Working Paper 1993:11

Pollution, Technology Transfer and Sustainable Growth

by

Hans W. Gottinger
Pollution, Technology Transfer and Sustainable Growth

Hans W. Gottinger

Abstract

We provide a model of international technology transfer in its linkage to pollution and economic growth. Technology is assumed to be transferred via international capital movements from the developed North to the developing South. In this model, free capital movements, as compared to autarky, have beneficial effects on an initially backward country’s technological change and sustainable growth rate. Policies prohibiting investment from the North to the South prevent the income gap, defined as per capita income ratio between the two regions, from narrowing.
Pollution, Technology Transfer and Sustainable Growth

1. Introduction

We consider a situation of international capital movements, technology transfer in a two-country or two-regions world, North and South. This is based on a dynamic model with exogenously specified savings rates and a description of technological change in a less developed country (LDC). The description assumes that the rate of technological improvement will be an increasing function of the amount of foreign capital invested and of the extent to which the foreign technology is much more advanced than that of the LDC. We assume that direct investment from the advanced North creates a positive externality on the production efficiency of the LDC firms, and a reduction of negative externalities of LDC emission rates contributing to a cumulative global greenhouse budget. From a growth and environmental perspective, attracting more direct foreign investment and technology transfer will have long-run benefits for both North and South.

As we approach neoclassical growth theory its assumptions on technology are the most critical. One concerns the assumption of constant returns to scale and the notion of exogenous technological change (Romer, 1986; Lucas, 1988). These economists revived the notion that some form of increasing returns to scale is essential to the understanding of long-run growth.

A distinct feature of this new focus on technological change is the emphasis on the role of technological knowledge (or human capital) and the spillover effects of investment in knowledge. The last point has been pursued in endogenizing technical change in the context of energy-economy-environmental (EEE) models (Gottinger, 1991).

In an interesting paper, in the spirit of the new growth theory, which relates growth, trade to international policy coordination, van der Ploeg (1991) raises the important issue whether in case of international knowledge spillover (or technology transfer) “international policy coordination may boost growth and harm environmental quality”. He uses differential game theory to derive the results. Our tools are more conventional. We show that this need not be the case if technology transfer would involve a substantial part of pollution abatement technology or new pollution saving production technology (clean technology). Thus a growth path securing sustainability in the long run could be assured.

In the next two sections we explore a dynamic two-country model with exogenously specified savings rates and a description of the rate of technological change in a LDC. This description assumes that the rate of technological improvement will be an increasing function of the amount of foreign capital invested in the LDC and of the extent to which the technology in the advanced country exceeds that in the LDC.

Technological improvement may come in two configurations. (i) increased production efficiency and/or (ii) new pollution saving devices. Any improvements in each of these configurations or both will result in desirable consequences, but only if both are clearly linked there would be sustainable growth. This model is rather modest, in deriving saving rates, technological improvement and productivities, but on the other hand it has the advantage to be simpler to manipulate and to answer specific questions, like what happens if savings increase in a LDC while the rate of diffusion of
technology stays the same.

2. A Model of Growth and Technology Transfer

In order to construct the model we first need some notation and definitions. Let the subscripts S and N denote South and North, respectively. The two economies are endowed with different amount of capital in both physical and human forms:

\[ k_S < k_N \quad \text{and} \quad h_S < h_N \]

Define the usual per capita quantities:

\[ y_i = Y_i / L_i, \quad k_i = K_i / L_i \]
\[ w_i = W_i / L_i, \quad z = Z / L_N, \quad i = S, N \]

where \( Y_i \) and \( W_i \) are national income (GNP) and wage bill in country \( i \), respectively, and \( Z \) is the amount of foreign capital located in the S-country.

Two fundamental variables are relative (physical) capital intensity \( k \) and technology gap \( q \) between the two regions:

\[ k = k_S / k_N, \quad q = h_S / h_N. \]

National income \( Y \) is equal to GDP \( Q \) plus or minus the foreign investment earning \( rZ \), where \( r \) is the rate of return of capital in the world. Thus in terms of the production functions for both regions we have

\[ Y_S = \Omega(h_S)f(k_S + z) - rz \]
\[ Y_N = \Omega(h_N)g(k_N - z) + rz \]

The two regions are linked together in an international capital market. The static equilibrium in this world is obtained when the rental rate of physical capital is equalized across regions, assuming (physical) capital is perfectly mobile. The arbitrage equation which should hold every moment is

\[ \Omega(h_S) f'(k_S + z) = \Omega(h_N) g'(k_N - z) = r, \quad (0) \]

with primes indicating derivatives.

We let the production function be Cobb-Douglas, i.e.

\[ Q = K^\beta (hL)^{1-\beta} h^\alpha, \quad \alpha \epsilon (0,1), \quad \beta \epsilon (0,1) \]

Per capita output is \( Q / L = (K / L)^\beta h^{1-\beta + \alpha} = f(K / L)\Omega(h) \) The contributions of human capital to production is summarized by the power function \( \Omega(h) = h^\nu, \quad \nu = 1 - \beta + \alpha \) where \( \alpha \) is the external effect of human capital.

To establish a dynamic model the accumulation functions of both physical and human capital need to be specified. The (physical) capital formation functions are the standard ones:

\[ k_i = \sigma y_i - \lambda_i - k_i, \quad i = S, N \quad (1) \]
where \( ^* \) indicates the time derivative of the variable and \( \sigma_S \) and \( \sigma_N \) are fixed savings propensities. Per capita gross investment is \( \sigma_y \).

To maintain the existing capital-labour ratio, \( \lambda_k \), amount of investment is required, where \( \lambda \) is the sum of the growth rate of labour and the capital depreciation rate.

How the system evolves over time hinges on our assumption about the dynamics of human capital, the spread of knowledge and skills.

Three different ways to model the augmentation of aggregate human capital can be traced from the literature:

1. the stock of knowledge is increased through a research or education production function which uses real resources (Uzawa, 1965, Shell, 1967, Romer, 1986),

2. learning by doing (Lucas, 1988)

3. learning from advanced countries through trade and technology transfer (Jensen and Thursby, 1986)

In what follows we emphasize the last point. It is assumed that both countries’ stock of human capital grows at constant rates \( \mu_S \) and \( \mu_N \) before trade occurs, and \( \hat{h}_S \) is an increasing function of the degree to which the Southern country is open to foreign investment, measured by the ratio of foreign investment to domestically owned capital, \( x \).

Then,

\[
\hat{h}_S = \mu_S \theta(x,q;\tau)h_S \tag{2}
\]

\[
\theta_1 \equiv d\theta / dx > 0, \quad \theta_2 \equiv d\theta / dq > 0,
\]

\[
\theta_3 \equiv d\theta / d\tau > 0, \quad \theta(0,1;\tau) = 1
\]

\[
\hat{h}_N = \mu_N h_N, \quad \mu_N \geq \mu_S \tag{3}
\]

The intuitive explanation of the term \( \theta(\cdot) \) is that a typical developing country wants foreign capital not only because it is capital but also because it embodies superior technology. The presence of foreign firms generate positive technology spillovers to the LDC firms. This is what we call technology transfer in this model. This linkage of the world economy has been empirically important. In fact, most developing countries in the world rely heavily on foreign technologies. This is a shifting parameter which can be interpreted as the technology diffusion rate or technology adaptive efficiency in the South. The growth rate of Northern human capital, \( \mu_N \), is assumed to be exogenous. Since strong evidence suggests that physical capital investment leads to simultaneous creation of new knowledge that spills over and has positive external effects (see Romer, 1987), one could link \( \hat{h}_N \) to the level of investment activities in the North. The present specification (3) is hence weakly justified. The assumption that \( \mu_S \leq \mu_N \) makes sense if one thinks that \( \mu_i \) is in some way related to \( k_i \) and \( h_i = i=S,N \). The better education and research facilities as well as higher average human capital thus enable the North to augment technical knowledge at
relatively faster pace than the South.

Equations (1)-(3) form our dynamic system. To obtain the solution we need the following preliminary results, in approximation:

Lemma 1. (Income Adjustment)

\[
y_k / k_S = C y_S(k,q), \quad d\tilde{y}_S / dk < 0, \quad d\tilde{y}_S / dq < 0 \quad (4)
\]
\[
y_N / k_N = C y_N(k,q), \quad d\tilde{y}_N / dk > 0, \quad d\tilde{y}_N / dq > 0 \quad (5)
\]

where \( C \equiv h^\beta k^{\beta-1} > 0, all t. \)

Proof. Appendix 1.

With the help of Lemma 1, we can convert the system (1) - (3) into another system of three differential equations in terms of \( k, q, \) and \( C. \) The \( k \) equation is obtained as follows. The definition of \( k \) implies

\[
\dot{k} / k = C[\sigma_S \tilde{y}_S(k,q) - \sigma_N \tilde{y}_N(k,q)]
\]

(6)

where \( \tilde{y}_i, \quad i=N,S \) and \( C \) are defined in Lemma 1. Similarly,\(^1\)

\[
\dot{q} / q = \mu_N - \mu_S \theta(x(k,q), q; \tau)
\]

(7)

\[
\dot{C} / C = \nu \mu_S \theta(x(k,q), q; \tau)(1 - \beta) C[\sigma_S \tilde{y}_S(k,q) - \lambda]
\]

(8)

There are two opposite effects on the growth rate of Southern human capital in the technology transfer function \( \theta(.) \) specified in (2). A decreasing \( q \) induces more foreign capital flows, which tends to drive up \( \dot{h}_S / h_S. \) However, the narrowing income gap also slows down the pace at which a developing country can catch up.

It is assumed that the gap effect dominates the induced capital inflow effect.

\[
\theta_2 > \theta_1 \quad *(dx / dq) \]

(9)

\(^1\)It is worthwhile to point out the correspondence between (7) and the innovation and technology transfer equations in the “product cycle” models of Krugman (1979) and Dollar (1986). Krugman’s technology transfer is exogenously fixed. Dollar’s is, like ours, motivated by the production cost differentials between the two regions. However, in Dollar’s dynamic analysis the world capital stock is fixed while it is growing in our model.
With inequality (8), it can be shown that the 3x3 system (6), (7), and (8) satisfies the Routh-Hurwitz stability conditions. (Brock and Malliaris, 1989, chap.3) The stability analysis is relegated to Appendix 2. The world steady state is characterized by \((k^*, q^*, C^*)\) such that

\[
C^* \varphi(k^*, q^*) = 0 \quad (10)
\]

\[
\psi(k^*, q^*) = 0 \quad (11)
\]

\[
\Gamma(k^*, q^*, C^*) = 0 \quad (12)
\]

It is obvious from (10)-(12) that the system (6), (7), and (8) can be decoupled, i.e., equations (6) and (7) are sufficient to determine the steady state values of \(k^*\) and \(q^*\). With \(k^*\) and \(q^*\) determined, there exist a unique constant \(C^*\) which satisfies (12). The sub-system (6) and (7) can be dissected by employing the phase diagram technique in the now familiar \(k-q\) plane. Figure 1 depicts the phase diagram of the subsystem (6) and (7).

The KK \((\dot{k} = 0)\) schedule is downward sloping. Given the value of \(k\), as \(q\) decreases, the South becomes more attractive to the foreign capital, the presence of foreign capital \(x\) will increase. From (4)-(6) we know \(\dot{k} > 0\). To keep \(\dot{k} = 0\), \(k\) has to increase to deter an increasing \(x\). The QQ \((\dot{q} = 0)\) schedule is upward sloping. With a constant growth rate of human capital in the North, the changes of \(q\) are determined by the accumulation of human capital in the South. Fixing \(k\), a reduction in the technology gap slows down the rate at which the South can catch up but also induces a higher \(x\). Because the gap effect dominates the induced capital inflow effect by assumption (9), \(\dot{q} < 0\). To restore \(\dot{q} = 0\), \(k\) needs to decrease to induce more foreign capital flows from the North to the South. The horizontal and vertical arrows indicate the direction of variables in the zone where they are drawn. The intersection of KK and QQ curves gives the steady state \((k^*, q^*)\) of the world economy. The superscripts * and o are used to denote the free capital mobility equilibrium and the initial values of the relevant variables, respectively.

The steady state of our model is quite a different concept from that of previous models. Driven by each country’s technological progress, each country’s “steady state” capital-labour ratio is not “steady”. A constant \(k^*\) implies both \(k_s^*\) and \(k_N^*\) grow at an equal “steady state” rate \((\nu / (1 - \beta)) \mu_N\). Both \(h_s\) and \(h_N\) also grow at the same pace set by \(\mu_N\) in the steady state. This is not the case in autarky where the developing country may persistently grow more slowly. Comparisons between the free capital mobility and the autarky steady states yield the following theorem.

Theorem 1: Perfect capital mobility raises steady state growth rates of wage rate, capital-labour ratio and per capita income in the South compared to autarky from \((\nu / (1 - \beta)) \mu_S\) to \((\nu / (1 - \beta)) \mu_S^*\), providing \(x^* > 0\). \(\mu_S < \mu_N\). The corresponding growth rates remain to be the same in the North.

---

2For instance, see Oniki and Uzawa (1965), Ruffin (1979) and Buiter (1981), where both countries share the identical fixed capital-labour ratio in the free trading steady state.
Proof. The autarky steady state growth rates of capital-labour ratio in both regions can be easily deduced from (1)-(3) and (0) by letting \( z \) be zero.

\[
\frac{\dot{k}_i}{k_i} = (\frac{\nu}{1-\beta}) \frac{\dot{h}_i}{h_i} \quad i=S,N
\]

(13)

Any steady state requires \( \theta_y \equiv k_i \) and \( w_i = y_i - r_i k_i \). The autarky rental rate on capital is \( r_i = \frac{W_i f'(k_i)}{k_i} = h_i (1-\beta) \), and eq. (13) implies the steady state constancy of \( r_i \). It follows that \( w_i, y_i \) and \( k_i \) all grow at the same rate \( (\frac{\nu}{1-\beta}) \mu_i \), \( i=S,N \). Clearly the South grows more slowly than the North in autarky if \( \mu_S < \mu_N \).

With perfect capital mobility \( \dot{k}_S^* / k_S^* = \dot{k}_N^* / k_N^* = (\frac{\nu}{1-\beta}) \mu_N \) in the long-run equilibrium \( (k^*,q^*,C^*) \). The world steady state rate of return on capital \( r^* \) is a constant and equal to \( C^* f'(1+x^*) \) from (0). It follows that \( w_i^*, y_i^* \) and \( k_i^* \) all grow at the rate \( (\frac{\nu}{1-\beta}) \mu_N \), \( i=S,N \).

Comparisons between the given initial conditions and the free capital mobility steady state generate the following.

**Theorem 2:** Open to foreign investment from the North raises the growth rate of the South during the transition period to the long run equilibrium. These temporary Southern growth gains do not depend on \( \mu_S < \mu_N \).

**Proof.** \( \theta_y \equiv k_i \) must hold in any steady state. Let \( y \equiv y_N / y_S \). The relations between \( y \) and \( k \) in any steady state therefore is

\[
y = (\sigma_S / \sigma_N) / k
\]

(14)

Given the initial condition \( k^0 \) assumed free capital mobility raises \( k^* \) above \( k^0 \) (see Figure 1). Thus \( y^* < y^0 \) by (14). It implies that \( \dot{y}_N / y_N > \dot{y}_S / y_S \) during the convergence transition.

**Corollary:** (i) The steady state per capita income gap \( y^* (y \equiv y_N / y_S) \) will be enlarging at the rate of \( (\frac{\nu}{1-\beta})(\mu_N - \mu_S) \) in autarky, if \( \mu_S < \mu_N \).

(ii) Shifting from autarky to perfect capital mobility narrows the income gap, which becomes a constant in the steady states. The steady state income gap is jointly determined by the parameters \( \sigma_i, \mu_i, i=S,N \), and \( \tau \).

**Proof.** Statements (i) and (ii) follow from Theorem 1 and Theorem 2, respectively. Since \( k^* \) depends on \( \sigma_i, \mu_i, i=S,N \), and \( \tau \), \( y^* \) must also depend on these parameters.
A possible trajectory is indicated as path AE in Figure 1. A developing country would start with the initial condition \((k_0, q_0)\), a low level of capital stock and a backward technology. With the inflow of foreign capital, advanced technology as well as managerial skill are transmitted through the presence of foreign firms. The growth rate of \(h_S\) increases. International capital movements promote (physical) capital accumulation in the South relative to that in the North mainly due to the Southern efficiency gains. An increasing \(h_S\) increases \(y_S\) through both its internal and external effects on Southern production. Given the fixed saving rate, \(\dot{k}_S\) tends to increase. The system eventually converges to the steady state. The curvature of the AE path suggests that the ratio of foreign capital to locally owned capital in a developing country would increase initially, then maintain a certain level once up to some point and eventually go down (Figure 2a).

Possibles time profile of \(z/k_S\), \(h_i\), \(k_i\) and \(y_i\), \(i=S,N\), along the path AE from \((k_0, q_0)\) to the steady state are displayed in Figure 2.

The assumption that \(\mu_S \leq \mu_N\) in (3) make the steady state \(x^*\) most likely to be positive. It implies that a developing country may be persistently in debt (a net capital and technology importer) in this model given the parameters of saving and technological progress in both countries. In the next section, we consider the impacts of altering these parameters on the long run equilibrium.

3. Shocks to the Long Run Equilibrium

We first study the long-run effects of exogenously shifting the technology parameters \(\mu_S\), \(\tau\) and \(\mu_N\). Then we consider the impacts of changing the saving propensities of both countries. The combination of repeatedly increasing \(\mu_S\), (or decreasing \(\mu_N\)) and decreasing \(\sigma_S\) (or increasing \(\sigma_N\)) suggests a hypothesis regarding the long run fluctuation on one country’s capital account, which we present at the end of this section.

The effects of comparative steady state can be studies formally with the help of equations (6) and (7). Differentiating the system totally and taking \(d\mu_S\) as increase in the autarky rate of technological progress of the South.

\[
\frac{dk}{d\mu_S} \bigg|_{k=0} = -\theta \varphi_q / \Delta > 0
\]

\[
\frac{dq}{d\mu_S} \bigg|_{q=0} = \theta \varphi_q / \Delta < 0
\]

where \(\Delta \equiv \psi_q \psi_q - \varphi_q \psi_q > 0\), \(\varphi_q\) and \(\psi_q\) denote the partial derivatives of eq. (6) and (7) with respect to its argument \(i\), \(i=k,q\). All the partial derivatives are evaluated at the steady state. The impact of changing \(\mu_S\) is clearly seen in Figure 3.

---

3Since the discriminant of the system (6), (7) and (8) is nonzero, we can rule out stable degenerate node. However, the sign of discriminant remains ambiguous. For a given initial condition the system has two possible convergence scenarios, stable degenerate node and stable node. I will confine the discussion in the rest of this paper to the case of stable node on the assumption that technology gap \(q\) changes monotonously during the transition to the steady state.
Since the parameters $\mu_i$, $i=S,N$, and $\tau$ only appear in the $\dot{q}$ equation, the KK schedule will remain unmoved in response to the variations of these parameters. An increase in $\mu_S$ shifts the $QQ$ schedule to the left ($Q'Q'$). In the new steady state the home country enjoys a higher capital-labour ratio and higher level of human capital relative to the rest of the world. During the transition the home country’s capital account will run a surplus. The reason for this result is that as the home country becomes more amiable for foreign investment, foreign capital flows in. The service payment also increases with the increasing stock of foreign capital located in the home country.

An increase in $\tau$, the technology diffusion rate of South, has similar effects on the world long run equilibrium, and hence on the North-South steady state income gap (see eq. (17) and (18)). The higher

$$\frac{dk}{d\tau} \bigg|_{\tau=0} = -\varphi_q \mu_S \theta_S / \Delta > 0$$

(17)

$$\frac{dq}{d\tau} \bigg|_{\tau=0} = \varphi_k \mu_S \theta_S / \Delta < 0$$

(18)

the $\tau$ or $\mu_S$ or both, the narrower the equilibrium per capita income gap between the North and the South.

An increase in $\mu_N$ has the opposite effects on the system. The $QQ$ schedule shifts to the right ($Q''Q''$) in Figure 3. $k$ is lower and $q$ higher in the new steady state than the original equilibrium. The booming innovative drive in the rich country may hold back the capital that would otherwise flow to the less developed countries. The division between the rich and the poor is not only perpetuated but also aggravated. It seems that the worst thing for an open developing economy in this world is to allow its growth rate of human capital to further lag behind the rich countries. Efforts to increase $\mu_S$ or $\tau$ can alleviate or offset the adverse effects of an increase in $\mu_N$ on the relative income between the two regions.

Now we consider the effects of altering saving propensities. Both $\sigma_S$ and $\sigma_N$ only appear in the $\dot{k}$ equation. The $QQ$ schedule will remain unmoved in this exercise. Formally we differentiate the system (6) and (7) and obtain the following partial derivatives.

$$\frac{dk}{d\sigma} \bigg|_{\sigma=0} = -C^* \tilde{\gamma}_S \psi_q / \Delta > 0$$

(19)

$$\frac{dq}{d\sigma} \bigg|_{\sigma=0} = C^* \tilde{\gamma}_S \psi_q / \Delta > 0$$

(20)

An exogenous increase in the saving propensity in the home country shifts the KK schedule upwards. The new steady state levels of $k$ and $q$ are higher than before. Foreign investment will be crowded out. For a developing country the deterrence of capital inflow retards its technological learning process. A drop in the foreign saving rate has a similar impact on the long run equilibrium. Conversely, a rise in foreign saving propensity has a similar effect as a decrease in the home saving rate, resulting in a lower equilibrium value of $k$ and $q$.

The combination of an increase in $\mu_S$ relative to $\mu_N$ and a drop in $\sigma_S$ relative to $\sigma_N$ is reported in Figure 4. The KK schedule shifts to
K’K’ and QQ to Q’Q’. The path AA in Figure 5 shows a country grows from a net international debtor to a net creditor due to a combination of higher $\mu_s$, $\tau$, and $\sigma_s$. In this process foreign capital and technology play an important role. Subsequent combination of higher $\mu_s$ (lower $\mu_N$) and lower $\sigma_s$ (higher $\sigma_N$) lead this now advanced country (though in the analysis it is still labeled South) to become a capital importer again.

4. Conclusions

We have presented a model to study the relationship between international capital movements, technology transfer, and growth. The model highlights the importance of human capital accumulation and its interaction with physical in the economic development process. The economic development process will move on a higher welfare level if technology is directed toward pollution abatement and environmentally sustainable production processes. From a developing country point of view, opening to direct foreign investment from more advanced countries has important implications on domestic technological change and hence on the rate of growth. Policies that increase domestic human capital accumulation, technology adaptive efficiency and saving rate reduce the equilibrium income gap between an initially backward country and the rich ones under the condition of free capital mobility. Policies of prohibiting investment from more advanced countries deprive the growth gains to the policy imposing LDC, and may lead to an increasing income gap between the rich and the poor in the world.

The study of comparative steady state generates a hypothesis about the long run fluctuation of a country’s capital account, which may also be relevant to the understanding of todays world.

Further extension of the model is possible. One might want to relax the assumptions (4),(5) to make the technology progress really endogenous and to derive saving rates from more basic assumptions. Issues concerning welfare policies which have not been addressed in this paper can also be investigated.
References:


Appendix 1. Proof of Lemma 1.

Let \( x = z / k_t = Z / K_s \) and \( x = x(k,q) \). i.e. the direction and quantity of foreign investment is uniquely determined by the configuration of initial endowments in both physical and human forms. By definition of \( C \equiv h_s^\beta k_s^{1-\beta} \), we obtain the following expressions.

\[
y_s / k_s = \Omega(h_s)k_s^{-1}\left[f(1+x) - f'(1+x)x\right] = C\gamma_s(k,q) \tag{A.1}
\]

\[
y_n / k_n = \Omega(h_s)k_s^{-1}\left[q'g(k^{-1} - x) + f'(1+x)k\right] = C\gamma_n(k,q) \tag{A.2}
\]

\[
d\gamma_s / dk = -f''x(dx / dk) < 0, \quad \text{for } x > 0 \tag{A.3}
\]

\[
d\gamma_n / dq = -f''x(dx / dq) < 0, \quad \text{for } x > 0 \tag{A.4}
\]

\[
d\gamma_n / d\gamma = q\left[g - (k^{-1} - x)g'\right] + f''xk(dk / dk) > 0, \quad x > 0 \tag{A.5}
\]

\[
d\gamma_n / dq = vq^{-x}g + f''xk(dq / dq) > 0, \quad x > 0 \tag{A.6}
\]

In deriving (A.5) and (A.6) note that \( f'(1+x) = q'g' (k^{-1} - x) \), where \( \Omega(h_s) / \Omega(h_s) = q' \). (A.1), (A.3) and (A.4) yield equation (4). (A.2) (A.5) and (A.6) imply (5). Given the initial conditions \((h_0, k_0, q_0)\), both \( h_s \) and \( k_s \) are growing over time \( C \equiv h_s^\beta / k_s^{1-\beta} \) therefore is always positive and becomes a constant \( h_s^0 / k_s^0 \) in the steady state due to (13).

Appendix 2: Stability of System (6), (7) and (8).

For analytical tractability, we consider only the local behaviour of the system, linearizing \( \varphi \), \( \psi \) and \( \Gamma \) in a neighbourhood of the long-run equilibrium \((k^*,q^*,C^*)\). This gives the linear approximation

\[
\frac{\dot{k}}{k^*} = C^*\phi_k \quad \frac{\dot{q}}{q^*} = \psi_k \quad \frac{\dot{C}}{C^*} = \Gamma_k \tag{A.7}
\]

where

\[
\phi_k = \left[\sigma_c(dy_s / dk) - \sigma_n(dy_n / dk)\right] < 0 \tag{A.8}
\]

\[
\phi_q = \left[\sigma_c(dy_s / dq) - \sigma_n(dy_n / dq)\right] < 0 \tag{A.9}
\]

\[
\psi_k = -\mu_s \theta_s (dx / dk) > 0 \tag{A.10}
\]

\[
\psi_q = -\mu_s \theta_q (dx / dq) > 0 \tag{A.11}
\]

\[
\Gamma_c = -(1-\beta)\sigma_s y_s < 0 \tag{A.12}
\]

all the partial derivatives are evaluated at the steady state \((k^*,q^*,C^*)\). (A.8) and (A.9) are the consequences of Lemma 1. (A.11) follows from assumption (9). It is transparent that the Routh-Hurwitz conditions for stability are satisfied. All three roots of the system have negative real parts (see Pontryagin 1962, pp.115-26).