constant (U takes the constant relative risk aversion form), the problem is much simpler. dF/dt and dM/dt are zero in equilibrium. If η is the exponential rate of increase in S, dS/dt divided by S equals γ and $\frac{dU'}{dt}$ divided by U' equals $-\beta\eta$.

Then from (3) we derive the most important equation:

$$-(\partial h/\partial F)\,/\,(\partial h/\partial M)=dM/dF=\frac{1}{(r+\alpha-\eta(1-\beta))} \tag{4}$$

dM/dF is defined along an isoquant of h. If the right hand side of (4) is interpreted as the ratio of the prices of fossil fuel increases and atmospheric CO_2 decreases, the long-run costs of CO_2 reductions are reduced by technical progress. We would expect such a cost reduction to cause substitution away from fossil fuel use and toward reduced CO_2 levels.

The sufficiency condition on q requires that the rate of increase in utility, $\eta(1-\beta)$, be less than the discount rate. This assures that the integral is finite and the maximum is well defined. It also assures that the slope of dM/dF will be less than the slope of the line along which αM equals F.

Using (4) we can construct Figure 7 in which the locus of solutions for (4) is drawn for two different values of technical progress, curves a_1b_1 and a_2b_2. η_2 which defines a_2b_2 is greater than η_1 which defines a_1b_1. Figure 7 shows that the higher the rate of progress the lower the steady state values of F and M.

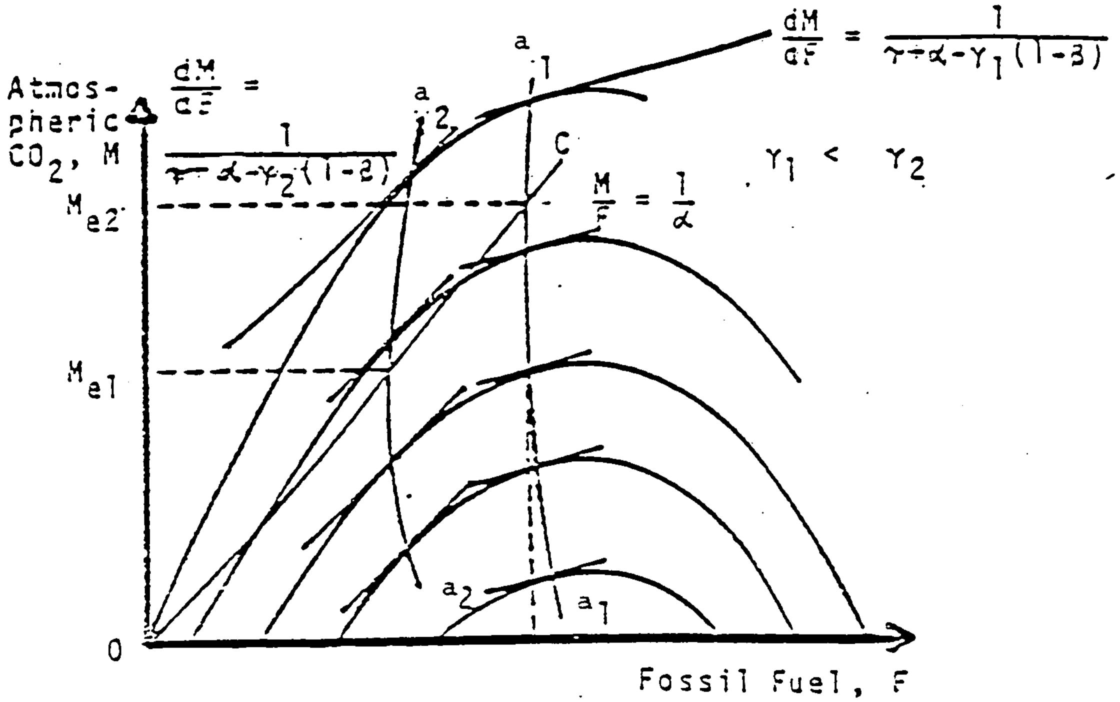

Figure 7: Exogenous, Neutral Technical Progress

2.11 EXPONENTIAL TECHNICAL PROGRESS IN A BENCHMARK MODEL

Exponential technical progress can be added to the benchmark type model developed in. In this case the production function is:

$$C=\begin{cases} F^{\epsilon}e^{\eta t}, & M\leq M_c \\ 0 & ,M>M_c \end{cases} \tag{1}$$

By looking at the rate of change in fossil fuel use and the initial fossil fuel use, equations 4(17) and 4(22), are restated

$$F=F(T)\,e^{A(T-t)} \tag{2}$$

$$F=A(M_c-M)+\alpha M \tag{3}$$

The variable A is redefined as

$$A=(r+\alpha-\eta(1-\gamma))/(1-\epsilon(1-\gamma)) \tag{4}$$

As we can see, the changes in F due to changes in the social discount rate, r, the fossil fuel productivity factor, ϵ, and the reabsorption rate, α, are as in the model without technical progress. Fossil fuel declines more slowly in this model because technical progress increases the productivity of fossil fuels. With technical progress what can we derive for the rate of growth of consumption (i.e. $(dC/dt)(1/C)=\Delta C)$?

From the present until time T when equilibrium is reached, the rate of growth in consumption is

$$\Delta C=-(\epsilon(r+\alpha-\eta(1-\gamma))/(1-\eta(1-\gamma))+\eta \tag{5}$$

The first term on the right is the rate of change in output from the energy sector, and the second term on the right is the rate of technical progress.

This expression can be simplified to

$$\Delta C = (\eta - \epsilon(r+\alpha))/(1-\epsilon(1-\gamma)) \qquad (6)$$

When the rate of growth in the economy is positive, increases in the consumption elasticity of utility, γ , tend to increase present energy use, and thereby decrease the growth rate of consumption. When the rate of growth in the economy is negative, γ has the opposite effect.

The results can be extended and partially generalized by taking other and more complicated forms of consumption equations.

In all cases model parameters affect the rate of change in energy use in the same fashion in the models with and without technical progress.

In summary we can conclude that increases in the new parameter, η , reduce both initial energy use and the rate of decline in energy use.

Equation (6) shows whether and how consumption increases or decreases over time. Technical progress tends to make consumption grow. Reabsorption, social discounting, and high fossil fuel productivity all encourage high initial energy use and make a consumption decline more likely.

2.12 INTERNATIONAL CO-OPERATION

International control agreements on CO_2 emissions would require many participants. Since temperature increases will impact countries in extremely varied ways, in some countries the impact may be beneficial. Thus international agreements on fossil fuel trade may be difficult to achieve. The basic premise is that critical CO_2 levels will be reached more quickly in a world without co-operation on CO_2 controls.

International co-operation on CO_2 emissions could be speeded up by acceleration of technology transfer between two countries or block of countries. If technology changes have a significant influence (or leverage) on critical CO_2 emissions technology transfer schemes could facilitate co-operative ventures that accelerate the innovation and diffusion of technologies for enhancing global welfare.

If CO_2 emissions and possible global warming are perceived as a threat to their survival, individual countries or a group of countries may wish to unilaterally alter the international use of fossil fuels through export and import controls such as taxes and subsidies. The general relation between global pollution and international trade has only recently received considerable attention, and the suggestion to extend the mechanism of tradeable permits to global issues, such as CO_2, has been followed up as part of the official US negotiation position on climate change (*Economist*, 1990). But the greatest part of work so far deals with local rather than transnational pollutants and is not particularly applicable to CO_2.

Within the class of models previously suggested we now consider the difficulties in the control of carbon dioxide emissions.

We set out from the benchmark (step) model and compare two cases: co-operative and non-co-operative, given by subscripts 'C0' and 'NC' respectively. For example, let F_{CO} refer to world emissions in the co-operative case, F_{NC} to world emissions in the non-co-operative case. q_{CO} refers to the tax or negative adjoint variable in the co-operative case, and T_{CO} to the time at which M equals M_c in the co-operative case. In the model two world regions, 1 and 2, exist. There are n and m factories in the two world regions. Let f be the production function of a single factory. The regions use F_1 and F_2 units of energy, respectively. The factories have decreasing returns and energy is used efficiently. Factories in Region 1 use F_1/n units of energy and factories in Region 2 use F_2/m units of energy. We assume a maximum level of energy use per factory, F_m, which is greater than $\alpha M_c/(n + m)$. Accordingly, we define the consumption in each region:

$$C_1=\begin{cases} nf(F_1/n), & M\le M_c \\ 0, & M>M_c \end{cases} \text{ and}$$

$$C_2=\begin{cases} mf(F_2/m), & M\le M_c \\ 0, & M>M_c \end{cases} \tag{1}$$

Co-operation is "complete" and the countries maximize the discounted sum of the consumption in the two regions being fully aware of future CO_2 effects.

The following setup describes the problem:

$$\underset{F_1,\,F_2}{Max}\ \int_0^{\infty} e^{-rt}(C_1+C_2)\,dt \tag{2}$$

$$dM/dt = F_1 + F_2 - \alpha M \tag{3}$$

Not surprisingly, the solution is similar to the pure step model. The negative of the adjoint, q_{CO}, rises at an exponential rate of $(r + \alpha)$ to equilibrium at $f'(\alpha M_c/(m+n))$. The marginal product of energy in every factory equals the negative of the adjoint variable, f' equals q_{CO}. The inverse of this function, say $\phi(q)$, determines F_{1CO} and F_{2CO}:

$$F_{1CO}(t)=n\phi(q_{CO}(t),\ F_{2CO}=m\phi(q_{CO}(t)) \tag{4}$$

T_{CO} is defined by

$$\int_0^{T_{CO}} e^{\alpha(T_{CO}-t)}(n+m)\,\phi(f'[\frac{\alpha M_c}{m+n}]\,e^{(r+\alpha)(T_{CO}-t)}\,dt-M_0e^{-\alpha T_{CO}}=M_c \tag{5}$$

The non-co-operative case differs considerably. Region 1 recognizes the future CO_2 impacts and control emissions, but Region 2 is unaware of or ignores the impacts until M equals M_c. Region 2 initially burns fuel at a level mF_m such that f' (F_m) equals 0.

For simplicity we assume that when M equals M_c co-operation begins and that F_{1NC}/n equals F_{1NC}/m and $F_{1NC} + F_{2NC}$ equals αM_c. Region 1 solves the following problem:

$$\max_{F_1} \int_0^{\infty} e^{-rt} C_1 \, dt \tag{6}$$

$$\frac{dM}{dt} = \begin{cases} F_1 + mF_m - \alpha M, & M \leq M_c \\ 0, & M > M_c \end{cases} \tag{7}$$

q_n rises exponentially at the rate $(r + \alpha)$ until equilibrium at f' $(\alpha M_c/(m + n))$. In Region 1 only, the marginal products of factories are held at q_n and emissions of these factories are equal to $\phi(q)$. F_{1NC} and F_{2NC} for $M < M_c$ are:

$$\begin{aligned} F_{1NC} &= n\phi(q_n(t)) \\ F_{2NC} &= mF_m \end{aligned} \tag{8}$$

T_{NC} is then given by

$$\int_0^{T_{NC}} e^{-\alpha(T_{NC}-t)} \{ n\phi [f' (\frac{\alpha M_c}{m+n}) e^{-(r+\alpha)(T_{NC}-t)}] + F_m \} dt - M_0 e^{-\alpha T_{NC}} = M_c \tag{9}$$

Thus we could state the theorem.

Theorem

A critical CO_2 emissions level is always reached in less time in the non-co-operative case than in the co-operative case. T_{NC} is less than T_{CO}.

In order to prove this result, we restate (5) and (9) respectively, as:

$$\int_0^{T_{CO}} e^{-\alpha t} \{ (n+m) \phi [f' [\frac{\alpha M_c}{m+n}] e^{-(r+\alpha)t}] - \alpha M_c \} dt + M_0 = M_c \tag{10}$$

and

$$\int_0^{T_{NC}} e^{-\alpha t} \{ n\phi [f^1 [\frac{\alpha M_c}{m+n}] e^{-(r+\alpha)t}] + mF_m - \alpha M_c \} dt + M_0 = M_c \tag{11}$$

Essentially the integration is now backwards in time. Because F_m is greater than $m\phi [f' (\alpha M_c / (m+n)) \exp(-(r+\alpha) t)]$ and the other terms of the integrands are equal, at each instant the integrand in (11) is greater than the integrand in (10). Since the integrand is always larger in (11), the integral must equal $M_c - M_0$ in a shorter time.

We have shown previously that q_{NC} (T_{NC}) equals q_{CO} (T_{CO}) equals f' $(\alpha M_c/(m + n))$.

If we single out the specific Region 1 and let T_{NC} be less than T_{CO}:

$$q_{NC}(t) = q_{NC}(T_{NC})\, e^{-(r+\alpha)(T_{NC}-t)} > q_{CO}(T_{CO})\, e^{-(r+\alpha)(T_{CO}-t)} = q_{CO}(t) \tag{12}$$

$$n\phi(q_{NC}(t)) = F_{1NC} < F_{1CO} = n\phi(q_{CO}(t)) \tag{13}$$

The emissions of the concerned region are lower in the non-co-operative case than they are in the co-operative case.

T_{NC} less than T_{CO} appears to imply that world fossil fuel use in the non-co-operative case is always higher than in the co-operative case; however, this is not necessarily true. There could be special situations identified by specific parametric configurations of the production function and other parameters, in which world emissions in the non-co-operative case are actually lower than in the co-operative case for a short initial period.

In summarizing the results, if co-operation is not feasible, regions concerned about carbon dioxide will lower emissions to compensate for regions not concerned about carbon dioxide, however, the maximum level of carbon dioxide concentration will be reached later with co-operation than without co-operation.

2.13 STRUCTURAL UNCERTAINTY

As we have seen before, various types of uncertainties are linked with the CO_2 problem. In view of planned or perceived policy actions on national or international levels, some or most of these uncertainties are not even quantifiable. Examples include costs of new and improved technologies that could substitute existing ones, or costs of incremental technologies that may be used to increase efficiencies and the environmental benign of existing fossil fuel use. But from the perspective of economic modelling we come across these intangible uncertainties and ask ourselves how to cope with uncertainties impacting the modelling process. We call this phenomenon "structural uncertainty". We will show that when dealing with structural uncertainty it is not enough to use expected (certainty equivalent) values of critical parameters rather a comprehensive treatment of uncertainty is called for, in view of assessing optimal present and future fossil fuel use.

First of all we observe that determinations of optimal fossil fuel use are similar in uncertainty when there is an uncertain, critical atmospheric carbon dioxide level as compared to a situation where the fossil fuel resource is uncertain. When we neglect absorption of atmospheric CO_2, the two situations are structurally equivalent in uncertainty.

The results of research on uncertain, limited natural resources are also pertinent to the CO_2 problem with minor changes in definitions.

2.13.1 Modelling Uncertainty about Critical CO_2 Levels as Uncertainty about a Critical, Limited Natural Resource

Let us assume first that the present level of CO_2 is M_0 and that $i = 1, ..., n$ possible critical carbon dioxide levels, M_i, exist. The prior probability that M_c equals M_i is π_i. J_i is the maximum expected value of future fossil fuel consumption when the current level of carbon dioxide in the atmosphere is M_i.

π_{ij} is the update probability that M_c equals M_j given that M_i has been reached and is not the critical level. E denotes an expectation. J_i is defined by

$$J_i = Max\ E\left[\int_O^{\infty} e^{-rt} U(C)\,dt\right] \tag{1}$$

such that

$$C = \begin{cases} f(F), & M \le M_c \\ 0, & M > M_c \end{cases} \tag{2}$$

$$M(t=O) = M_i \tag{3}$$

$$\pi_{ij} = Prob\{M_j = M_c \mid M_i \ne M_c\} = \begin{cases} \pi_k / \sum_{j=i+1}^{n} \pi_j & k = i+1, \ldots, n \\ O, & otherwise \end{cases} \tag{4}$$

Let T_i be the time to move between carbon dioxide levels M_i and M_{i+1}, given by

$$\int_O^{T_i} e^{-\alpha(T_i - t)} F(t)\,dt = M_{i+1} - e^{-\alpha T_i} M_i \tag{5}$$

This formulation is an adaptation of Gilbert's (1979) analysis to the CO_2 problem, and most of his results are pertinent to this analysis.

In view of the definition of T_i, we consider a slightly different optimization problem, as the value $J(M_i, M_{i+1}, T_i)$, the value obtained by moving from a CO_2 concentration of M_i to M_{i+1}, in time T_i, defined as:

$$J(M_i, M_{i+1}, T_i) = \underset{F}{Max} \int_O^{T_i} e^{-rt} U(C)\,dt \tag{6}$$

such that

$$C = f(F) \tag{7}$$

$$dM/dt = F - \alpha M \tag{8}$$

$$M(O) = M_i \quad \text{and} \quad M(T_i) = M_{i+1} \tag{9}$$

The results of Gilbert (1979) can be immediately transferred to the determination of optimal fossil fuel use given structural uncertainty about the critical level of atmospheric carbon dioxide. The solution algorithm for (6) requires maximization over T_i of three terms:

$$J_i = \underset{T_i}{Max}\{J(M_i, M_{i+1}, T_i) + e^{-rT_i}[\pi_{i,i+1}(U(C(\alpha M_{i+1}))/r = (1-\pi_{i,i+1})J_{i+1}]\} \tag{10}$$

In the order of appearance the maximand consists of

(i) the value J gained while raising the CO_2 level from M_i to M_{i+1}.

(ii) the discounted expected value of consuming fuel at a rate that maintains the CO_2 level at M_{i+1} weighted by the probability that M_{i+1} is the critical CO_2 level.

(iii) the discounted expected value of raising the CO_2 concentration above M_{i+1} weighted by the probability that M_{i+1} is not the critical level.

Another result is that a "certainty equivalent" critical level of CO_2 exists, i.e. M_{ce}, which produces the same initial emissions as the algorithm in (10). It can be shown that M_{ce} is greater than $E(M_c)$. If so, it follows that the calculated optimal current fossil fuel use, when probabilities are fully taken care of, is lower than the optimal current fossil fuel use when expected values are treated as certainty equivalents.

2.13.2 Treating Structural Uncertainty

We now show on the basis of the previous results that the way in which uncertainty is treated in such models can significantly alter the calculated response to CO_2. For reasons of simplicity, let us first assume that no (observable) CO_2 impacts occur and no information is gathered about CO_2 impacts until the CO_2 level reaches a threshold M_*. At that time all uncertainty regarding the future impacts of CO_2 is resolved.

This simplifies the previous model because all learning occurs at a single CO_2 level rather than at several levels.

In this form, the problem can be stated as

$$J_O=\max_{T}\{J(M_O,M_a,T)+e^{-rT}E(J_a)\} \qquad (11)$$

T is the time at which the CO_2 level equals M_a. $E(J_a)$ is the expected utility gain after the CO_2 level reaches M_a and uncertainty is resolved. J_a does not depend on T.

From the necessary conditions, we derive

$$F(t)=F_O^{-At} \text{ for } t\leq T \qquad (12)$$

Using (5) and (12) we determine F_O as a function of T:

$$F_O=(M_a e^{\alpha T}-M_O)(A-\alpha)/(1-e^{-(A-\alpha)T}) \qquad (13)$$

The utility gained while increasing the CO_2 level from M_O to M_a, $J(M_O, M_a, T)$ is certain and a function of T:

$$J(M_O,M_a,T)=\int_O^T e^{-rt}u(C)\,dt=[F_O^{e(1-\gamma)}/(1-\gamma)(A-\alpha)](1-e^{-(A-\alpha)T}) \qquad (14)$$

The solution algorithm from equation (11) then takes the form

$$J_O=\max_{T}\{[F_O^{e(1-\gamma)}/(1-\gamma)(A-\alpha)](1-e^{-(A-\alpha)T})+e^{-rT}E(J_a)\} \qquad (15)$$

To determine the maximum, the derivative with respect to T is set to zero, recalling that F_O is a function of T:

$$dJ_O/dT = 0 \qquad (16)$$

Thus, for (15)

$$F_O^{e(1-\gamma)}\{[e/(A-\alpha)F_O](1-e^{-(A-\alpha)T})(dF_O/dT)+ [1/(1-\gamma)]e^{-(A-\alpha)T}\}-re^{rT}E(J_a)=0 \qquad (17)$$

If, from (13), the derivative of F_0 with respect to T is substituted into (17), we find

$$F_0^{\epsilon(1-\gamma)}\{[\delta\alpha M_a/F_0]e^{\alpha T}+[1-\epsilon(1-\gamma)/(1-\gamma)]e^{-(A-\alpha)T}\}-re^{rT}E(J_a)=0 \qquad (18)$$

Since we can define

$$F_a=F_0e^{-AT} \qquad (19)$$

we can simplify (18), as

$$[F_a^{\epsilon(1-\gamma)}/r]\,[\delta\alpha M_a/F_a)+(1-\epsilon(1-\gamma)/(1-\gamma))]=E(J_a) \qquad (20)$$

(20) can be solved by numerical methods for any value of $E(J_a)$. It gives rise to a correct, appropriate treatment of uncertainty. Thus, T and the optimal present emissions can be found.

In comparing two different treatments of uncertainty, we distinguish the "certainty equivalent" case from the comprehensive treatment of uncertainty, as shown by the formulae above culminating in equation (20).

To show even more specific results we use our general benchmark model and calculate changes in present fossil fuel use under the two treatments of uncertainty for a variety of parameter settings.

Let us start with the familiar framework: M_c equals M_1 with probability π and M_2 with probability $(1 - \pi)$. If the critical CO_2 level is assumed equal to the expected critical

$$M_{ca}=\pi M_1+(1-\pi)M_2 \qquad (21)$$

where M_{ca} is the assumed certain critical level. The problem can now be treated like a deterministic problem and the results obtained in Section 4 directly apply.

In the second approach to uncertainty, we make use of the probability estimates and the algorithm developed in the previous section.

In this case, the CO_2 level certainly can increase from M_0 to M_1. $J(M_0, M_1, T)$, the value gained in this transition, is certain and is given by (14). As could be seen from (15), $E(J_a)$ represents the expected utility gathered after M_1 is reached. $E(J_a)$ is the weighted sum of the utility gained by staying at M_1 and the utility gained by going to M_2 and staying at that level of CO_2.

Let $J_s(M)$ be the utility gathered when the CO_2 level is maintained at a constant M from the present into infinity.

The following equation defines $J_s(M)$ for a general production and for the production function F^5, in particular:

$$J_s(M) = U(f(\alpha M, M))/r = (\alpha M)^{\epsilon(1-\gamma)}/r(1-\gamma) \tag{22}$$

If M_1 is the maximum level of CO_2 concentration, the utility is specified by (22) with M equals M_1. If M_2 is the maximum level of CO_2 concentration, $J(M_1, M_2, T^*)$ defined by (14), is collected going from M_1 to M_2, and $J_s(M_2)$ is collected after M_2 is reached. T^* is the optimal time to go from M_1 to M_2. With M_O equal to M_1 and M_c equal to M_2, we use 4(19), to determine T^*.

$$[\alpha M_2/(A-\alpha)M_1]\,[e^{(A-\alpha)T*}-1]+e^{-\alpha T*}-1=0 \tag{23}$$

After T^* being defined by (23), $E(J_s)$ can now be calculated as

$$E(J_a) = \pi J_s(M_1) + (1-\pi)\{J(M_1, M_2, T*) + e^{-rT*}J_s(M_2)\} \tag{24}$$

In more complete, specific parametric form $E(J_s)$ is

$$E(J_s) = \pi[(\alpha M_1)^{\epsilon(1-\gamma)}/r(1-\gamma)] + (1-\pi)e^{-rT*}[(\alpha M_2)^{\epsilon(1-\gamma)}/r(1-\gamma)] \\ [1+(r/(A-\alpha))(e^{A-\alpha)T*}-1)] \tag{25}$$

To find the optimal F_O under this full treatment of uncertainty (FTU), the above equation for $E(J_s)$ is used in conjunction with (20), the latter dealing with the simple case.

2.13.3 Numerical Calculations

We have presented two alternatives to dealing with uncertainty and choosing the appropriate emissions level. The results show so far that emissions are higher if the "certainty equivalent" (CE) case is used. However, to determine the magnitude of the differences we have to resort to numerical examples, and we turn to this next. We did some limited calculations for specific examples presented in Tables 1 and 2.

In the first three columns of each table we list the parameters used in each run or set of calculations. In the fourth column we list the percentage increase in initial emissions when we change from the FTU case to the CE case. In the column CE appears the present optimal fossil fuel use when the expected value of M_c is assumed certain. In the final column FTU is the value of F(O) when uncertainty is fully treated.

In all calculations we assume $M_O = 1$, $\pi = 0.5$, production equals F^δ for M less than the critical level of CO_2.

In Table 2 the uncertain levels are $M_1 = 2$ and $M_2 = 4$. At this level of uncertainty the estimates of the proper emission level are close together. In Table 2 a much wider range of uncertainty is considered: $M_1 = 2$ and $M_2 = 8$. Therefore the differences in initial emission rates are more significant.

The emissions when a CE critical level of carbon dioxide is assumed are on the average over 30 per cent higher than in the case of FTU.

These examples add to the claim that an appropriate treatment of uncertainty is important in modelling carbon dioxide.

Table 2: Comparison of CE and FTU with Low Variance

Basis: $M_O = 1$, $M_1 = 2$, $M_2 = 4$, $\pi = 0.5$

$\epsilon(1-\gamma)$	r	α	% change in F_O	CE F_O	FTU F_O
0.5	0.02	0.001	12.2	0.087	0.077
0.75	0.04	0.001	7.2	0.332	0.309
0.95	0.06	0.001	0.4	2.420	2.410
0.5	0.02	0.002	5.5	0.094	0.089
0.75	0.04	0.002	4.8	0.342	0.326
0.95	0.06	0.002	1.1	2.486	2.460

Table 3: Comparison of CE and FTU with High Variance

Basis: $M_0 = 1$, $M_1 = 2$, $M_2 = 8$, $\pi = 0.5$

$e(1-\gamma)$	r	α	% change in F_0	CE	FTU
				F_0	
0.5	0.02	0.001	46.4	0.215	0.134
0.75	0.04	0.001	34.2	0.825	0.584
0.95	0.06	0.001	24.3	6.105	4.780
0.5	0.02	0.002	39.6	0.230	0.154
0.75	0.04	0.002	32.1	0.850	0.615
0.95	0.06	0.002	24.0	6.210	4.880

2.14 CONCLUSIONS AND PERSPECTIVES

The choice of simple models proposed so far have been examined under three major issues: technical progress, international co-operation and structural uncertainty. We show that for this class of models depending on the assumptions regarding technical progress, the optimal steady state CO_2 concentration may rise or fall with increases in the steady state level of progress. More specifically, an improved substitute for fossil fuels always reduces the long-run level of CO_2; while an improvement in fossil fuel productivity may increase or decrease the level of CO_2.

Also we show that the solutions of a model with neutral, constant and ongoing technical progress and the general (static) model are very similar. In the model with technical progress, higher levels of technical progress lead to lower long-run optimal levels of CO_2.

By examining two cases of international co-operation in controlling CO_2 accumulation, we first considered a reference case of complete co-operation between two regions in maximizing consumption with full awareness of the CO_2 problem. This case is compared with a situation in which no co-operation takes place until a critical CO_2 level is reached. A most interesting finding is that in the non-co-operative situation the critical level is reached sooner, even though one region (concerned about CO_2) always emits less carbon than in the co-operative case. A non-intuitive situation can occur in which initial emissions of the two regions are lower in the non-co-operative case than in the co-operative case.

The final applications are to uncertainty. Here we show that the results from studies of the optimal use of natural resource being in limited supply can be naturally extended to the CO_2 problem. Then, using numerical examples we show that an inappropriate treatment of structural uncertainty can lead to a significantly higher than optimal estimate of the desirable level of fossil fuel use.

Some final comments should be devoted to the general philosophy of modelling complex, highly interactive energy-environmental situations. Our task has been ambitious, namely to exhibit single type models of optimal control that are able to identify major structural parameters of the CO_2 problem as seen from an economist's perspective.

In models which optimize over an infinite horizon, future effects may change current policies, and feedback effects are always of importance. However, in these same models feedback effects may make solution much more difficult. In this class of optimizing models inclusion of feedback effects usually lowers the optimal initial use of fossil fuels. The long-run changes in fossil fuel use due to feedback effects are more uncertain and dependent on the model.

Given the uncertainty in the severity and timing of feedback effects, the sensitivity of individual models to variations in feedback effects is of much interest. In this class of models we find that current optimal fossil fuel use is significantly affected by different critical levels of CO_2.

Including optimization in models expands their applicability but may cause analytic problems and controversy. As with many social policy problems, involving welfare judgements, an acceptable objective function for CO_2 control problems is difficult to define.

Any definition will seem both inadequate and overly precise and certainly will be controversial. This may be the reason we will find very few optimizing models in this area. However, such a model will not hide useful results otherwise obtained and by a carefully done sensitivity analysis may add many new insights not otherwise obtained. I concur with R. E. Lucas (1987, chap. 2) that "useful policy discussions are ultimately based on models".

Optimization raises several new issues in these models. For example, pollution impoverishes but technical progress enriches the future.

The models show that curvature of the utility function, determined by the consumption elasticity of utility in our models, tends to smooth or even out wealth over time. Without an objective function being stated, the importance of this redistribution effect in determining fossil fuel use policy can not be examined.

APPENDIX A

Existence and Stability of Equilibrium

In this appendix we show that the equilibrium solution of Section 4 holds providing in detail the assumptions which assure our solution holds.

Several assumptions assure that a solution to 4(25') exists when F equals αM. As with the sufficiency conditions, the general existence proof is only applicable if the production function has continuous first derivatives. Continuous first derivatives assure that the left-hand side (lhs) of 4(25') is continuous except in any region where $\partial f / \partial M$ equals zero. Experience has shown that CO_2 accumulation has caused little damage to date and that energy is valuable in production; therefore, we assume that at zero energy use and zero atmospheric CO_2, the marginal product of energy use is greater in magnitude than the marginal harm due to an increase in CO_2. It follows that the lhs of 4(25') is very large when F and αM equal zero.

Existence of the equilibrium is then assured if at some level of atmospheric CO_2 concentration the marginal product of energy use is less in magnitude than the marginal harm due to an increase in atmospheric CO_2 i.e. the lhs of 4(25') is less than one. This can be assured by two reasonable assumptions:

$$\frac{\partial f(0,M)}{\partial F} \le \frac{\partial f(0,0)}{\partial F} \tag{A1}$$

$$\frac{\partial f(\alpha M,M)}{\partial M} > \frac{\partial f(0,0)}{\partial F} \tag{A2}$$

as M goes to infinity.

The first assumption is that the marginal product of fossil fuels is greatest at zero fossil fuel use and atmospheric CO_2 accumulation.

The second condition states that, as atmospheric CO_2 increases, at some point the marginal harm is greater than the maximum marginal product of fossil fuels. More stringent assumptions, which assure that equations (A1) and (A2) are satisfied, are that the second partial derivative of f with respect to F and M are negative and that the marginal harm due to CO_2 goes to infinity.

By taking the total differential of 4(25') we can show that, if the second partial derivative of f with respect to F and M are negative, we are also assured that curve ab in Figure 1 slopes downward as shown.

Although we have discussed the conditions which show that the equilibrium is optimal and exists, we still have to show that an optimal path to the equilibrium exists. If the equilibrium is stable or, in other words, a saddle point, a unique shadow price exists for each level of CO_2 concentration which if chosen initially will cause convergence to the equilibrium along the optimal path. If the equilibrium is unstable, the equilibrium is maintained when CO_2 is initially at the equilibrium level; but no optimal path to the equilibrium exists if CO_2 starts at any other level.

Our stability analysis follows Kamien and Schwartz (1981, p 160). The proof of stability depends on the particular structure of our simple model. When F is greater than zero, 4(15) is an equation of the form $G(F, M, q) = 0$. We earlier assumed that we could express F as a function of M and q. By the implicit function theorem, this function for G exists if the partial derivative of G with respect to F does not equal zero for any F, M, and q that satisfy 4(15). The partial derivative of f with respect to F and the derivative of U with respect to C have already been assumed to be non-zero for values of F in the range of interest, therefore, the function for F in terms of q and M exists.

The next step in the stability analysis is to linearize equations 4(24) and 4(25) about the equilibrium. If the equilibrium values of M and q are designated $\overline{M}$ and $\bar{q}$, then 4(24) and 4(25) can be approximated.

$$-\left[\alpha+\frac{U_{fm}}{U_{ff}}\right](M-\overline{M})+\frac{1}{U_{ff}}(q-\bar{q})=0 \tag{A3}$$

$$-\left[\frac{U_{fm}^2-U_{ff}U_{mm}}{U_{ff}}\right](M-\overline{M})+\left[r+\left(\alpha+\frac{U_{fm}}{U_{ff}}\right)\right](q-\bar{q})=0 \tag{A4}$$

A linear system, as in (A3) and (A4) has a characteristic equation. If the roots of the equation are real and of opposite signs, the equilibrium is a saddle point and the solution is stable. We state the derived stability condition without further discussion of the characteristic equation. For further details we refer to the discussion of stability analysis in Kamien and Schwartz (1981). If the following inequality holds, the equilibrium is stable:

$$U_{mm}+(r\alpha+\alpha^2)U_{ff}+(r+2\alpha)U_{fm}<0$$

In our discussion of sufficiency we assumed that U_{ff} and U_{mm} were both negative, we again make these assumptions. The final assumption is that U_{fm} is less than or equal to zero and consequently that the equilibrium is stable. This assumption can only be true if the second partial derivative of production with respect to fossil fuel use and atmospheric CO_2 is less than or equal to zero, see equation 4(20). This assumption makes sure that the marginal product of fossil fuel use does not increase with increases in atmospheric CO_2. Note, the relation in (A3) may hold even if U_{fm} is greater than zero.

In our analysis of the equilibrium we have found two key conditions. First, for the equilibrium itself to be optimal, the second partial derivatives of production with respect to both fossil fuel use and atmospheric carbon dioxide must be negative, and the condition in 4(23) must hold. Second, for a simple and general proof that an optimal path to the equilibrium exists, U_{fm} in 4(20) must be negative. This implies that the second partial derivative of production with respect to fossil fuel use and atmospheric carbon dioxide must be negative. We must emphasize that these conditions are not necessary, they only allow easy proofs of sufficiency and stability. If these conditions hold and M is initially lower than the equilibrium, CO_2 increases, the shadow price increases, and fossil fuel use decreases with time until equilibrium.

APPENDIX B

Proposition

If f in Section 5(1) has no maximum or the maximum occurs at F greater than αM_c and M(0) is less than M_c there is a unique finite time T at which M equals M_c.

Proof. If it exists, the time at which M equals M_c, T, is defined by:

$$M_c=\int_0^T e^{-\alpha(T-t)}g(q(t))dt+e^{-\alpha T}M_o \qquad \text{(B1)}$$

When M equals M_c or t is greater than or equal to T, q equals $U'f'(\alpha M_c)$.

From 5(3), when M is less than M_c or t is less than T, q equals $q_o\exp(r + \alpha)t)$ where q_o is the value of q at t equals 0. Continuity of q requires that, for an optimal path:

$$U'f'(\alpha M_c)=q_o e^{(r+\alpha)}T \qquad \text{(B2)}$$

If a q_o and T satisfying (B1) and (B2) can be found the paths of q, F and M are all determined.

(B2) can be solved for q_o in terms of T. (B1) can then be restated as an equation in which T is the single unknown variable. After rearranging terms:

$$\int_0^T e^{-\alpha T}\{g(U'f'(\alpha M_c)e^{-(\tau+\alpha)(T-t)})-\alpha M_c\}dt-e^{-\alpha T}(M_c-M_o)=0 \qquad \text{(B3)}$$

At T equals zero the integral in (B3) is zero and at T equals infinity the integral is infinity. The function is continuous, therefore, a solution exists and is finite. The derivative of the left-hand side with respect to T is positive, therefore the solution is unique. q.e.d.

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CHAPTER 3

LONG-RUN INVESTMENT AND TECHNICAL PROGRESS: DYNAMIC AND VINTAGE-TYPE MODELS

3.1 INTRODUCTION

In this chapter we set out to model the economic aspects of the CO_2 problem under endogenous technical progress. Such models appear more natural and provide increased flexibility and realism for policy-making purposes.

We begin with a comparison of the models with those developed in Chapter 2. The analysis considers possible long-run equilibrium solutions and possible approach paths to equilibrium.

We present a solution for a constant, long-run growth path of the model and an examination of the effects of parameters on the ratio of "knowledge" to capital and on the growth rate along this path.

The model introduced here bears a strong resemblance to the simple model with exogenous technical progress in Chapter 2; however, here we specify the manner in which technical progress occurs. In this model the economy does not acquire technical progress for free but must invest in knowledge and physical capital. There is an additional economic rationale for including capital accumulation and knowledge in the analysis of the CO_2 problem, that is, a compensation argument for intertemporal or intergenerational equity (Spash and d'Arge, 1989). If a given fossil fuel consumption and possible ensuing greenhouse warming cannot be avoided in this generation or the next, then at least part of the capital and knowledge should be put to mitigate the effects, or to create technologies which future generations could use to protect themselves against any harmful effects. This is part of an insurance policy on CO_2 strategies (Manne, 1990; Schelling, 1991).

This model is more flexible because the level of investment is not fixed in advance but is determined within the optimization process. A limitation of this model is that only neutral technical progress is allowed, but its usefulness can be defended on the aggregate level, as intended (V. K. Smith, 1974, Ch. 2). The three control variables in the model are the level of fossil fuel use, F; the level of investment, I, and the distribution of investment, θ . Atmospheric CO_2, M; knowledge, S, and capital, K, are all state variables. F, M, S and K all determine production, Y.

As in previous models the objective is to maximize utility discounted at the social discount rate, r, over an infinite horizon.

We assume that the utility function has a constant consumption elasticity of utility with elasticity γ as shown below.

$$U(C)=\begin{matrix}(C^{1-\gamma})\ (1-\gamma),\gamma\neq 1\\ lnC,\gamma=1\end{matrix} \tag{1}$$

The effects of CO_2 and energy use are determined by the function h(F,M) and are separable from the effects of knowledge and capital. A Cobb-Douglas type term determines the impact of knowledge and capital on production. In this term the elasticities of production with respect to knowledge and capital are μ and ν, respectively. Production is:

$$Y=h(F,M)\,S^{\mu}K^{\nu} \tag{2}$$

Because of the possibility of investment, this model is significantly different from those presented before. Consumption is C = Y - I, and investment is divided between knowledge and capital. θI is invested in knowledge and $(1-\theta)I$ is invested in capital. We assume that the control variables F and I are both greater than or equal to zero. θ is limited between zero and one which means that knowledge cannot be changed to capital, capital cannot be changed to knowledge and neither knowledge nor capital can be consumed. The atmospheric CO_2 level changes as in previous models,

$$dM/dt=F-\alpha M$$

Capital and knowledge differ in two major ways. First, capital depreciates at the rate ρ while knowledge does not depreciate. Second, the change in capital is a linear function of capital investment, but the change in knowledge is a non-linear function of knowledge investment. These assumptions are specified by

$$dS/dt=(\theta I)^{\sigma} \tag{3}$$

where σ can be seen as a "society-dependent" transformation parameter of human capital investment into knowledge.

$$dK/dt=(1-\theta)I-\rho K \tag{4}$$

For reasons of convenience, we refer to the derivative of the right-hand side of (3) with respect to I divided by θ as the *effective knowledge investment*, ESI. ESI is the marginal change in dS/dt per unit of investment in knowledge.

$$ESI=\sigma(\theta I)^{\sigma-1} \tag{5}$$

3.2 FIRST-ORDER NECESSARY CONDITIONS

The Hamiltonian, H, for this problem is

$$H=U(C)-q(f-\alpha M)+\zeta(\theta I)^2+\kappa((1-\theta)I-\rho K \tag{1}$$

Using optimal control we derive the necessary conditions in integral form. The first three equations determine the values of the adjoint variables or the shadow prices of fossil fuels, knowledge and capital. The shadow price of fossil fuels, q, is

$$q=-\int_t^{\infty} e^{-(r+\alpha)(\tau-t)} C^{-\gamma}(Y/h(F,M))(\delta h/\delta M)\,d\tau \tag{2}$$

As in earlier models the shadow price of energy at time t, q(t), represents the total future loss due to a one unit increase in atmospheric CO_2. The marginal disutility of increased CO_2 is the marginal utility of consumption times the marginal decrease in production with an increase in atmospheric CO_2. Equation (1) shows that the total future loss is the marginal disutility of increased CO_2 discounted at the social discount rate plus the absorption rate of CO_2 and summed over time.

$$\zeta=\int_t^{\infty} e^{-r(\tau-t)} C^{-\gamma}\mu(\eta/s)\,d\tau \tag{3}$$

$$\kappa=\int_t^{\infty} e^{-(r+\rho)(\tau-t)} C^{-\gamma}\nu(Y/K)\,d\tau \tag{4}$$

ζ and κ represent the long-run values of a unit increase in knowledge and capital, respectively. ζ is the value of a unit increase in knowledge as its marginal utility discounted at the social discount rate and summed over time. In a similar way, we define κ as the value of a unit increase in capital, discounted at the social discount plus the depreciation rate. The marginal utility times the marginal productivity of energy must be less than or equal to the shadow price of CO_2, q

$$C^{-\gamma}(Y/h(F,M))(\delta h/\delta F)\leq q \tag{5}$$

with a strict inequality holding if F=0. Similar relations hold between the adjoint variables ζ, κ and the marginal utility of consumption. The reasoning goes like this. Because capital changes as a linear function of investment, the value of capital also measures the value of capital investment. However, the value of investment in knowledge is measured by the effective knowledge investment times the value of an increase in knowledge. Thus we can state the sum of the values of investment in knowledge and capital weighted by the distribution of investment in each must be less than or equal to the marginal utility of consumption.

$$\theta(ESI)\zeta+(1-\theta)\kappa\le C^{-\gamma} \tag{6}$$

If the 'less than' condition holds, I equals 0. Furthermore, if σ were less than one and I were equal to 0 the effective knowledge investment would be infinite and (6) would be violated. Therefore, if σ is less than one then I is always greater than 0,

$$\begin{aligned}&(ESI)\zeta>\kappa,\ \ \theta=1\\&(ESI)\zeta=\kappa,\ \ 0\le\theta\le1\\&(ESI)\zeta<\kappa,\ \ \theta=0\end{aligned} \tag{7}$$

(7) assures that we invest in both knowledge and capital only when it is equally effective in each. If the value of investment is unequal in knowledge and capital, we invest only in the more valuable factor. Adjoint variables must be continuous except at the boundary values of the associated state variables. This implies that C is continuous, and it follows that Y, I, M, K, S and F are continuous.

3.3 LONG-RUN EQUILIBRIUM

We first establish the existence and characteristics of two possible long-run equilibria. In one equilibrium, consumption growth is zero, in the second equilibrium, the economy grows exponentially. In equilibrium the energy-CO_2 sector of the economy behaves as in the simple static model with given exogenous technical process (Chapter 2).

Even if the economy grows, F and M are constant in the equilibrium. The necessary conditions can be considerably simplified. We first examine the conditions on the energy CO_2 sector. If F is greater than 0, then from 2(2) and 2(5):

$$q^* = [r+\alpha+(\delta h/\delta M)]\,/\,(\delta h/\delta F) \tag{1}$$

As previously, rates are designated by *. If $\theta < 1$, from 2(7), $(ESI)\zeta \leq \kappa$. We can easily solve 2(6) for κ:

$$\kappa = C^{-\gamma} \text{ for } \theta<1 \tag{2}$$

Let g denote the rate of consumption growth. By differentiating (2) with respect to time combining (2) with 2(4) and restating the equation in terms of rates of change

$$-\gamma g = r+\rho-\nu(Y/K) \text{ for } \theta<1 \tag{3}$$

If $\theta > 0$, then

$$\zeta = C^{-\gamma}/ESI \text{ for } \theta>0 \tag{4}$$

and

$$-\gamma g(1-\sigma)\ (I^*+\theta^*) = r-\mu(ESI)\ (Y/S) \text{ for } \theta<0 \tag{5}$$

The system of necessary conditions has been considerably simplified. Both ζ and κ have now been solved out of the system. In our proposed equilibria, g equals zero or is constant and greater than zero; C,Y, and I all grow at g, and θ is constant.

The equal growth rates of C,Y, and I allow equation 1(3) to be satisfied. Equations 1(4), 2(5) and (1) can be satisfied if q increases at a constant rate of g $(1-\gamma)$ and F and M are constant. As noted earlier, we assume that h has the same properties as f in the simple model. It follows that the pair of equations 1(4) and (1), can be solved in the same manner as the similar equations for the simple model with exogenous exponential technical progress. F, M and h(F,M) are constant, and the following relation holds:

$$M = \alpha F \tag{6}$$

Along a line in the F-M plane where h is constant

$$dM/dE = 1/(r+\alpha-g(1-\gamma)) \tag{7}$$

The familiar equations in (6) and (7) are very important, the intersection of the lines defined by these equations determines the equilibrium values of F and M in all cases.

If no investment is made in the capital, the ratio of knowledge to capital will grow as capital stock shrinks, consequently the marginal product of capital will grow; and when the marginal product of capital is large enough, it will always be optimal to sacrifice some consumption for investment in capital. In equilibrium, investment must be greater than 0 and θ must be greater than 0.

We can now define the static equilibrium. In the static equilibrium g equals zero, and investment is made in capital only. I equals ρK. The ratio of S^{μ} to K^{ν} is treated as a single variable and (3), (6), (7) form a system of three equations in three unknowns which can be solved for the equilibrium.

We now examine an equilibrium in which $g > 0$. By our previous argument, investment in capital must be greater then zero. By a similar argument, if the economy and capital stock are growing, investment must occur in knowledge or the marginal product of knowledge goes to infinity. A growth equilibrium is, therefore, also a balanced growth equilibrium in which knowledge and capital both receive investment and both (3) and (5) hold. Next we look back at 1(5) and 1(6). If θ is constant, S increases at a rate of $g\sigma$ and

K increases at a rate of g. Examining the production function 1(2) we reach the conclusion that a balanced growth path equilibrium exists only if a critical condition is met.

$$\textit{Assume} \quad \sigma\mu+\nu = 1 \tag{8}$$

This assumption might be more easily understood by comparing the effects of a constant rate of investment in economies where (8) does and does not hold. Suppose that current inputs and the rate of investment are constant in each of the following cases:

(i) If $\sigma\mu+\nu$ equals one, the economy grows uniformly at the investment rate. Most importantly, the marginal products of knowledge and capital and the fraction of production invested are constant.

(ii) If $\sigma\mu+\nu$ is greater than one, the marginal products of knowledge and capital continually rise and the fraction of production invested continually falls.

(iii) If $\sigma\mu+\nu$ is less than one, the opposite effects would occur.

A better consideration of the reasonableness of the assumption would require data on historical trends in output, current input productivity and use, and knowledge and capital investment. For example, if such data showed that historically production, research and capital investment all grew at a common rate but that the productivity of current inputs stagnated, the assumption that $\sigma\mu+\nu$ equals one would be supported. On the other hand, if production, research, capital investment, and the productivity of current inputs all grew at a common rate, this would suggest that $\sigma\mu+\nu$ is less than one. After this digression, using the relation in (8) we can state

$$Y/K = h(F,M) \quad [S/K^{\sigma}]^{\mu} \tag{9}$$

and

$$(ESI)\quad (Y/S) = h(F,M)\ \sigma(\mu\sigma g)^{(\sigma-1)/\sigma}\ (K^{\sigma}/S)^{\mu/\sigma} \tag{10}$$

(3) and (5) can be rewritten as:

$$-\gamma g = r+\rho+\gamma h(F,M)\ (S/K^{\sigma})^{\mu} \tag{11}$$

$$-\gamma g = r+(1-\sigma)g - \mu\sigma(\sigma g)^{(\sigma-1)/\sigma}\ h(F,M)\ (K/S)^{\nu/\sigma} \tag{12}$$

Equations (11) and (12) derived from (3) and (5) have straight-forward interpretations.

In both we have one term giving the marginal value of investment. For capital, this equals the marginal product of capital; and for knowledge, this equals the marginal product of knowledge times the marginal effectiveness of investment. The investment in both cases must have an equal return to the drop over time in output value and production. Output value falls due to discounting and the change in marginal utility with growth in consumption. Production from a unit of capital disappears due to depreciation. Equations (6), (7), (11), (12) are a system of four equations in four unknowns M, F, g, and the ratio of K^{σ} to S. These equations define the growth equilibrium, in order to lend it to policy directed computational support we have to further specify and simplify the model. Such growth equilibrium paths essentially depend on the Cobb-Douglas technology (Dixit, 1977, 1990).

3.4 SIMPLIFICATION OF MODEL FOR COMPUTATIONAL PURPOSES

A simple model allows us to examine the two equilibria in more detail and determine which of the equilibria is appropriate.

Again we use the step model of CO_2 impacts. We assume that a critical CO_2 level exists, M_c, below which atmospheric CO_2 has no effect upon production and above which production drops to zero. Further we assume that there is a fossil fuel productivity factor (ε) constant at δ and the fossil fuel sector is multiplied by a scaling factor λ (λ is only introduced for the computational analysis, it is of no importance to the economic analysis). In this model the equilibrium use of energy is αM_c.

To simplify notation, the variable ω is equal to the energy sector output in equilibrium, defined as $\lambda(\alpha M_c)^\delta$. We also assume that, as with capital, knowledge increases approximately linearly with investment or research; therefore, σ equals one. This follows from the assumption made in 3(8) that $\nu = 1\text{-}\mu$. These assumptions are expressed as

$$\begin{array}{ll} Y=\lambda F^\delta S^\mu K^{1-\mu}, & M\leq M_c \\ 0 \qquad\qquad , & M>M_c \end{array} \tag{1}$$

$$\frac{dS}{dt} = \theta I \tag{2}$$

With these simplifications 3(11) and 3(12) can be restated as

$$\lambda F^\delta (1-\mu)\,[S/K]^\mu = r+\rho+\gamma g \tag{3}$$

$$\lambda F^\delta \mu\,[K/S]^{1-\mu} = r+\gamma g \tag{4}$$

3.5 CLASSIFICATION OF EQUILIBRIA

3.5.1 Stationary Equilibria

In stationary equilibrium only capital receives investment, $\theta = 0$, the CO_2 level equals M_c, and investment equals K. 3(6) and 4(3) form a two-equation system in two unknowns, F and K/S. The equilibrium is similar to that in many growth models in that the ratio of the stocks determines the equilibrium. F equals αM_c. The ratio of K/S is determined by the equation 4(3). The specific level of knowledge, capital and consumption at the equilibrium is determined by the initial point.

A comparative statics analysis of equilibrium can be made. The equilibrium level of fossil fuel sector output, ω, increases with increases in α, M_c, and δ. These parameters either make fossil fuels more productive or CO_2 limits less severe. When g equals zero from 4(3), four parameters, ω, μ, ρ and r affect the equilibrium value of K/S. Because greater productivity reduces the importance of capital depreciation, increases in ω increase the marginal product of capital at any given ratio of capital to knowledge. This causes an increase in the level of the capital relative to knowledge. Conversely, increases in the social discount rate, r, increase the value of current consumption versus investment in capital and increase the shadow cost of capital. Increases in depreciation, ρ, cause capital to disappear more rapidly and likewise increase the shadow cost of capital. Increases in r or ρ result in a lower relative use of capital. The impacts of changes in μ are uncertain.

3.5.2 Balanced Growth Equilibrium

In the balanced growth equilibrium F again equals αM. Both 4(3) and 4(4) hold and these two equations can be solved for the ratio of K to S and for the rate of economic growth, g. Production Y equals $\omega S^{\mu} K^{1-\mu}$. Using 1(3) and 1(4), I and ω can be found.

By taking the total differential of 4(3) and 4(4) we can examine changes in the equilibrium values of g and the ratio of K to S with respect to changes in parameter values. Again the impact of μ on either value is uncertain. An increase in the productivity of the energy sector as measured by ω causes both the rate of economic growth, g, and the ratio of capital to knowledge to increase. Again, a more productive economy makes depreciation less important. An increase in consumption elasticity of utility, γ , causes the economy to grow more slowly but has no effect on the capital to knowledge ratio. An increase in the social discount rate has the same effects as an increase in γ . An increase in the capital depreciation rate lowers the equilibrium growth rate and the capital to knowledge ratio.

3.5.3 Optimality of Equilibrium

We assume that optimal behaviour leads to an equilibrium. However, the proofs of stability and optimality, used before, cannot be applied to this problem, because the Hamiltonian is not differentiable with respect to atmospheric CO_2 at the equilibrium. We know that for optimality the fossil fuel use and the atmospheric CO_2 level must converge to αM_c and M_c , respectively.

If a path does not raise CO_2 to the critical level, it leaves unused capacity in the economy. For any such path we can construct a dominant path which does raise CO_2 to the critical level. If the CO_2 level remains at the critical level, the problem is reduced to a capital two-sector growth model (Gehrels, 1975) where it will be optimal for knowledge and capital to converge to an equilibrium. If it is not optimal for CO_2 to remain at the critical level then oscillations down from and back to the critical level must be optimal. Such oscillations seem highly unlikely to be optimal, because of both discounting and the curvature of the utility function. Oscillations cause periods of reduced consumption, discounting tends to move higher levels of consumption to earlier time periods. The greater the curvature of the utility function the greater the loss in marginal value as consumption moves from low points to high points. In general, the curvature of the utility function tends to smooth out the consumption stream and eliminate cycles in consumption.

Within our means we were unable to prove that cycles are not optimal though tools along this line could be put to test (Goodwin, 1990). Instead, in what follows, I will use a simple graphical representation to argue that at least one of the equilibria classified represents long-run optimal behaviour if an optimum exists.

If the economy is very productive, an optimum defined by regular maximisation does not exist. If the utility growth rate, $g(1-\gamma)$, is greater than the social discount rate, r, welfare will be infinite. In this case the investment required to maintain the growth rate will be greater than the production.

Figures 1 and 2 illustrate how the appropriate equilibrium is determined. The figures show levels of knowledge, OA, and of capital, OB, both as lines in the knowledge-capital plane. Along the knowledge line the marginal product of knowledge (MPS) equals the social discount rate, r, and along the capital line the marginal product of capital (MPK) equals the sum of the social discount rate, r, plus the depreciation rate, ρ

$$MPK = \lambda F^{\delta} (S/K)^{\mu}, \quad MPS = \lambda F^{\delta} (K/S)^{1-\mu}$$

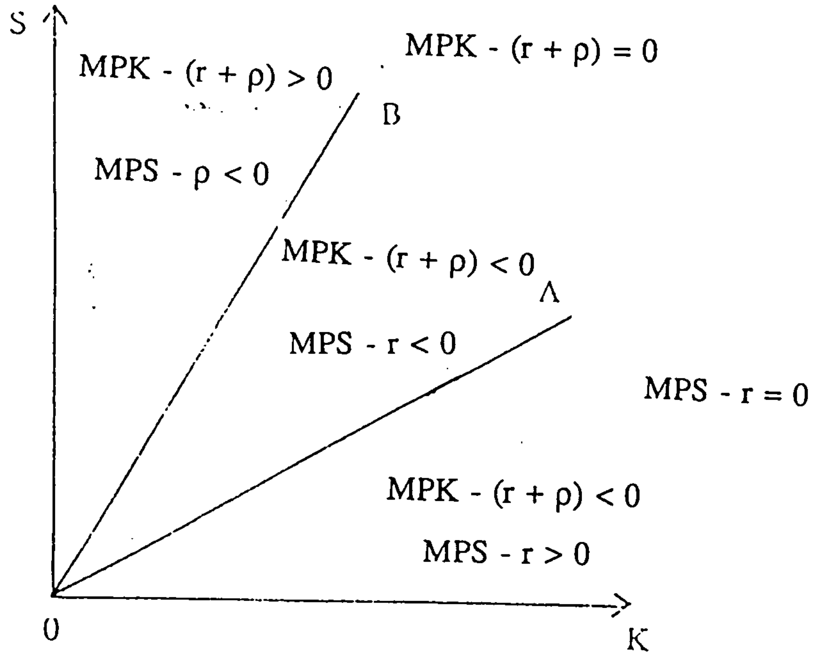

Figure 1: Stationary Equilibrium

In Fig. 1 the capital line lies above the knowledge line. If MPK is greater than or equal to r+ρ, MPS must be less than r. In this situation g along the equilibrium path is negative. Since it is never optimal for knowledge to decrease this is clearly not an optimal equilibrium.

$$MPK = \lambda F^{\delta} (S/K)^{\mu}, \quad MPS = \lambda F^{\delta} (K/S)^{1-\mu}$$

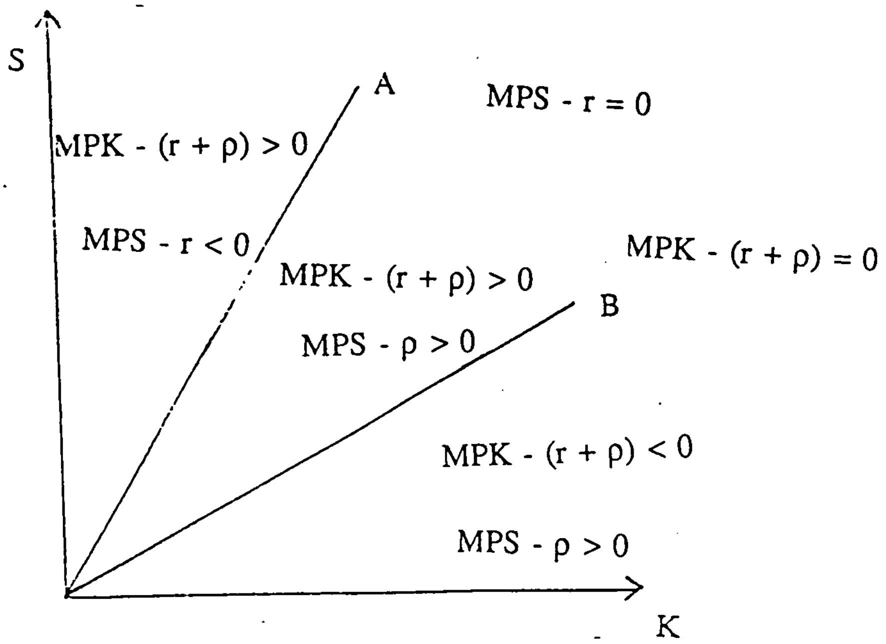

Figure 2: Possible Growth Equilibrium

In Fig. 2 the knowledge line, OA, lies above the capital line, OB, and a line exists along which both the marginal product of knowledge minus the social discount rate and the marginal product of capital minus the sum of the social discount rate plus the depreciation rate are equal and greater than zero. In this case g is positive and the balanced growth equilibrium is optimal.

3.6 CHARACTERIZATION OF EQUILIBRIUM

We now analytically characterize the rates of change of energy use, knowledge, and capital as the equilibria are reached. The solution of these equations requires the continuity of consumption and investment.

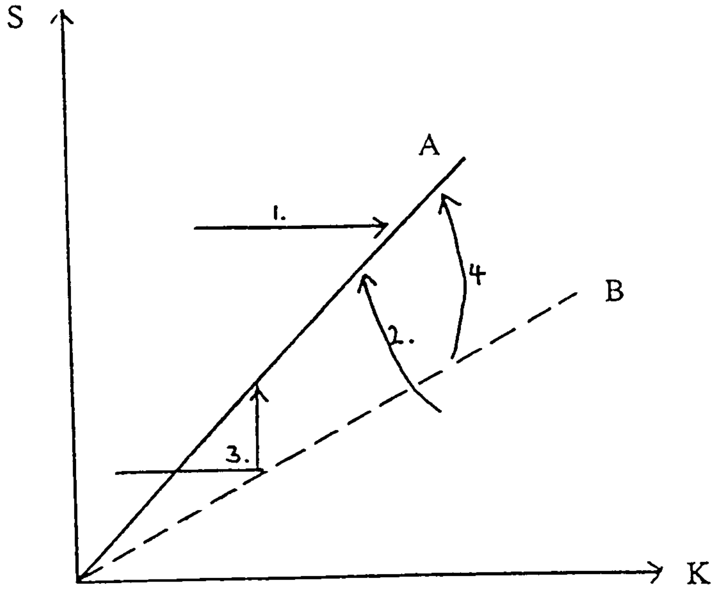

Figure 3: Transition Paths to Optimal Growth

Fig. 3 illustrates possible transition paths to a balanced growth equilibrium, labelled 1 to 4. When fossil fuel use equals αM_c, OA in the figure is the line along which balanced investment takes place. When the level of output from the fossil fuel sector is greater than the equilibrium output, OB is the line along which balanced investment will proceed.

As indicated by the figure the line OB lies below and moves up to coincide with OA as fossil fuel use moves toward equilibrium. Above line OA the ratio of knowledge to capital is too high and investment takes place in capital only, path 1, below OB the opposite results in path 2.

The possibility exists of combined paths in which we first invest only in capital and then invest only in knowledge, path 3. Along path four the ratio of knowledge and capital, by reaching OA, stays on OA as OA moves. The possibility of being on such a path will be further examined later on.

Paths to a stationary equilibrium would be similar. However, when converging to a stationary equilibrium a region exists in which it is not optimal to invest in either knowledge or capital, the region between OA and OB in Fig. 1. In this region the capital stock is allowed to shrink as the equilibrium is approached.

At the moment equilibrium is reached, the continuity of production and consumption determine investment and the continuity of the left hand sides of 4(3) and 4(4) determine g. Knowing g and I allows us to solve for the rates of change of the variables just as the equilibrium is reached and allows the analysis of this small portion of the transition path. Because of the similarity of the approach to the growth and stationary equilibrium, we will only discuss the approach to the growth equilibrium.

From 3(1) we derive a condition on fossil fuel use which holds when CO_2 is not at its critical level:

$$C^{-\gamma}\partial(Y/F)=q_o^{\theta} \tag{1}$$

By differentiating both sides of (1) and dividing out common elements, we obtain:

$$-\gamma g+\eta^{*}-F^{*}=r+\alpha \tag{2}$$

4(1) provides an expression for Y in terms of F, S and K, and it can be differentiated with respect to time and substituted into (2) obtaining

$$\mu S^{*}+(1-\mu)K^{*}-(1-\delta)F^{*}=\alpha+r+\gamma g \tag{3}$$

Because investment is continuous, the total investment in knowledge and capital, the sum of the right-hand sides of 1(4) and 4(2) must be equal just before and after the equilibrium is reached. We can then derive the following equation which holds as equilibrium is reached.

$$SS^{*}+KK^{*}=g(S+K) \tag{4}$$

Along regular transition paths to optimal growth, investment is *either* in knowledge *or* capital alone when equilibrium is reached. Knowledge and capital reach their equilibrium ratio after or, less likely, just as fossil fuel use reaches its equilibrium level. In all of these cases, (3) and (4) are sufficient to determine the rates of change of all variables. It is much more difficult to determine the possibility of balanced investment equilibrium. If equilibrium can be reached with investment in both knowledge and capital, we can equate 3(11) and 3(12), differentiate with respect to time, and develop

$$(\mu\rho+r+\gamma g)\,S^{*}-(\mu\rho+r+\gamma g)\,K^{*}+\delta\rho F^{*}=0 \tag{5}$$

(3) - (5) form a system of three linear equations in three unknowns. One method of solving this system is to invert the matrix of coefficients of the three variables S^*, K^* and F^* and multiply the vector of the right-hand sides of these equations. A first step in finding the inverse is to find the determinant of the matrix, that is

$$D=\delta\rho S-(S+K)\,(\mu\rho+(1-\delta)\,(r+\gamma g)) \tag{6}$$

We then can calculate three equations to determine S^*, K^* and F^* as the equilibrium is approached.

$$S^{*}=g-[\delta\rho K(\alpha+r-(1-\gamma)\,g)]\,/\,[\delta\rho S-(S+K)\,(\mu\rho+(1-\delta)\,(r+\gamma g))] \tag{7}$$

$$K^{*}=g+[\delta\rho S(\alpha+r-(1-\gamma)\,g]\,/\delta\rho S-(S+K)\,(\mu\rho+(1-\delta)\,(r+\gamma g))] \tag{8}$$

$$F^{*}=[(K+S)\,(\mu\rho+r+\gamma g)\,(\alpha+r-(1-\gamma)\,g)]\,/\,[\delta\rho S-(S+K)\,(\mu\rho+(1-\delta)\,(r+\gamma g))] \tag{9}$$

It should be clear that if D is positive, the equilibrium cannot be reached along a balanced investment path. If D is positive, then fossil fuel use is increasing. To reach equilibrium, fossil fuel use must be greater than αM_c and increasing at equilibrium, CO_2 must either exceed the critical level or the fossil fuel path must be discontinuous. Neither of these can be optimal. D must therefore not be positive. A high social discount rate encourages high early use of fossil fuels and makes the possibility of a positive D less likely. High values of the consumption elasticity of utility encourage high early fossil fuel use to level out consumption in a growing economy and affect D in the same manner as the social discount rate. The role of the fossil fuel productivity factor, δ , and depreciation, ρ , are less clear. (5) shows that if δ and ρ are high the ratio of knowledge to capital must change rapidly along the balanced growth path. This suggests that, when fossil fuel use is dropping and δ and ρ are large, the investment required to maintain a balanced approach path is too high to be optimal. If D is zero, an infinity of solutions or no solutions may exist. Further, very small variations in parameters, while allowing us to find unique solutions to the problem, are likely to cause completely varying answers.

3.7 NUMERICAL EXAMPLES

Further study of this model requires the use of numerical examples. The model is not detailed or realistic enough to allow an estimation of parameters, therefore we examine the behaviour of the model over a wide range of parameter values. Because our purpose is to discover anomalies in the model's behaviour and to examine the shapes of possible optimal paths, the ability to scale the model is not of immediate concern. One major feature of equilibrium discussion has been the balanced growth equilibrium. In this context we see that the individual parameters of the energy sector, λ, α, M_c and δ do not affect the equilibrium. It is only necessary to specify the total output of the energy sector in equilibrium, ω . The other parameters we need to specify are the consumption elasticity of utility, γ , the discount rate, r; the elasticity of production with respect to knowledge, μ , and the depreciation rate of capital, ρ . In order to reduce the number of cases examined and because the value of the capital depreciation rate is less uncertain than the others, the depreciation rate of capital was set at 0.1 in all cases. The range of values of the parameters are listed:

$\rho=0.1$
$\mu=0.1-0.4, r=0.01-0.1, \gamma=0.5-0.95$
$\omega=0.1-0.4$

All combinations of the extreme values of the parameters provide sixteen cases which were examined for possible balanced equilibria. In four cases the optimum is not defined because $(1-\gamma)g$ is greater than r. These cases are characterized by high equilibrium energy sector output, ω , which results in a high growth rate. When this is coupled with either a low discount rate or low relative risk aversion, a finite optimum does not exist. In six more cases the balanced growth rate is negative, this condition is associated with low fossil fuel sector output. This result suggests that, when fossil fuel use is severely limited due to CO_2 and substitutes are not available, the economy may stagnate even if opportunities for investment exist. The cases in which the balanced growth path represents a possible optimal equilibrium are listed below:

Table 1: Cases, Growth Rates, K/S Ratios

	Cases					
Parameter	1	2	3	4	5	6
μ	0.10	0.40	0.40	0.40	0.40	0.40
r	0.01	0.01	0.01	0.01	0.10	0.10
γ	0.95	0.50	0.95	0.95	0.50	0.95
ω	0.40	0.10	0.10	0.40	0.40	0.40
Growth g(%)	10.60	1.10	0.54	14.85	10.10	5.34
K/S	1.55	0.20	0.20	0.90	0.90	0.90

The examples show a wide variation in both the equilibrium growth rate and the equilibrium capital to knowledge ratio. Cases 2 and 3 again illustrate that low energy sector output causes low equilibrium growth rates. Cases 5 and 6 show how sensitive the model is to the curvature of the utility function. When the consumption elasticity of utility, γ, nearly doubles, the optimal growth rate is cut nearly by half. If progress and growth continue in spite of CO_2, the present generation is the poorest generation. The optimality of lowering fossil fuel use and/or consumption in the present to promote future growth is tempered by our desire to raise consumption for the poorest consumers, ourselves. Therefore, a high consumption elasticity leads to higher present consumption and lower growth.

Other relationships in Table 1 are not as transparent. As shown in 4(3) and 4(4) the marginal product of knowledge minus the marginal product of capital must equal the rate of depreciation ρ, in equilibrium. Increases in μ raise the marginal product of knowledge and lower the marginal product of capital, therefore, to maintain a difference in the marginal products equal to ρ, the K/S ratio must fall when μ rises. This is reflected in cases 1 and 4. The level of energy sector output is low, the K/S ratio becomes very low to maintain the required difference in marginal products, as in cases 2 and 3. Capital depreciation is in a sense a fixed cost of maintaining capital. When the energy sector is large this fixed cost is not too important. However, when energy sector output and growth is low, the fixed cost is important and the level of capital fails to raise the marginal product of capital. This suggests a rule of thumb that if the economy slows down due to CO_2 or other influences, a shift away from inputs with substantial fixed costs would possibly occur. Furthermore, we can conclude: when the fossil fuel sector output is shrinking the K/S ratio must be changing to maintain the difference in the marginal products. It follows that the rate of change in capital and knowledge will be very different in these cases.

If the fossil fuel output shrinks rapidly to low levels, the structure of the economy must change very rapidly. Non-depreciating assets such as knowledge must increase rapidly compared to regular capital assets. A fundamental shift to a service economy?

3.8 MORE ON ENDOGENOUS TECHNICAL PROGRESS

With a view to generalizing the class of models on optimal energy use, as presented in Chapter 2, we develop a unique model in which prices influence the pattern of technical progress and furthermore non-neutral technical progress is possible. This model has some more realistic features.

First, it is a vintage-type model; that is the fossil energy input required by a piece of capital (equipment) is determined by the year in which it is purchased and cannot be changed after the capital is in place (Solow, 1964).

Second, research can be performed to improve the productivity of new capital or to reduce its energy requirements. The output required, the research budget, and cost of energy and capital are all exogenous. The objective is to minimize the cost of production.

The first push toward generalization involves the departure from neutrality of technical progress. The assumption of neutral technical progress is quite common. Neutral technical progress may be seen as a natural process, as a continuation of a past trend, or may be assumed for its simplicity.

If we look for economic factors as sources of explanation for the pattern of technical progress, we should take into account the different age structure of the capital stock. A machine in place usually has a limited capacity for changes in its mix of inputs.

In order to substantially change the ratio between the inputs to the production process, the capital stock must change. A vintage model of capital, in which the date of purchase determines the operating characteristics of capital, accounts for the inflexibility of capital.

In what follows, we develop two models which differ mainly in their fossil fuel requirements. In each model, a special or preferred ratio of technical coefficients exists and particular price patterns cause research to move the technology toward this ratio. We examine how this ratio changes under different price patterns. Finally, we examine a variation from these basic models which allows us to discuss CO_2 limits.

The remainder of the this chapter is organized as follows:

We first develop several models which reasonably represent the influence of economic factors on technical change. We show the sensitivity of the optimal research pattern to assumptions about the relation of energy use to output and the research budget.

Such models could well form an essential part of policy-oriented energy models in which capital stock vintages generate major changes in technology and productivity (Scherago et al, 1990, Jorgenson, 1989; Ingham et al., 1987).

3.9 A VINTAGE MODEL

A basic feature of this model is that the portion of research devoted to each input, energy and capital, is an endogenous variable. As technical progress occurs, the characteristics of machines change. Each machine is distinguished by its vintage. T(t) is the age of the oldest machine or equipment in use at time t. M(t) is the life of a machine bought at time t. By definition,

$$M(t)=T(t+M(t)) \tag{1}$$

Let the scale parameters for the oldest equipment and the equipment in use be defined by:

$$dT/dt=\tau,\ dM/dt=\lambda \tag{2}$$

Y(k) is the production from all machines bought at time k. I(k) is the number of machines purchased at time k. F(k) is the energy used by machines of vintage k. The critical energy assumption made is that energy use is proportional to the output level.

$$Y(k)=Min(A_1(k)I(k),A_2(k)F(k)) \tag{3}$$

$A_1(k)$ and $A_2(k)$ are the Leontief type technical coefficients.

If the technology is used efficiently the conditions below hold:

$$Y(k)=\left\{\begin{matrix} A_1(k)I(k), & k\geq t-T(t) \\ 0 & ,\ k<t-T(t) \end{matrix}\right\} \tag{4}$$

$$F(k)=\left\{\begin{matrix} \dfrac{Y(k)}{A_2(k)}, & k\geq t-T(t) \\ 0 & ,k<t-T(t) \end{matrix}\right\} \tag{5}$$

Because, in this context, we emphasize the nature of technological progress, the production level Y*(t) and the total research budget R*(t) are assumed exogenous. The objective is to minimize all future discounted costs. The problem is described in the following equations:

$$J = \underset{Y,\tau,\theta}{Min} \int_{t^*}^{\infty} \exp(-r(t-t^*)) \left\{ p_1(t)\,\frac{Y(t)}{S_1} + p_2(t) \int_{t-T(t)}^{t} F(k)\,dk \right\} dt \tag{6}$$

Future costs have two components. Machines sell at a price of p_1 and the first term in the bracket of (6) is the expenditure on machines at time t.

Energy costs p_2 per unit, and the second term is the cost of energy for all machines operating at time t.

We require that:

$$\int_{t-T(t)}^{t} Y(k)\,dk - Y^*(t) \geq 0 \tag{7}$$

The relation (7) states that the production from all machines must meet the output goal. Furthermore, research can increase the productivity of machines and reduce their energy requirements.

$$dA_1/dt = a_1\theta R^*(t)\,,\; dA_2/dt = a_2(1-\theta)R^*(t) \qquad 0 \leq \theta \leq 1 \tag{8}$$

where θ designates the portion of research funds devoted to each input factor.

$$\tau \leq 1 \quad \text{and} \quad \lambda + 1 \geq 0 \tag{9}$$

(9) implies that older machines are taken out before newer machines.

The first step in the analysis incorporates (7) into the objective function. This requires the use of the Lagrange multiplier L. The problem is also changed from minimization to maximization and restated below:

$$\underset{Y,\tau,\theta}{Max} \int_{t^*}^{\infty} e^{-r(t-t^*)} [-p_1(Y/A_1) - p_2\int_{t-T(t)}^{t} (Y/A_2)\,dk + L\int_{t-T(t)}^{t} Y dk - LY^*]\,dt = -J \tag{10}$$

As stated in (10) the outer integral sums over time while the inner integrals sum over vintages operating in a given year.

An equivalent statement is possible with the outer integral being over vintages and the inner over years of operation. The equivalent form is given by

$$\begin{aligned} -J = \max_{Y,\lambda,\theta} \int_{t^*-T(t^*)}^{\infty} & e^{-r(t-t^*)} [-\Omega(t)(Y/A_1)p_1 - (Y/A_2)\int_t^{t+M(t)} \Omega(x)\, e \\ & p_2(x)\,dx + Y \int_t^{t+M(t)} \Omega(x)\, e^{-r(x-t)} L(x)\,dx - \Omega(t) L(t) Y^*]\,dt \qquad (11) \\ & = \int_{t^*-T(t^*)}^{\infty} e^{-r(t-t^*)} F(Y, A_1, A_2, M)\,dt \end{aligned}$$

The function Ω(t) gets rid of costs previous to t^*. It is defined by

$$\Omega(t) = \begin{cases} 1, & t \geq t^* \\ 0, & t < t^* \end{cases} \qquad (12)$$

To form the current value Hamiltonian (V. L. Smith, 1977) we use the adjoint variables η, μ_1, and μ_2:

$$H = F(Y, A_1, A_2, M) + \eta\lambda + \mu_1 a_1 \theta R^* + \mu_2 a_2 (1-\theta) R^* \qquad (13)$$

3.10 THE ANALYSIS OF NECESSARY CONDITIONS

We derive the necessary conditions of the above vintage model and provide for each a suitable economic interpretation in the context of the proposed model.

For $t \geq t^*$ we have

$$\int_t^{t+M(t)} e^{-r(x-t)} L(x)\, dx - \frac{p_1}{A_1} - \frac{1}{A_2} \int_t^{t+M(t)} e^{-r(x-t)} p_2(x)\, dx \leq 0 \tag{1}$$

Equality holds in (1) if Y(t) is greater than zero. L can be interpreted as the current marginal value of a new unit of production at time t. When Y(t) is greater than zero, the discounted value of a new unit of production over a machine's lifetime, must equal the capital cost per unit of production plus the discounted lifetime energy costs per unit of production.

$$\eta \leq 0 \tag{2}$$

η is the value of retiring newer units before old. A closer examination of the problem shows that η always equals zero. The nature of technical progress, as described in 9(8), assures that A_1 and A_2 never decrease. New technologies are better in every way than old; therefore, a newer technology never will be retired before an older. (If new machines were improved in one characteristic but worse in another, this would not necessarily be the case).

$$\begin{gathered} (\mu_1 a_1 - \mu_2 a_2) R^* < 0,\ \theta = 0 \\ (\mu_1 a_1 - \mu_2 a_2) R^* = 0,\ 0 \leq \theta \leq 1 \\ (\mu_1 a_1 - \mu_2 a_2) R^* > 0,\ \theta = 1 \end{gathered} \tag{3}$$

μ_1 and μ_2 can be interpreted as the values of improvements in capital and energy use, respectively. (3) assures that research is devoted to the input which provides the greatest value, and that research is only done on both inputs when equally valuable.

$$\frac{d\eta}{dt} - r\eta + e^{-rM(t)} Y(t) \left[L(t+M(t) \frac{p_2(t+M(t))}{A_2(t)}\right] \leq 0 \tag{4}$$

Equality holds in (4) if M is greater than zero. M equal to zero practically does not occur, because purchasing and never using a machine cannot be optimal.

$$\frac{d\mu}{dt} r\mu_1 + \frac{p_1}{(A_1)^2} Y = 0 \tag{5}$$

$$\frac{d\mu_2}{dt} r\mu_2 + \frac{Y}{(A_2)^2} \int_t^{t+M(t)} e^{-r(x-t)} p_2(x)\, dx = 0 \tag{6}$$

(5) and (6) state that the values of research on capital and energy use decrease at the social discount rate and increase with the marginal value of an improvement in the technical coefficient. Since η equals zero, $\frac{d\eta}{dt}$ equals zero and (4) can be simplified and rewritten as

$$L(t+M(t)) = \frac{p_2(t+M(t))}{A_2(t)} \quad \text{or} \quad L(t) = \frac{p_2(t)}{A_2(t-T(t))} \tag{7}$$

L(t) is the current marginal value of a new unit of production. (7) states that a machine is retired when its operating cost per unit of production equals the current marginal value of a new unit of production. L(t) is an important quantity; but to investigate it further, additional assumptions are required. If Y(t) is greater than zero, (1) holds with equality and can be restated as:

$$\int_t^{t+M(t)} e^{-rx} L(x)\, dx - \frac{p_1}{A_1} e^{-rt} - \frac{1}{A_2} \int_t^{t+M(t)} e^{-r(x+t)} p_2(x)\, dx = 0 \tag{8}$$

By taking the time derivative of (8) and using (7) to simplify the result, we derive

$$L(t) = \frac{1}{A_1}\left[p_1(r+A_1^*) - \frac{dp_1}{dt}\right] + \frac{1}{A_2}\left[p_2 + A_2^* \int_t^{t+M(t)} e^{-r(x-t)} p_2(x)\, dx\right] \tag{9}$$

(9) simply states that the marginal value of a new unit of production must equal its marginal cost.

The first parenthesis on the right-hand side is divided by A_1 to put capital related costs on a per unit of output basis. The first term within this parenthesis shows the capital cost being amortized at the discount rate plus the rate of capital improvement. The next term is the capital savings lost by purchasing now rather than later. The second pair of large parentheses contain energy related costs and are placed on a per unit basis by dividing by A_2. Inside are the current energy cost plus the savings in lifetime energy costs which are lost by purchasing now rather than later. An interesting point is that L(t) does not depend on $Y^*(t)$, except that $Y^*(t)$ must be great enough since $Y(t)$ is greater than zero.

Necessary conditions (3) - (6) determine the research balance. Here we examine the case in which both technical coefficients improve. When this occurs the value of research in capital and energy must be equal over a period of time. If $\mu_1 a_1$ equals $\mu_2 a_2$ over time, then the change of each with time must also be equal:

$$a_1 \frac{d\mu_1}{dt} = a_2 \frac{d\mu_2}{dt} \tag{10}$$

Using (5) and (6) this condition is met only if a_1 times the marginal change in costs when A_1 is improved equals a_2 times the marginal change in costs when A_2 is improved.

$$a_1 \frac{p_1}{(A_1)^2} Y = a_2 \frac{Y}{(A_2)^2} \int_t^{t+M(t)} e^{-r(x-t)} p_2(x)\, dx \tag{11}$$

(11) is perhaps the most important derivation in this part, and has significant implications. It simply states that the value of research in each input must be equal if we commit research funds to each, a_1 and a_2 measure the effectiveness of research in improving the efficiency of capital and energy, respectively.

The marginal change in costs when the efficiency of an input is improved measures the value of changing A_1 and A_2.

(11) can be rearranged to give an expression in terms of the ratio of the technical coefficients. The ratio of the technical coefficients is determined by the ratio of the lifecycle costs times the ratio of the research efficiencies.

$$\left[\frac{A_2}{A_1}\right]^2 = \frac{a_2}{a_1} \int_t^{t+M(t)} e^{-r(x-t)} \frac{p_2(x)}{p_1} dx \tag{12}$$

3.11 NEUTRAL TECHNICAL PROGRESS

Improving the performance of the economy while not changing the ratio of the technical coefficients is the Leontief technology equivalent of neutral technical progress. The following proposition underlines the existence of such a ratio:

Proposition 1

Given the vintage model 9(1) to 9(9), if p_1 and p_2 are constant and $R^*(t)$ equals $R^* \exp(\gamma t)$ there is a unique ratio of technical coefficients A_1^* to A_2^* such that θ and M are constant along an optimal path. The ratio is expressed in

$$\left[\frac{A_2^*}{A_1^*}\right]^2 = \frac{a_2}{a_1}\frac{p_2}{p_1}\frac{(1-e^{-rM})}{r} \qquad (1)$$

Proof

If A_1 and A_2 are invested in at a constant ratio, each will change at the rate γ. By substituting from 10(7) into 10(9):

$$e^{\gamma T} = \frac{A_1}{A_2}\frac{p_1}{p_2}(r+\gamma)+1+\gamma\frac{(1-e^{-rM})}{r} \qquad (2)$$

This equation can be solved for the ratio of A_2 to A_1 and substituted into 10(12) to yield.

$$\left[\frac{p_2}{p_1}\frac{1}{(r+\gamma)}\left(e^{\gamma T}-1-\gamma\frac{(1-e^{-rM})}{r}\right)\right]^2 \frac{a_2}{a_1}\frac{p_2}{p_1}\frac{(1-e^{-rM})}{r}=0 \qquad (3)$$

Assuming M is constant, M equals T. At M equals zero, the left-hand side of (3) is zero and the first derivative of the left-hand side with respect to M is negative. At M equals infinity, the left-hand side of (3) is infinite. The second derivative of the left-hand side with respect to M is positive for all values of M greater than zero. From this we can conclude that there is a single, strictly positive value of M which satisfies (3).

Once the lifetime which satisfied (3), M^*, is found, (3) can be solved for the ratio of A_1 to A_2.

To prove that the necessary conditions are also sufficient, we would need to first show that $F(Y, A_1, A_2, M)$ in 9(11) is concave in both control and state variables. We have examined the matrix which determines concavity of $F(Y, A_1, A_2, M)$, and found that concavity depends on parameter values.

3.12 INDUCED PRICE CHANGES

The existence of a CO_2 problem adds on additional cost to the use of fossil fuels. In previous models this is captured through a shadow price, q. In this analysis we examine the response of the above equilibrium to an increase in the price of fossil fuel, which might be caused by the CO_2 induced shadow price. More precisely, we examine an increase in the ratio between the fossil fuel and capital prices. We find that an increase in the relative price of an input results in increased research in that input and a corresponding fall in the purchases of that input relative to the other input. The calculation is complicated because the optimal machine life changes when prices change. We first derive a relationship between the change in the equilibrium ratio of technical coefficients, A_2^*/A_1^*, to the change in the constant machine lifetime, M^*. Rearranging terms in 11(1), we derive

$$\frac{p_1}{A_1^*} = \frac{a_2}{a_1}\frac{A_1^*}{A_2^*}\frac{p_2}{A_2^*}\frac{(1-e^{-rM^*})}{r} \tag{1}$$

If we substitute from (1) and 10(7) into 10(9) and rearrange terms:

$$e^{\gamma M^*} = \frac{a_2}{a_2}\frac{A_1^*}{A_2^*}\frac{(1-e^{-rM^*})}{r}(r+\gamma) + \left[1+\gamma\frac{(1-e^{-rM^*})}{r}\right. \tag{2}$$

We further rearrange (2) so that A_1^*/A_2^* and M^* each appear in separate terms.

$$\frac{a_2}{a_1}\frac{A_1^*}{A_2^*}(r+\gamma) + \gamma - r\frac{(e^{\gamma M^*}-1)}{(1-e^{-rM^*})} = 0 \tag{3}$$

By taking the total derivative of (3), we can find the direction of change in the optimal lifetime when the optimal ratio of technical coefficients changes. The total derivative is:

$$rG/[1-e^{-rM^*}]^2 dM^* - \frac{a_2}{a_1}(r+\gamma)\, d\left[\frac{A_1^*}{A_2^*}\right] = 0 \tag{4}$$

G is defined by

$$G = \gamma e^{\gamma M^*} + r e^{-rM^*} - (\gamma+r)\, e^{(\gamma-r)M^*} \tag{5}$$

At M^* equal to zero, G is zero and the derivative of G with respect to M^* is positive; therefore, G is positive. From (4) we can then conclude that the optimal ratio of technical coefficients and the lifetime change in the same direction.

We complete the analysis of the impact of price changes by taking the total derivative of 4(1). The total derivative is

$$d\left[\frac{p_1}{p_2}\right]-\left[2\frac{a_2}{a_1}\frac{A_1^*}{A_2^*}\frac{(1-e^{-rM^*})}{r}\right] \times d\left[\frac{A_1^*}{A_2^*}\right]-\left[\frac{a_2}{a_1}\left[\frac{A_1^*}{A_2^*}\right]^2 e^{-rM^*}\right]dM^*=0 \qquad (6)$$

The multipliers of $d(A_1^*/A_2^*)$ and dM^*, the changes in the ratio of technical coefficients and the lifetime respectively, have the same sign. From (6) we then can conclude the following rule: when the ratio of the equipment price, p_1, to the fossil fuel price, p_2, increases, the level of research in capital relative to fossil fuel increases. Conversely, when the fossil fuel price goes up, relatively more research is being done on the efficiency of fossil fuel use. The research (and development) allows the economy to substitute equipment for energy in production.

3.13 CONSIDERATIONS OF MODEL WORKABILITY

Up to this point we have been concerned with model responses to specific patterns of shadow price increases. But it is difficult to examine even simple changes with this model. We look for an equilibrium when carbon dioxide causes the fossil fuel price to rise along an approximately exponential path. We find that the research response in this model to such a price rise is complex and cannot be described in simple terms.

It is natural to examine three possible equilibrium investment patterns: (i) balanced investment, (ii) all investment in A_1, and (iii) all investment in A_2.

If we examine the equation governing the balanced growth path, 10(12), we see that a balanced growth path with constant lifetimes and exponentially increasing fossil fuel costs is not possible. According to 10(9) and 10(7) lifetimes cannot vary in any simple way so that a balanced growth path is possible. Putting all our research in A_1 also cannot satisfy the necessary conditions. With all research in A_1, 10(9) and 10(7) give two different expressions for L. Neither can all research in A_2 produce a simple solution. From 10(9), when all investment is in A_2, the lifetime can only be constant if the rate of increase in research and the price is the same. If these rates are the same and the lifetime is constant, μ_1 is constant and μ_2 decreases at an exponential rate. The value of research in capital must eventually become higher than the value of research in fossil fuel. If all research is in A_2 and the lifetime is not constant, we cannot show that the necessary conditions are not satisfied, but any solution would be quite complex.

In order to accommodate some of these limitations, we will change the model in two important ways.

First, we assume that energy use is proportional to the number of machines in use and *not* the level of production. In the previous model, if no research was done on energy use but if machines were made twice as productive, each machine would consume twice as much energy. We now move to a situation, where machines may be made more productive without increasing the energy consumption per machine. In this modified model, the energy use of each vintage is:

$$F(k) - I(k)/A_2(k) \qquad (1)$$

Second, we assume that over time the rate of growth in research funds, R, and the price of fossil fuels, p_2, is the same. This seems to be reasonable if the research budget and the harm due to CO_2 are both proportional to an exponentially growing economy.

We conclude that given the above assumptions, technical progress is neutral when the cost of energy rises exponentially. In this regard, the model appears less realistic than the previous model. However, this model's behaviour may give an indication of the performance of similar, more realistic, but more complex models.

3.14 SPECIFIC MODEL CHARACTERISTICS

The necessary conditions are quite similar to those of the previous model. The major difference is that the output per unit of energy input is the product of A_1 and A_2. The Lagrangean equations are replaced by

$$L(t+M(t)) = \frac{p_2(t+M(t))}{A_1(t)A_2(t)} \tag{1}$$

$$L(t) = \frac{1}{A_1(t)}\left[p_1(r+A_1^*) - \frac{dp_1}{dt}\right] + \frac{1}{A_1(t)A_2(t)} \left[p_2 + (A_1^*+A_2^*)\int_t^{t+M} e^{-r(x-t)} p_2(x)\,dx\right] \tag{2}$$

(1) states that a machine is retired when the value of production from a new machine equals the operating cost per unit of production of a machine to be retired. (1) can be interpreted just as 10(7). L is again the present marginal value of a new unit of production.

The first pair of parentheses on the right-hand side of (2) contains the capital costs per unit of output, and the second pair contains the energy costs per unit of output.

Proposition 2

Given the energy requirement in 13(1), if p_1 is constant and p_2 and $R^*(t)$ grow at the rate γ, there is a unique ratio of technical coefficients, A_1^* to A_2^*, such that θ and M are constant along an optimal path.

The ratio is expressed as:

$$\frac{A_2^*}{A_1^*} = \left[\frac{a_2}{a_1} - \frac{A_2^*}{A_1^*}\right] \frac{p_2(0)\,(e^{(\gamma-r)M^*}-1)}{A_2(0)\,p_1} \tag{3}$$

Proof

The proof is very similar to Proposition 1. (3) is derived by equating the rates of change of the value of research in each technical coefficient. The equation which allows us to solve for the constant lifetime, M^*, is:

$$e^{2\gamma M^*} = \frac{A_2(0)}{p_2(0)} p_1(r+\gamma) + \frac{2\gamma}{r}(1-e^{-rM^*})+1 \tag{4}$$

These two models illustrate the difficulties under which the relationship of energy to production is specified in order to determine the proper research response to increasing CO_2.

3.15 CONCLUSIONS

By referring to the step model of CO_2 impacts we observe that in such a model a critical CO_2 level imposes a long-run limit on fossil fuel use, and this limit is reflected in the shadow price of fossil fuel.

We assume that fossil fuels have no cost except for the shadow price.

In our reference model the level of output was constant and exogeneously specified. The level of fossil fuel use continually falls along an exponential path as efficiency in fuel use is gained. Therefore, in such a model, long-run limitations on fossil fuel use are irrelevant.

In the two models presented here such limitations are relevant. Let us analyse this property more closely.

In the first model, demand Y^*, and the research budget, R^*, grow at the same rate.

The solution is very similar to the initial model presented. We gain one unknown in the shadow price, but the added condition that emissions be less than or equal to the absorption rate times the critical CO_2 level allows us to solve for this unknown. A balanced research policy with a fixed ratio of knowledge about capital and fossil fuel use satisfies the necessary conditions. The shadow price in this model is constant.

In the second model rather than meeting a set production goal, we use the utility maximization framework. The constant relative risk aversion utility function is assumed. Output can only be raised by adding additional machines, there is no research on improved machine productivity. The level of research on the efficiency of fossil fuel use is an endogenous variable. Maximization is over the investment in new machines, the level of research on fossil fuel use, and the machine lifetimes. The economy, consumption, production, investment, research, all grow at a single internally determined rate. The shadow price rises at the rate of growth of utility which is the growth rate times one minus the consumption elasticity of utility.

The main conclusion of this part of the paper is that economic factors may have dramatic effects on the pattern of technical development. We have found that the neutral pattern of technical development, often assumed in energy environmental economic studies can be the outcome of several different sets of economic assumptions. A second major conclusion is that the type of impact is highly dependent on which model structure is considered most appropriate.

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CHAPTER 4

ENERGY - ECONOMY - ENVIRONMENTAL MODELS WITH SPECIAL REFERENCE TO CO_2 EMISSION CONTROL

Abstract

Following the simple aggregative models of long-run growth and environmental constraints, developed in previous papers, we now focus on a discussion of large-scale models of energy, economy and CO_2 interactions (EEE models).

Our primary conclusions are that CO_2 feedback effects and simple forms of optimization could be added to conventional EEE models without much difficulty. They would greatly enhance the usefulness of these models.

The advantages and difficulties of including such features as optimization, CO_2 feedback, and control of knowledge and physical capital stocks are considered. The structure and results of each model are discussed separately.

4.1 INTRODUCTION

As a consequence of the modelling exercise in previous Chapters 2 and 3 the question arises as to how these models could be extended and improved for large- scale use so as to analyse more detailed, more specific policy options.

Before we suggest extensions, generalizations and ramifications of such models for more policy-directed use we review the state of the art of energy modelling today and possibly derive the lessons that can be learnt to date. As a natural starting point we examine the world energy and carbon dioxide model by Edmonds and Reilly (1983) and the model by Nordhaus and Yohe (1983).

4.2 REVIEW OF ENERGY - ECONOMY - ENVIRONMENTAL (EEE) MODELS

We will start with some early modelling approaches of W. D. Nordhaus (1979, 1980), in the context of carbon dioxide policies, involving an activity analysis model adapted to the CO_2 problem.

This is an instructive example of the application of simple optimization models to a quantitative, qualitative and integrated analysis of CO_2 strategies. The analysis contains the major ingredients of EEE models of this kind:

(i) the dynamics of the CO_2 cycle, the sources of CO_2 and the diffusion of atmospheric CO_2 and the limits of CO_2 concentrations;

(ii) the CO_2 energy model, e.g. a multi-sector activity analysis model which involves step-wise linear programming type optimization over a set of equidistant periods (ten periods each with twenty years);

(iii) the development of control strategies based on shadow prices of CO_2 emissions and costs of abatement.

There are some obvious links between this approach and the models presented in Chapters 2 and 3: an explicit statement of an objective function and an equation for change in atmospheric CO_2. Nordhaus' consumption function includes two terms, a term dependent on fossil energy use alone minus a term dependent on CO_2 alone. Of particular interest is Nordhaus' formulation of a 'carbon tax' on fossil fuel emissions, as an outcome of deriving CO_2 control strategies. Otherwise, his study appears to be more narrowly focused, and since his model is not truly dynamic he does not address issues such as uncertainty, various types of technological progress, intergenerational equity and international co-operation, or changes in capital formation.

A much more detailed and more extensive EEE modelling effort has been initiated by Edmonds and Reilly (1983, 1985). However, there are still weak points. The model is not optimizing, feedback effects are not considered, capital investment is not traced and uncertainty is neglected.

The model matches the demand and supply for energy at discrete points in time in individual world regions. The model structure is relatively simple, but its disaggregation of data both increases realism and makes the results difficult to track. The model considers nine world regions, six major primary fuel categories, and three energy demand sectors. Energy demand is first projected based on population, economic activity, technological change, energy prices, and energy taxes and tariffs.

The model determines the supply of primary energy in three different ways. Resource constrained fuels, such as conventional oil and gas, are produced according to resource depletion curves and their supply is unresponsive to price. Hydro power and biomass are considered resource constrained renewable sources which reach and maintain a specified production level. Their production is also insensitive to price. Finally, sources such as coal, nuclear, solar and unconventional oil and gas are classified as unconstrained. The production levels of these sources are not limited but depend on price. The primary fuels are converted to secondary fuels and then to energy services. The prices of energy services depend on the price of primary energy, the mix of energy sources providing the service, and transportation and conversion costs. The initial fuel mix is exogenous, but the mix is then modified in light of energy prices. The final energy use is determined by an iterative process which changes prices, balances energy trade among regions and balances energy supply and demand for each region. As energy prices vary, demand and supply adjust in response to exogenously specified demand elasticities and supply functions. Based on the mix and level of fossil fuel use, the carbon dioxide emissions are calculated. The retention of carbon dioxide in the atmosphere and the consequent climate changes are determined by a sophisticated model which considers other greenhouse gases, ocean absorption of CO_2 and other factors.

The Edmonds and Reilly (ER) model is part of an integrated U.S. EPA report (1983). The report's major conclusions are that little can be done to delay a greenhouse warming and that major uncertainties include the effects of other greenhouse gases and the temperature sensitivity of the atmosphere. The report emphasizes the importance of coal use in determining the CO_2 level and of the trace gases in determining the eventual temperature increase. In the ER model, logistic curve shaped paths are exogenously specified for several new energy technologies. A related MIT study, Rose et al (1983), modifies these paths and adds additional technologies. It finds that the adoption of realistic but CO_2 benign technologies, while not eliminating a significant CO_2 warming, could increase the CO_2 doubling time to several centuries.

Another major and comprehensive report is that of the National Research Council

(1983). It uses energy-economy, climate and agricultural models to predict future impacts of carbon dioxide and trace gas accumulation. The major conclusions are that no radical action should be taken, that increases in carbon dioxide are likely, which will cause measurable rises in the world's average temperature, and that more research is necessary.

Referring to the energy-economy model by Nordhaus and Yohe (NY), contained therein, one can note that the technology development and elasticity of substitution parameters critically affect the model's results. More specifically, the NY model maximizes consumption in individual time periods. A highly aggregate function determines production - the model considers no world regions and only two fuel types, carbon and noncarbon. The model is not optimizing over time, does not include feedback effects, and does not consider capital stocks. The most significant characteristic of this model is the estimation of probability distributions for important future variables.

Furthermore, overall technical progress is assumed at a constant rate. The production function relates the energy and labour sectors in a Cobb-Douglas fashion; however, a modified form of this function is used. In the usual Cobb-Douglas formulation, the production elasticity of an input is constant and equal to the share of GNP assigned to the input. Further, in the usual formulation the elasticity of substitution between inputs is always -1. These rigidities are a major drawback to the use of the Cobb-Douglas function.

In the NY model the payments to inputs are adjusted in each time period. The adjustment is consistent with a changing, exogenously set elasticity of substitution between labour and energy. The reason is to overcome the limitation placed on substitution elasticities by the Cobb-Douglas function. Within the energy sector there is a constant elasticity of substitution between the fossil and non-fossil inputs.

The price of noncarbon based fuel equals the sum of distribution costs and production costs. The distribution costs are constant, but the production costs change exponentially over time. The rate of change is the sum of a term representing the technical change in the energy industry and a term representing a bias towards noncarbon energy.

The equation for carbon fuel prices is similar but somewhat more complex. The first term combines both production costs and costs due to fuel depletion. Further, the second term is multiplied by the exponential change in energy industry technology, but there is no bias term. Finally, a tax on carbon fuels may be added. The emissions of carbon into the atmosphere per unit of carbon fuel is assumed to grow over time, because the mix of fuels includes more high carbon content fuels.

They assume that parameters describing the current economy can be determined accurately, although they wish to account for an inherent uncertainty in future values of parameters.

The authors estimate the probability distributions on important parameters using the distribution of published predictions.

4.3 TREATMENT OF NEW ENERGY TECHNOLOGIES

Because the levels of fossil and non-fossil fuel development will have enormous impacts on the severity of the CO_2 problem, we find that technical change and technology substitution are specified exogenously or modelled in a simple manner.

In some technology development models, for example Marchetti (1975), long development cycles over several decades are given for the introduction of significant levels of non-fossil fuels. In these models forecasts on the use of new fuels are based on their growth immediately following introduction and the assumption of a logistic growth path in the future. The logistic assumption is very popular with technologists because of its reasonable looking symmetric shape and good fit with the historical growth of new technologies.

Peterka (1977) and Spinrad (1980) consider the logistic process as very natural for the proliferation of energy technologies.

This approach, however, has two specific problems which we seek to avoid in our description of technology: the observations do not have a good theoretical basis and the technologies appear exogenously rather than through a process of research (Chapter 2).

In studies of CO_2 by Perry et al (1982) the authors first assume an energy demand level and a fossil fuel use pattern. Fossil fuel use follows a logistic curve between the present and an assumed ultimate level of use. The rate of non-fossil energy growth needed to fill the gap between fossil energy use and an assumed total energy demand is then examined. Their work emphasizes the importance of analysing the investment needed in non-fossil energy to fill this gap but does not present a model of the substitution process. In our models, the substitution process is a direct result of our maximization of welfare.

In these studies we see an agreement about the importance of technological progress but a lack of detailed modelling. None of them show the model-theoretic connection between economic factors and changing technologies developed in our models.

As a prototype model for treating technical progress and capital investment in energy-economy interactions we can consider the ETA-MACRO model by A. S. Manne (1977). This model can be described as a multi-sector 'look ahead' model simulating a market economy through a dynamic, non-linear optimization process. It examines consumption and investment policies and their impact on national welfare. National welfare is measured by discounting utility from the present to a distant future. ETA-MACRO consists of two models - a macro-model of the whole economy and a more detailed model of the energy sector.

In this model future changes in energy technologies and input-output coefficients may be exogenously specified. It assumes that capital in place requires fixed inputs of energy, labour etc, but that new technology uses the most efficient combination of inputs. This is the 'putty-clay' assumption. The model assumes a single capital good can be used to help produce any type of energy. ETA-MACRO uses a single energy aggregate distributed to the rest of the economy.

The model seems to be more sophisticated in its treatment of capital and the determination of the desirable level of energy use than the models presently used in CO_2 analysis. However, in its present form it is unsuitable for examining international problems such as CO_2. It also lacks a treatment of endogenous technical progress.

Such a model has desirable features for EEE interactions. It is optimizing and considers costs and benefits of capital investment.

More recently, A. S. Manne (1990) has proposed substantial modifications of his ETA-MACRO model for CO_2-energy-economy interactions integrated into his Global 2100 model. Global 2100 is a multi-regional model in which each region, pursuing its own interests, is a contributor to global carbon emissions. Since each region is likely to pursue its own individual interests rather than the global welfare, such a model could be solved within a computable general equilibrium framework.

We examine the effects of technical change on the economy and CO_2 accumulation in both single and multiple state models. The simplest type of technical change is a finite or limited improvement in a technology.

The key difference in the single and multiple state variable models is the ability to control technical change. In the single state models technical change is exogenous; in the multiple state models, the rate of technical change is determined by decisions on investment.

Another interesting extension of ETA-MACRO, albeit in its impact assessment limited to the U.S., is the recently established GEMINI model (Scheraga et al, 1990).

4.4 ECONOMIC MODELS OF POLLUTION AND CONTROL

A different class of models originating in pollution and environmental economics deals with the intertemporal optimization of welfare functions under resource constraints relating shadow prices to optimal taxes. Unfortunately, to our knowledge, there has been no attempt so far to link these models to EEE models and thus provide more sophisticated explanations for policy responses to global environmental issues. The general static model of pollution considers both consumers and producers. The model is solved for the conditions which govern Pareto optimality. Baumol and Oates (1975) make clear the distinction between pollutants that impinge on utility rather than production. Many types of pollution affect utility directly such as air pollution; however, the major CO_2 impact will most likely affect production.

They also show that tax rates which achieve a desired reduction in pollutants will satisfy the necessary conditions for Pareto optimality. By reviewing optimal control models of pollution problems we observe that none of these models are appropriate for the analysis of CO_2.

Still there are some key elements that have to be integrated into general EEE models for global environmental problems.

In other respects, specific distinctions remain. For example, D'Arge and Kogiku (1973) introduced optimal control models with two important features: they assumed that pollution affected utility directly and their models consider a finite horizon. They found several non-intuitive results in their models, many caused by finite horizon assumptions. In the context of the issues pursued we are convinced that infinite horizon models make more sense. Three specific optimal control models, for different reasons, are of some interest for our modelling exercise.

Forster (1980) develops three models of pollution from a source which is in limited supply. In all three models, the pollution affects utility directly and acts as a flow. He is mainly concerned with the existence of equilibria. Asako (1980) and Becker (1982) are both concerned with issues of intergenerational equity and the contrast between maximin solutions and utility maximization. In models with technical progress, maximin solutions lead to higher current emissions than utility maximization (in a comprehensive sensitivity analysis for large-scale models other decision criteria could be used).

As a rough approximation this suggests that utility maximization with discounting is not a particularly short-sighted approach to planning.

Another string of models and research results of resource economics (on the depletion of non-renewable resources) could be applied with simple modifications to the CO_2 problem.

Under two crucial assumptions the problem of fossil fuel use in the face of increasing carbon dioxide is parallel to the problem of consumption of a limited resource. The first assumption is that the carbon dioxide absorption rate is sufficiently small to be ignored. The second is that CO_2 impacts follow a 'step' pattern, that is, CO_2 (as a pollution stock) has no impact on productivity until a critical level, M_c, is reached; then if the CO_2 level exceeds M_c, production drops sharply (or more extremely, falls to zero).

A model with endogenous neutral technical progress, as in chapter 3, is proposed to provide a better explanation of technical changes used to date in EEE models. Such a model originates from a similar attempt by Chiarella (1980). He proves the existence of a steady state growth path and a simple rule governing the rate of investment in research. Research investment along the optimal path should be carried out until the growth rate in the marginal accumulation of technology equals the difference between the marginal product due to an extra unit of research investment and the marginal product of capital.

Another issue is uncertainty. Here again there is a link with models of resource use for a limited, non-renewable resource when the reserve of the resource is unknown.

The key finding of models by Loury (1978) and Gilbert (1979) is that plans for resource use based on the expected level of a resource will be overly optimistic. Gilbert's model is conducive to our models of fossil fuel use when the critical CO_2 level is uncertain. Under the above assumptions this problem is equivalent to determining the rate of fossil fuel use when the critical concentration of atmospheric carbon dioxide is unknown. Their results show that the optimal use of fossil fuel is lower when uncertainty is properly considered than when the expected values are assumed to be certainty equivalents.

A significant additional element is the possibility of undertaking exploration to find new reserves. The parallel in the CO_2 problem is R & D to increase the probability of finding a technology for the removal of CO_2 from the atmosphere.

Deshmukh and Pliska (1980, 1983) find that in the periods between discoveries or research breakthroughs, fossil fuel use and consumption fall, but if R & D is very successful, long-run fuel use may rise.

There are three major issues that have been addressed in our models but they have not been dealt with in a systematic way in all the major EEE models proposed and implemented. These are international or multi-regional issues, technology issues and issues of uncertainty and risk. In a paradigmatic way we go through each of those issues and offer some solutions in the context of our models. We are still far from offering an integrated view.

4.5 INTERNATIONAL ISSUES

Much of the work on international relations, trade and pollution deals with legal issues and the form of international control agreements. Mathematical modelling has concentrated particularly on balance of payments and terms of trade when a country controls pollution within its own boundaries. Even in modern textbooks on trade theory and international economics, pollution problems are barely mentioned (Dixit and Norman, 1980).

Transnational pollution involving adjacent nations where one is the polluter and the other receives the pollution has received little attention, and global pollution where all or many nations contribute has received even less (Baumol and Oates, 1975, Chapter 1, 16) but some recent work (Ulph, 1990) points to the right direction.

The difficulties just mentioned suggest that a single country or a group of countries may wish to unilaterally alter the international use of fossil fuels through export and import controls such as taxes and subsidies. We approach this problem as part of the modelling effort in Chapter 2. The model assumes there is a shadow cost, q, associated with the use of fossil fuels. The results are equally applicable if this is a static cost or a changing cost in a dynamic model. In the model the government wishes to control two variables, F, domestic fuel use, and F_e, exports of fossil fuels. Domestic production of fossil fuels, F_d, equals F + F_e. First assume U(C) equals C and that atmospheric CO_2 does not affect the economy ($q = 0$):

$$C=f(F)-c(F_d)+p_iF_e \tag{1}$$

f is the goods production function, c is the fossil energy cost function, and p_i is the international price of fossil fuels. In particular cases p_i may be a function of F_e. The two necessary conditions for maximizing consumption are:

$$f'=c' \tag{2}$$

$$c'=p_i+\frac{\partial p_i}{dF_e}F_e \tag{3}$$

In a competitive international market the partial derivative of p_i with respect to fossil fuel exports is zero, and the national government has no control over the international price, p_i, or the international use of fossil fuels. Production occurs until the marginal cost equals the international price.

Now consider a situation in which CO_2 accumulation has an adverse impact, but the impacts are not observed outside the country of interest.

Let F_i be the international fossil fuel consumption. Consumption can now be expressed as below.

$$C=f(F)-c(F_d)+p_iF_e-q(F+F_i) \tag{4}$$

q is the domestic shadow price of world, domestic plus international, fossil fuel use.

The necessary conditions can be stated as:

$$f'=c'+q \tag{5}$$

$$c'=p_i\frac{\partial p_i}{\partial F_e}\left[F_e-q\frac{\partial F_i}{\partial p_i}\right] \tag{6}$$

[Maximization of (4) gives the same necessary conditions as the optimal control problem with an export sector, except for the import difference that we have no information on the magnitude and rate of change in q. Because we only use the fact that q is positive, we can use the much simpler problem statement in (4).]

First, consider the competitive international case. Comparing (2) with (5) it is seen that f' is unequal in the two situations; therefore, domestic use of fossil fuel changes. A tax of q, which is greater than zero and changes over time in dynamic problems, can be placed on domestic fossil fuel use to reduce domestic use to the optimum. Since non-domestic (international) use does not change this will cause a decrease in world emissions. In the competitive case the partial derivative of the international price with respect to fossil fuel exports is zero. Equations (3) and (6) are identical in this case, therefore, domestic production and international use of fossil fuels are the same with or without CO_2 problems.

Because net exports equal domestic production minus domestic consumption, concern over CO_2 in a competitive environment has the odd effect of increasing exports of fossil based fuels.

A more realistic model would consider that even in a highly competitive market any increased exports would lower the world price, reducing world production and increasing consumption outside the concerned region. This more complex model, however, would not change the basic conclusion that, in a highly competitive market, exports from the concerned nation should increase.

Now consider the even more realistic case of a non-competitive world, a world in which the country or countries of concern are large enough producers of fossil fuels to affect the international price and use of fuels. In this case, changes in exports do alter the international price and consequently international consumption.

From (3) it can be concluded that even in the absence of CO_2 problems it will be optimal to tax exports at a rate equal to the partial derivative of the international price with respect to exports times the level of exports. This drives up the price and allows the domestic economy to take advantage of its quasi-monopoly position. When concern exists about CO_2 in a non-competitive market (6) shows that a still higher tax may be placed on exports. This tax further reduces domestic production and international fossil fuel use.

Equation (5) has the same form in both the competitive and monopoly cases: therefore, a domestic use tax may be used to achieve optimum domestic use. When markets are non-competitive we cannot determine if the reduction in domestic fossil fuel use is greater or less than the reduction in fossil fuel production; consequently, it cannot be determined if exports increase or decrease. Note that in a dynamic model, q will be different in competitive and non-competitive markets because of differences in the levels of emissions over time.

Another interesting case occurs when the country or countries concerned about CO_2 are major exporters of a fossil fuel substitute. If the market is very nearly competitive and the fossil fuel substitute is priced very close to fossil fuels, a small export subsidy may cause a very large switch to the substitute. As in the case of a fossil fuel export, if a monopoly position is realized there exists an incentive to tax exports of the fossil substitute and raise its international price. Because use of the substitute reduces CO_2 output, there may be opposing reasons to subsidize or tax international sales of the substitute. Only in specific cases can it be determined whether the net result will be a subsidy or tax on exports.

More sophisticated results have recently been obtained by M. Hoel (1990).