Working Paper 1994:1

Principal-Agency Control of Wastes

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PRINCIPAL-AGENCY CONTROL OF WASTES

Hans W. Gottinger

ABSTRACT
We analyze performance and outcome of principal-agency relationships in an environment with pollution externalities and technological progress.

We assume that firms may not purposely violate the pollution control regulations but nonetheless generate some pollution due to negligence. The models allow firms two possible actions: either increase the level of treated waste or pay an expected penalty if illegal pollution is detected.

We show that in a world with pollution externalities, technological progress does not guarantee increases in the welfare level. Most important for policy purposes, the analysis shows the trade-offs between the policy instruments: penalties, taxes/subsidies and treatment costs in a world where technological progress occurs and firms may violate the law.
1 INTRODUCTION

The problem of externalities and its theoretical solution has received considerable attention. It is known, for example, that pollution levels should be controlled in such a way that marginal social costs equal marginal social benefits. But implementing this rule is not easy. (Jones, 1989; Russell et al., 1986). In general, firms do not pay the full amount of the social cost they cause in generating pollution.

If monitoring the sources of pollution were not expensive, then it would be possible to identify firms which pollute, and to fine them an amount equal to the social cost of the activity. Firms would pollute only if their private benefits exceeded the social cost. However, in most situations it is costly to catch firms and impose a penalty. The agency responsible of enforcing pollution control measures chooses the level of resources devoted to enforcement that equates marginal benefit to marginal cost of enforcement. Usually this enforcement level is such that the probability of firms being detected violating the law is less than one.

The level of the firm's compliance is influenced by the effectiveness of the authority enforcing the regulation. The probability of detecting a firm that violates the law can be so low that the level of firm compliance is almost null. Then subsidizing waste treatment can be more effective. The waste that a firm generates in the production process can be treated or recycled rather than dumped. Waste treatment, is more expensive for the firm, than dumping, but usually cheaper for society. In line with previous attempts to model regulation and control of hazardous wastes, Gottinger (1993 a, b), we model a situation where both possibilities are available, the agency can either enforce an expected penalty function or subsidize treatment cost. These two alternatives define two types of pollution: legal or treated and illegal or untreated pollution. Both, expected penalty and treatment cost, can be implemented through an economic incentives regime or quantity standards one. The use of a subsidy reduces the treatment cost for the firms, so that there are less incentives to dump waste illegally, the amount of treated waste increases. An increase in the expected penalty function increases the cost of illegal waste making treated waste cheaper for the firm. (Lee, 1984).

Two general policy settings are compared: a taxes and a standards regime, the first as a representative of incentive-based type, the latter as one of command and control type regulation. In the first one, if firms want to increase the level of waste generated, they are
able to choose between treating the additional waste, paying a price \( t \) per unit of treated waste, or dumping it illegally and facing a penalty if they are discovered. In the second regime, the possibility of increasing the level of treated waste does not exist: firms are constrained by a standard and any increase in the level of waste must be done illegally. In both cases, it is assumed that technological progress does take place.

In this context a novel feature on which we focus is a treatment of technological progress. It is assumed that technological change is a feature of the firm's operating environment. Most of the emphasis on technological change and pollution control (e.g., Magat, 1978; Downing and White, 1987, Macculey et al., 1993) has focused on the role of the incentives for a firm to invest in research and development to reduce its pollution levels in the face of effluent taxes rather than a standards regime. Most of the work considering technological progress assumes that technological progress reduces the generation of waste in the production process. This paper, however, considers that technological progress is not explicitly devoted to reduce pollution, technological progress is assumed only to make production more efficient.

One of the arguments most broadly used to encourage the use of economic incentives is that this type of policy measure is an incentive for the introduction of innovations in the production process which will substitute more abundant resources for expensive resources. But a problem may appear when we consider that usually the use of the environment, that is, the deposition of waste on it, is underpriced. If the correct incentives are not applied, there is no reason to expect that the forthcoming technological progress will reduce the generation of waste in the production process.

We analyze the problems that the presence of an externality may pose when there is technological progress. Two main questions are addressed. First, we examine the consequences of technological progress in a model with externalities: the characteristics of the optimal allocations, and the way in which the welfare level is affected. And second, which is the best policy instrument that can be used in such a situation. The two policy setting compared are the taxes and the standards regime proposed above.

This paper takes into account other features of the real world. The first one is that the central agency cannot continuously adjust its policies with respect to either the desirable discharge standard or the desired effluent tax schedule. Another major feature of the model is that firms are assumed to be willing to violate the pollution control regulation (if it is in
their profit interest to do so) by either discharging more than the quantity allowed by legal standards, or by failing to pay effluent taxes on the entire quantity discharged.

Given these assumptions several results are derived. First, contrary to expectations, it is shown that there is no guarantee that an increase in technological progress results in an increase in the welfare level. Second, the willingness to violate the pollution control regulation by a profit maximizing firm defines an optimal tax rate which is lower than the optimal one in a first best world where firms cannot violate regulations. Third, the way the firm determines its waste level differs under the taxes and standards regime. Under a taxes regime, the firm equates the marginal cost of violation to the tax rate. Under a standards regime, the firm equates marginal cost of violation to its waste marginal benefit. Any change in the economy that changes the marginal benefit schedule (as technological progress does) will also change the level of violation in a standards regime. Yet violations will remain constant under a taxes regime.

Another set of results obtained tells us under which circumstances, a tax rate regime is better than a standards one, as technological progress takes place. Essentially, if there were no externality problems the rate of increase of welfare will always be larger under a taxes regime than under a standards one. But when an externality problem is introduced, the better policy type depends on how marginal social cost differs from marginal private cost under each regime.
2 MODEL DESCRIPTION

There are two types of agents in this model, firms and a central agency. Each firm produces an output $y$ that is sold in a competitive market at a price $p$. As a byproduct, the firm produces a waste $w$. Total waste is composed of two types: i) a legal amount $x$, the waste amount that the firm generates without violating the legal regulations, and ii) an illegal amount $v$, the amount of waste that exceeds the allowed limit. Thus $w = (x+v)$.

Firms choose the level of output and waste that maximizes profits. The cost of producing $y$ depends not only on the amount of good produced, but also on the amount of waste emitted. The more waste the firm generates in the production process, the cheaper it is to produce a given amount of output.

Another type of decision faced by the firm is the proportion of waste that is legally declared and the proportion that is illegally dumped. Under a taxes regime the firm has the possibility of declaring the full amount of waste that the firm generates. The declared proportion of waste is taxed at a price $t$ per unit of waste. Under the standards regime the firm faces a fixed cost that is supposed to account for the treatment cost of the waste quota assigned by the agency. If it is in the firm’s interest to generate more waste than the allowed amount, it will have to do it illegally.

The production of illegal waste entails a penalty cost (or exceeded penalty) imposed by the agency to discourage violation. The (expected) penalty function is represented by $H(v)$. It is assumed twice continuously differentiable, and it increases at an increasing rate with the violation level: $H'(v) > 0$, $H''(v) > 0$. If the agency were able to fully enforce the law, this penalty function could be set to reflect the full social damage caused by violation. For a given positive probability of catching a firm, the fine function could be set higher than the actual cost in such a way that expected penalty still equates social cost. We assume however that the agency is unable to fully enforce the law, because the nature of the argument setting a large penalty function, is neither realistic nor fair. It is not realistic because firms are rarely, if ever, fined an amount equal or larger than the damage caused; and it is not fair because, in most of the cases, only a small number of the polluters (the polluters that are detected) would end up paying a fine. Any failure to monitor and fully enforce pollution control will be represented by the differences between the expected penalty function $H(v)$ and a social damage function $J(v)$. This social damage function represents the true violation cost for society. It includes not only the damage caused by pollution but it can also include the
cost of cleaning up the spill, and the cost of monitoring and enforcing the penalty, if any. The social damage increases at an increasing rate with illegal pollution $J_s(v) > 0$ and $J_{as}(v) > 0$.

The central agency’s goal is to attain the highest welfare level given the expected firm behaviour. The welfare function is the sum of consumer surplus $S(y)$ and producer surplus less the social damage caused by untreated waste $J(v)$ and the treatment cost of the legally declared pollution $T(x)$. This treatment cost is assumed to be known for the agency; it exhibits constant returns to scale, $T_s(x) > 0$ and $T_{as}(x) = 0$. Also, it is assumed that $J(v) > T(x)$ otherwise it would not be worth to treat pollution.

We model a situation where the marginal benefit of waste increases with technological progress. Technological progress is modelled in two ways: output augmenting technological progress and waste reducing technological progress. These situations will be described by a labour requirement function:

$$L = f \left( \frac{y}{M}, x + v \right),$$

in the output augmenting case, and

$$L = f(y, N(x + v)),$$

in the waste reducing case, where $y$ is output, $x$ is legal waste and $v$ is illegal waste.

The effective products, $\frac{y}{M}$ (output) and $N(x + v)$ (waste) in these two versions of the model are measured relative to the labour input required to produce them. This is in contrast to the usual augmentation models in which the effective inputs are considered relative to the output they produce. This type of functions were first used by Magat (1978). In this model, an improvement in the output production and effluent abatement technologies is represented by increases in $M$ and $N$ respectively. This result stems from the fact that: i) for the same labour and waste rate a larger level of $M$ allows a higher output rate; and ii) for the same labour and output rate a higher level of $N$ allows a lower effluent discharge rate.

The difference between these two labour requirement functions resides in the initial incidence of the technological progress. Output augmenting technological progress provides a proportional increase in the level of output for the same levels of labour and waste. Waste
reducing technological progress gives a proportionate decrease in the waste generated for a given level of output and labour. Both types of modelling enable the firm to modify the profit maximizing combination of inputs, waste, and output. The labour requirement functions \( f \left( \frac{y}{M} (x+v) \right) \) and \( f (y, N(x+v)) \) are assumed to be twice continuously differentiable and quasi-convex. It is customary to assume that this function is a positive value-increasing one, but because in this case one of the outputs is waste effluent, we assume that production cost will decrease as the use of waste is increased.

Formally, we assume the following conditions:

\[
\begin{align*}
    f_1 &> 0, \quad f_{11} > 0, \quad f_{12} < 0 \\
    f_2 &< 0, \quad f_{22} > 0
\end{align*}
\]  

(3)

Here is the outline of what follows: First, we look at the firm’s behaviour, at the way in which the agency determines the optimal level of policy instruments, and at the behaviour of both the firms and the agency, when technological changes take place. Since the conclusions are similar under both types of technological progress, we look in detail only at the case of output augmenting technological progress. Second, we look at how the welfare level changes when technological progress takes place. And third, we compare the rate at which welfare changes under a taxes or a standards regime.
3 AGENTS BEHAVIOUR

3.1 Firms' Behaviour

The firm's problem in the output augmentation technological change case may be expressed as:

$$\max_{y,x,v} \pi = py - w(\frac{y}{M},(x+v)) - tx - H(v),$$

(4)

under the tax regime, and

$$\max_{y,v} \pi = py - w(\frac{y}{M},(\bar{x}+v)) - H(v) - \text{Fix Cost}$$

(4a)

under the standards regime.

Under both regimes, firms want to maximize profits subject to the same production and penalty functions. Under a taxes regime, firms have three choice variables: production level $y$, legal waste $x$, and illegal waste $v$. In this regime the legal proportion of waste will be taxed at a price $t$ per unit of waste. Under the standards regime the firm has only two choice variables: output level $y$ and illegal waste level $v$, and it faces a fixed cost which is supposed to account for the treatment cost of the waste quota $\bar{x}$ assigned by the agency.

The government agency under a tax regime levies a tax rate $t$ upon the firm. Under a standards regime the agency chooses the level $\bar{x}$ that allows to attain the same allocation than under a taxes regime.

Under a taxes regime, assuming an interior maximum, the firm's first order conditions require that the profit-maximizing values of output, and of legal and illegal waste, satisfy:

$$p = f_1(\frac{y}{M},(x+v)) - \frac{1}{M}$$

(5)

$$t = f_2(\frac{y}{M},(x+v))$$

(6)

$$H_v(v) = -f_2(\frac{y}{M},(x+v))$$

(7)
And under the standards regime, the firm's F.O.C. require:

\[ P = f_1 \left( \frac{v}{M} (x+v) \right) \frac{1}{M} \]  \hspace{1cm} (5a)

\[ H_\nu (\nu) = -f_2 \left( \frac{v}{M} (x+v) \right) \]  \hspace{1cm} (7a)

The allocation is the same under both regimes if the agency chooses \( \hat{t} \) and \( \hat{x} \) such that:

\[ t = f_2 \left( \frac{v}{M} (x+v) \right), \]  \hspace{1cm} (8)

### 3.2 Agency's Behaviour

The goal of the agency is to attain the highest welfare level for society, so it chooses the tax rate \( \hat{t} \) or the standard level \( \hat{x} \) that maximizes welfare. The agency takes into account the damage caused by the pollutant, represented by \( J(\nu) \), the expected penalty function \( H(\nu) \), and the cost of treating pollution \( T(x) \). It is assumed that these three functions are already determined when the agency decides the optimal tax rate or standard level. The treatment cost and social damage are exogenously given, the first by the available treatment technology and the second by the environmental damage caused by the specific pollutant under consideration. It is assumed that the resources necessary to enforce the penalty function are limited, so that, the expected penalty function does not equate the damage caused by waste and we have an externality problem in the model.

Thus, the problem for the agency is to attain the highest welfare level choosing the level of pollution control instruments, taxes or standards:

\[ \text{MAX. } W(t) = S(y) - f \left( \frac{v}{M} x+v \right) - T(x) - J(\nu). \]  \hspace{1cm} (9)

\[ t \]

\[ \text{s.t. } P = f_1 \left( \frac{v}{M} (x+v) \right). \]  \hspace{1cm} (5)

\[ t = f_2 \left( \frac{v}{M} (x+v) \right) \]  \hspace{1cm} (6)

\[ H_\nu (\nu) = -f_2 \left( \frac{v}{M} (x+v) \right). \]  \hspace{1cm} (7)
$S(y)$ is a consumer surplus measure of the total social value of the output $y$; $f(M, x + v)$ is the input requirement function; $T(x)$ is the legal waste treatment cost; and $J(v)$ is the social damage function of non-treated waste.

**Proposition 1**: Given the above assumptions, the optimal tax rate $\hat{i}$ is

$$\hat{i} = T_x(x) - [J_x(v) - T_x] \frac{(f_{11} f_{22} - f_{12})}{(f_{11} H_w)}.$$

(10)

**Proof**: Solving for the F.O.C. of the agency and given the firm constraints and market clearing condition, we have:

$$\frac{\partial W}{\partial t} = \left[ \frac{\partial S}{\partial y} - f_1 \frac{1}{M} \right] \frac{\partial v}{\partial t} - [T_x(x) + f_{22}] \frac{\partial x}{\partial t} - [J_x(v) + f_2] \frac{\partial v}{\partial t} = 0.$$

Given the firm behaviour constraints and market clearing condition we have:

$$\frac{\partial W}{\partial t} = -[t + T_x(x)] \frac{\partial x}{\partial t} - [t + J_x(v)] \frac{\partial v}{\partial t} = 0.$$

Substituting for the comparative static results of the agency, i.e.,

$$\frac{\partial v}{\partial t} = \frac{1}{H_w} > 0,$$

and

$$\frac{\partial x}{\partial t} = -\frac{1}{H_w(v)} - \frac{f_{11}}{(f_{11} f_{22} - f_{12})} < 0,$$

and solving for $\hat{i}$ we get as a result (10).

This optimal tax rate is smaller than the one corresponding to a first best situation namely $t = T_x(x)$. In a first best model, without externalities, the welfare maximization rule is the traditional price equals marginal cost. Without externalities private and social cost are the same, firms face the marginal social damage of illegal waste $J_x(v)$ and the marginal treatment cost $T_x(x)$ functions. By profit maximization, firms choose the levels of illegal $v$ and treated $x$ waste that maximizes profits. Firms use each type of waste until the marginal cost are equated and equal to the marginal benefits $T_x(x) = J_x(v) = -f_2\left(\frac{v}{M}, x + v\right)$. When there are no externalities, the private and social optimum coincide, profit maximization leads to welfare maximization.
The optimal solution to the problem changes when there is an externality. Firms no longer face the marginal social cost of pollution, thus they do not pay the marginal social cost that they cause. Firms pay only the part of the social cost assessed by the expected penalty function or by the tax rate. The firm's first order conditions imply that

\[ t = H_v(v) = -f_2 \left( -\frac{v}{M_2(x+v)} \right) \]

Firms equate the marginal private cost of both types of pollution to the private marginal benefit. But now the firms do not pay the social damage that they cause generating waste. Condition (10) gives us the welfare maximizing tax rate. If the agency wants to decrease the level of illegal waste used in the production process, it should decrease the price of the treated waste, thus, it should decrease \( t \). The optimal tax rate depends directly on the marginal treatment cost function and, inversely, on the marginal social damage.

The larger the marginal damage caused by a unit of waste in the environment \( J_2(v) \), the lower the optimal tax rate \( \hat{t} \), or the larger the optimal subsidy \( \hat{s} = (T_2(x) - \hat{t}) \) per unit of waste. There is a trade-off between legal and illegal waste. Treated and illegal waste play the same role in the production process. If the agency wants to decrease the level of illegal waste used in the production process, it should decrease the price of the treated waste. The decrease in the price of treated waste has two effects: i) first, the level of illegal waste decreases, the firm substitutes legal waste for illegal in the production process, and ii) second, the demand of treated waste increases more than the decrease in illegal waste due to the reduction in the price of waste.

**Proposition 2:** The demand for treated waste is more responsive to changes in \( \hat{t} \) than the demand for illegal waste.

**Proof:** From the comparative static results of the agency problem we have:

\[ \frac{\partial v}{\partial t} = \frac{1}{H_v} \]

\[ \frac{\partial x}{\partial t} = -\frac{1}{H_v(v)} - \frac{f_{11}}{(f_{11} f_{22} - f_{12})} < 0. \]

So that:

10
\[ \left| \frac{\partial v}{\partial t} \right| > \left| \frac{\partial x}{\partial t} \right|. \]

Agency chooses \( \hat{t} \) and the firm chooses the profit maximization levels of output, and of legal and illegal waste. Next we consider how these optimal output and waste levels change when technological progress takes place.
4 COMPARATIVE STATIC RESULTS WITH TECHNOLOGICAL PROGRESS

4.1 Output Augmenting Technological Progress Case

In this context we develop an optimal tax model in characteristic form (Mirrlees, 1986).

Assume exogenous technological progress takes place. The firm is assumed small enough not to invest a significant amount in research, but it is able to copy innovations from elsewhere. Nor do we consider any cost of innovations carried out. The agency does not adjust continuously its policy measures as technological progress takes place. So it is assumed that both taxes and standards remains constant, although technological progress takes place.

The case of output augmenting technological progress is represented by an increase in the parameter M. The firm’s reaction under the two regimes is different. Under a taxes regime the illegal amount of waste stays constant while the treated amount increases. The opposite is true under a standards regime.

In the case of a taxes regime, firms are allowed to generate as much waste as they want, as long as they treat the pollution they cause: they pay a tax rate t per unit of treated waste. Both untreated (or illegally dumped) and treated (legal) waste are used in the same way in the production process; the marginal benefit that the firm is able to obtain is the same from legal and illegal waste, the only difference appears in the price that a firm pays for each type of waste. A tax rate t is paid for the treated or legal waste and the marginal expected penalty $H_0(v)$ is faced by the firms in the case of illegal pollution. Under a taxes regime, the firm will equate the marginal benefit of waste and the marginal cost of illegal pollution to the tax rate. The level of violations (i.e. untreated, illegal waste) will remain constant under a taxes regime as long as the tax rate does not change.

However, in a standards regime, any change in the economy that changes the marginal benefit of waste as technological progress does, also changes the level of illegal waste in a standards regime. The firm increases illegal pollution under a standards regime because it cannot increase the treated pollution level, since the marginal benefits of additional pollution exceed the marginal expected penalty cost. If the standard $\bar{x}$ is not automatically adjusted to the optimal waste level, the increase in illegal waste is the profit maximizing solution for the firm when technological progress increases the productivity of pollution. The use of waste reduces the cost of output production so that more output is produced. These results
can be summarized as:

**Taxes Policy**

\[
\frac{\partial v}{\partial M} T = \frac{v}{M} + \frac{f_1 f_{22}}{(f_{11} f_{22} - f_{12}^2)} > 0. \tag{13}
\]

\[
\frac{\partial x}{\partial M} T = \frac{v}{M} + \frac{f_1 f_{12}}{M (f_{11} f_{22} - f_{12}^2)} > 0. \tag{14}
\]

\[
\frac{\partial v}{\partial M} T = 0. \tag{15}
\]

**Standards Policy**

\[
\frac{\partial y}{\partial M} s = \frac{v}{M} + \frac{f_1 f_{22} H_w f_1}{(f_{11} f_{22} - f_{12}^2) + H_w f_{11}} > 0. \tag{16}
\]

\[
\frac{\partial v}{\partial M} s = -\frac{1}{M} + \left(\frac{f_1 f_{12}}{(f_{11} f_{22} - f_{12}^2) + H_w f_{11}}\right) > 0. \tag{17}
\]

\[
\frac{\partial x}{\partial M} s = 0 \tag{18}
\]

Technological progress increases the marginal benefit of waste used in the production process. The marginal benefits that the firm obtain from both types of waste are the same, but the marginal cost is not. The firm, under a taxes regime, increases legal waste. Under a standards regime the firm increases the illegal waste level.

### 4.2 Waste Reducing Technological Progress Case

The same type of exercise can be repeated with waste reducing technological progress. There are few differences between this case and the previous output augmenting technological change case. The firm's problem in the same as it was before, i.e. it wants to maximize profits. Also, in this case the firm can be subjected to two types of regulation: taxes and standards.
The firm's problem is expressed as:

$$\max_{y,x,v} \pi = py - f(y, N(x+v)) - tx - H(v),$$  \hspace{1cm} (19)

under the taxes regime, and by:

$$\max_{y,v} \pi = py - f(y, N(x+v)) - H(v) - \text{Fix Cost}$$  \hspace{1cm} (20)

Figure 1: Pollution Tax Regime

Legend: Here $-w_{f_1}$ and $-w_{f_{22}}$ are the marginal benefits of waste before and after technological progress, and $\hat{x}, \hat{x}_{1}, \hat{v}$ and $\hat{v}_{2}$ are optimal quantities of legal and illegal waste before and after technological progress.

under the standards regime.

Under a taxes regime, assuming an interior maximum, the first order conditions require that the profit maximizing values satisfy:

$$p = f_1(y, N(x+v))$$  \hspace{1cm} (21)

$$t = -f_2(y, N(x+v)) N$$  \hspace{1cm} (22)

$$H_u(v) = -f_2(y, N(x+v)) N.$$  \hspace{1cm} (23)
Figure 2: Command and Control Standards Regime

Legend: Here \(-w_f\) and \(-w_{f2}\) are the marginal benefits of waste before and after technological progress, and \(\dot{x}, \dot{x}_2, \dot{y}\) and \(\dot{y}_2\) are optimal quantities of legal and illegal waste before and after technological progress.

As before, under the standards regime, the F.O.C. require:

\[ p = f_1(y, N(x+y)) \]  \hspace{1cm} (24)

\[ H_s(v) = f_2(y, N(x+y))N \]  \hspace{1cm} (25)

The agency’s goal also remains unchanged, so the welfare maximization problem faced by the agency is:

\[ \text{MAX } W = S(y) - f(y, N(x+y)) - T(x) - J(v). \]  \hspace{1cm} (26)
\begin{align}
\text{s.t. } p &= f_1(y, N(x+v)) \\
 t &= -f_2(y, N(x+v)) N \\
 H_v(v) &= f_2(y, N(x+v)) N
\end{align}

Now, consider an exogenous waste reducing technological change. As before, we assume that the agency cannot adjust its policies in response to a technological change. A waste reducing technological change represented by an increase of N. The firm’s reaction differs under the two regimes. Although the optimal output rate increases under both regimes, the optimal illegal amount of waste stays constant under a taxes regime but not under a standards regime. On the other hand, the legal amount of waste stays constant under the standards regime by assumption, but not in a taxes regime. These are summarized by:

\begin{align}
\frac{\partial v}{\partial N} T &= \frac{f_{12} f_2}{f_{11} f_{22} - f_{12}^2} N > 0. \\
\frac{\partial x}{\partial N} T &= -\left[ \frac{(x+v) f_{12} f_{22} - f_{12}^2 + f_{12}^2}{N^2 (f_{11} f_{22} - f_{12}^2)} \right]. \\
\frac{\partial v}{\partial N} T &= 0.
\end{align}

**Standards Regime**

\begin{align}
\frac{\partial v}{\partial N} s &= \frac{-f_{12}(H_{vw}(x+v) - Nf_2)}{N^2 (f_{11} f_{22} - f_{12}^2) + f_{11} H_{vw}} > 0. \\
\frac{\partial v}{\partial N} s &= -\frac{N(x+v)(f_{11} f_{22} + f_{12}^2) - f_{12} f_{11}}{N^2 (f_{11} f_{22} - f_{12}^2) + f_{11} H_{vw}}. \\
\frac{\partial x}{\partial N} s &= 0
\end{align}
where,

\[
\frac{\partial v}{\partial N} s > 0 \quad \text{and} \quad \frac{\partial x}{\partial N} T > 0 \quad \text{if and only if}^{2}
\]

\[| f_2 f_{11} | > | N(x+y)(f_{11} f_{22} - f_{12}^2) |.\]

The main difference between the two types of technological progress is that the change in optimal waste level has no definite sign under waste reducing technological progress, but it always increases under output augmenting technological progress.

Under output augmenting technological progress, a firm that maximizes profits never decreases the level of waste. Under waste reducing technological progress, the decision of the firm depends on condition (33). Under output augmenting technological progress as technological change takes place, the output level per unit of waste increases, and so the marginal benefit for a given amount of waste. The output produced and the waste generated increase. After technological progress, even if every unit of waste becomes more productive, more waste is generated.

But in the case of waste reducing technological change, two effects have to be taken into account: First, a waste reducing technological progress (an increase in \(N\)) increases the output level per unit of waste that can be produced; waste becomes more productive, and as before, the marginal benefit of waste increases. As in the output augmenting case, together with an increase in the output level, the waste level increases. But second, waste reducing technological progress increases the effective waste level in efficiency units, \(N(x+y)\), so technological progress acts as if we were increasing the waste level used. Only the waste reducing technological progress allow for increases in the waste level in efficiency units. The increase in waste in efficiency units allows for not having to increase the waste level to reduce the production cost. In this case, it is possible that the total amount of waste decrease. Equation (33) shows the condition that determines the increase or decrease in the level of waste once waste reducing technological progress has taken place. The larger the rate of increase of the marginal cost of output the more likely the level of waste has to be increased to maximize profits.
5  WELFARE UNDER TECHNOLOGICAL PROGRESS

Now we examine the consequences of technological change on the welfare level. First it is shown that an increase in technological progress does not guarantee an increase in welfare level. And second we show which condition would guarantee such an increase.

Proposition 3: An increase in output augmenting technological progress (an increase in M) does not have a definite effect on the welfare level. It can decrease or increase the welfare level, independently from the policy instrument that the agency uses to control pollution.

Proof: Given output augmenting technological progress (increase in M), the change in welfare, the changes in consumer surplus plus producer surplus, is represented under a taxes regime and under a standards regime, respectively by:

\[
\frac{\partial W}{\partial M} T = \frac{py}{M} - (T_x(x) - t) \frac{\partial x}{\partial M} T. \tag{34}
\]

\[
\frac{\partial W}{\partial M} s = \frac{py}{M} - (J_\nu(v) - H_\nu(v)) \frac{\partial v}{\partial M} s. \tag{35}
\]

These expressions have no definite sign, so that output augmenting technological progress has no definite sign on the welfare level.

Proposition 4: An increase in waste reducing technological progress (an increase in N), does not have a definite effect on welfare level. The welfare level increases if any only if

\[ |f_{21}f_{11}| > |N(X+\nu)(f_{11}f_{22} - f_{12}^2)|. \]

Otherwise the sign in the change of the welfare level is undetermined.

Proof: Given waste reducing technological progress, the change in welfare level is represented under a taxes regime and under a standards regime, respectively by:

\[
\frac{\partial W}{\partial N} T = \frac{t(x+\nu)}{N} - (T_x(x) - t) \frac{\partial x}{\partial N} T. \tag{36}
\]

\[
\frac{\partial W}{\partial N} s = \frac{t(x+\nu)}{N} - (J_\nu(v) - H_\nu(v)) \frac{\partial v}{\partial N} s. \tag{37}
\]

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If \( \frac{\partial x}{\partial N} < 0 \) and \( \frac{\partial v}{\partial N} < 0 \) then \( \frac{\partial W}{\partial N} > 0 \) and \( \frac{\partial W}{\partial N} > 0 \). But if \( \frac{\partial x}{\partial N} > 0 \) and \( \frac{\partial v}{\partial N} > 0 \), then the welfare has no definite sign.

Technological change reduces the marginal cost of producing a given output level. For a given output price, this leads a profit maximizing firm to increase the output level. As waste is a byproduct of the production process, technological progress may end up increasing the amount of waste used in a production process.

Two effects are represented in the above expressions, the output and the waste effect. The output effect is represented by the first term in each equation. It is positive and represents the increase in the output value due to technological progress. The second term represents the effect of technological progress on the social cost and amount of waste. It is negative and represents the price that society ultimately pays for the increments in the waste level.

Firms only pay a part of the cost of waste. They pay \( t \) or \( H_s(v) \) per unit of waste, in the case of a taxes or a standards regime, respectively. The difference between social and private marginal cost is left for society to pay; \( (T_s(x) - t) \) or \( (J_s(v) - H_s(v)) \), respectively.

If firms were faced with the true marginal social cost, the share of the price left for society would be zero. It is important to realize that the social cost left to be paid by society depends on the damage and treatment cost functions and that those functions are exogenous to both the firm and the agency.

If there were no externality effects, that is, if \( J_s(v) = H_s(v) \) and \( t = T_s(x) \), technological progress will always tend to increase the welfare level. So the externality effect is responsible for technological progress to have a negative effect on the welfare level. The marginal social cost of waste left without paying due to the externality, \( (T_s(x) - t) \) or \( (J_s(v) - H_s(v)) \), can be high enough to eliminate the positive effect that the increase in output has on the welfare level. Thus, in an environment with externalities, technological progress does not guarantee increase in welfare.

**Proposition 5:** In an environment with externalities an output augmenting technological progress increases the welfare level if the increase in the firm’s profits due to technological progress is larger than the increase in social cost also due to technological progress.
Proof: From (34 and 35) it can be shown that:

\[
\frac{\partial W}{\partial M} \quad T > 0 \quad \text{if and only if} \quad \frac{M}{(x+v)} \cdot \frac{\partial x}{\partial M} \quad T < \frac{py}{s(x+v)}.
\] (38)

in the taxes regime, and

\[
\frac{\partial W}{\partial M} \quad s > 0 \quad \text{if and only if} \quad \frac{M}{(x+v)} \cdot \frac{\partial v}{\partial M} \quad s < \frac{py}{(J_v(v)-H_v(v))(x+v)}.
\] (39)

in the standards regime, where \(s\) stands for subsidy \(s=(T_s(x)-t)\).

Proposition 6: Waste reducing technological progress increase welfare if the optimal level of waste decreases with technological progress. If optimal waste increases with waste reducing technological progress that is, if condition (33) is satisfied, waste reducing technological progress increases the level of welfare if the increase in the firm benefit from technological progress is larger than the increase in social cost also due to technological progress.

Proof:

\[
\frac{\partial W}{\partial N} \quad T > 0 \quad \text{if and only if} \quad \frac{N}{(x+v)} \cdot \frac{\partial x}{\partial N} \quad T < \frac{t(x+v)}{s(x+v)}.
\] (40)

in the treatment regime, and

\[
\frac{\partial W}{\partial M} \quad s > 0 \quad \text{if and only if} \quad \frac{N}{(x+v)} \cdot \frac{\partial v}{\partial M} \quad s < \frac{H_s(x+v)}{(J_s(v)-H_s(v))(x+v)}.
\] (41)

in the penalty regime.

In the waste reducing case, the welfare level tends to increase if the elasticity of waste with respect to technological progress is smaller than the ratio between price paid by firms \((H_s(v)\ or\ t)\), and the price paid by society per unit of waste \((s\ or\ (J_s(v)-H_s(v))\)).

Thus, in the case of output augmenting technological progress, if the elasticity of waste with respect to technological progress is smaller than the proportion that the firm
revenue \( (py) \) represents in the amount of waste treatment cost subsidized by society \( s(x+v) \), the welfare level will tend to increase with technological progress. If the revenue that the firm obtains for the production of output (and waste) is large, the more likely it is that technological progress increases welfare.

The waste elasticity with respect to both types of technological progress has to be smaller than the proportion between the benefits that the use of waste represents for the firm, and the cost that the same waste represents for society. In the output augmenting case, a change in technological progress increases the firm’s revenue represented by \( py \). In the waste reducing case the increase in benefits will be represented by \( t(x+v) \), the marginal benefit (to the firm) obtained from the use of waste. The larger the revenue obtained by firms from the use of waste, the more likely it is that technological progress increases welfare.

The price that society pays for the waste depends also on the damage or on the treatment cost function, thus on the characteristics of the pollutant. The more socially costly a pollutant is, the less likely it is that technological progress increases welfare. Thus, technological progress is more likely to increase welfare if the output produced with the use of waste is highly desirable. This suggests that the use of the same type of pollution can be justified in some production processes but not in others. The optimal standard of a pollutant depends not only on the damage that it causes but also on the value of the output that it produces. Also, as the externality becomes smaller, the larger the likelihood that technological progress increases welfare.

This paper is interested in finding out if technological progress is able to guarantee an increase in the social welfare. Our study reveals that in a model with externalities when technological progress takes place, it is not possible to guarantee an increase in welfare. If the difference between the marginal social and private cost of pollution is enough, an increase in the amount of waste generated due to technological progress can give rise to an increase in the marginal social cost, which can be larger than the benefits obtained from the increase in output due also to technological progress. Thus, expressions (34) and (35) can ultimately have a negative sign.

With these two specific types of technological progress (it maybe possible to imagine other types of technological progress that completely change the production process), there is enough evidence to show that technological progress in an environment with externalities does not automatically guarantee: i) reductions in the waste level release into the environment; ii) reductions on the level of damage caused by waste; nor iii) an increase in social welfare.
6.1 Effects of Technological Progress on the Welfare Level

Up to now we have been looking at the agents' behaviour under a taxes and a standards regime, and how welfare changes when technological progress takes place. No detailed comparative analyses have been done of the consequences of these two types of policy regimes on the welfare level when technological progress takes place.

Proposition 7: At the second best optimal allocation \((\hat{\ell}, \hat{y}, \hat{x}, \hat{v})\), the changes in welfare due to marginal increase in technological progress is the same under a taxes than a standards regime.

Proof: To know which policy regime is better, up to a first approximation, we will compare the differences between the increase in welfare in each regime. Subtracting (35) from (34) we have:

\[
\frac{\partial W}{\partial M} T - \frac{\partial W}{\partial M} s = -(T_x(x) - \hat{t}) \frac{\partial x}{\partial M} T + (J_y(y) - H_z(z)) \frac{\partial v}{\partial M} s.
\] (42)

And from the firm's first order conditions and comparative static results we have:

\[
\frac{\partial x}{\partial M} T = - \frac{\partial v}{\partial M} s \left[ \begin{array}{c} \frac{\partial x}{\partial t} \\ \frac{\partial v}{\partial t} \end{array} \right].
\] (43)

Substituting (43) into (42) gives:

\[
\frac{\partial W}{\partial M} s = - \frac{\partial w}{\partial M} s = - \left[ -(T_x(x) - \hat{t}) + (J_y(y) - H_z(z)) \frac{\partial x}{\partial t} \right] \frac{\partial v}{\partial M} T.
\] (44)

From the agency welfare maximization problem we have:

\[
\frac{\partial W}{\partial t} = -(T_x(x) - \hat{t}) \frac{\partial x}{\partial t} - (J_y(y) - \hat{t}) \frac{\partial v}{\partial t}.
\] (45)

Thus, the difference between the rate of increase of welfare can thus be expressed as:
\[ \frac{\partial W}{\partial M} T - \frac{\partial W}{\partial M} s = - \frac{\partial W}{\partial t} \begin{bmatrix} \frac{\partial x}{\partial t} \\ \frac{\partial M}{\partial t} \\ \frac{\partial x}{\partial t} \end{bmatrix}. \]  

(46)

And \( \hat{\ell} \) maximizes welfare if and only if \( \frac{\partial W}{\partial t} = 0 \), thus, at \( \hat{\ell} \) it will be also true that:

\[ \frac{\partial W}{\partial M} T - \frac{\partial W}{\partial M} s = 0. \]  

(47)

### 6.2 Effects of Technological Progress on the Welfare Rate of Change

We examine now the effect of output augmenting technological progress on the welfare rate of change. To compare the performance of these policy regimes we compare the differences between rates of increase of the welfare function. Under a treatment regime the welfare rate of increase is:

\[ \frac{\partial^2 W}{\partial M^2} T = \frac{P}{M} \left[ \frac{\partial y}{\partial M} T - \frac{y}{M} \right] - (T_x(x) - \hat{t}) \frac{\partial^2 x}{\partial M^2} T. \]  

(48)

and under standards regime:

\[ \frac{\partial^2 W}{\partial M^2} s = \frac{P}{M} \left[ \frac{\partial y}{\partial M} s - \frac{y}{M} \right] \]  

(49)

\[
- (J_{\nu}(v) - H_{\nu}(v)) \left[ \frac{\partial y}{\partial M} s \right]^2 - (J_{\nu}(v) - H_{\nu}(v)) \frac{\partial^2 x}{\partial M^2} s.
\]

The difference is:

\[
\frac{\partial^2 W}{\partial M^2} T - \frac{\partial^2 W}{\partial M^2} s = \frac{P}{M} \left[ \frac{\partial y}{\partial M} T - \frac{\partial y}{\partial M} s \right] - \text{lineup} (J_{\nu}(v) - H_{\nu}(v)) \left[ \frac{\partial y}{\partial M} s \right]^2 - (T_x(x) - \hat{t}) \frac{\partial^2 x}{\partial M^2} T + (J_{\nu}(v) - H_{\nu}(v)) \frac{\partial^2 y}{\partial M^2} s.
\]  

(50)
There are two types of effects that can be analyzed from (50). The first one represents the
effect of technological progress on the output level. The second represents its effect on the
waste level. We analyze the meaning of each of these two effects and the assumptions and
elements on which they depend.

6.3 Effects of Technological Progress on the Output Level

The differences between the change in output under each regime is represented by the first
term of (50). It can be represented in elasticity form by:

$$\frac{p}{M} \left[ \frac{\partial y}{\partial M} \frac{M}{y} T - \frac{\partial y}{\partial M} \frac{M}{y} s \right] = \left[ \frac{p^2 f_{21}^2 H_w}{(f_{11} f_{22} - f_{21}^2) (f_{11} f_{22} - f_{21}^2) + H_w f_{11} + H_{w}(v)} \right] > 0. \quad (51)$$

Once technological progress has taken place, the output level that can be produced with the
same inputs amount increases. But responsiveness of output to technological progress differs
between regimes. This difference is measured through the elasticity of output with respect
to technological progress (51). The difference between the elasticities increases with $H_{w}(v)$.
If $H_{w}(v) = 0$ for all $v$, the output level would increase the same under both regimes.\(^5\) Thus,
not only is the slope of the expected penalty is important in determining the optimal output
amount but so is the rate at which this slope increases, since larger increases in waste allow
larger decreases in the marginal cost of production. Given $H_{w}(v) > 0$, the quantity produced
under a standards regime.

The second important element is the effect of waste in the production process,
represented by $f_{21}$ -- the responsiveness of the marginal cost of production to changes in the
waste level. The larger the decrease in the marginal production cost due to the use of waste,
the larger the difference in elasticities. If marginal cost were not responsive at all to the
generation of waste in the production process, (ie. $f_{21} = 0$) the waste level in the production
process would not increase with technological progress in either regime, and the output level
that could be produced after technological progress will be the same under each regime.

We have seen that the increase in the output produced after technological progress is
larger in a taxes than in a standards regime. The difference between the increase in the value
of output under each regime accounts for the increase in the marginal cost of waste paid by
the firm. A taxes regime will give rise to a larger increase in output as long as $H_{w}(v) > 0$,\(^6\)
but also - and this is crucial - it will give rise for the same reason to a larger increase in
waste. Notice that up to now, the social damage that waste causes has played no role in determining the waste amount; the crucial elements are the tax rate and the marginal expected penalty function.

6.4 Effects of the Technological Progress on the Waste Level

The externality problem becomes crucial. If there were no externalities in the model, the firms would pay the real cost of pollution, the profit maximization allocation would be the same as the welfare maximization one, the real cost of waste would already be taken into account in the production process, and so the increase in the waste level will be the optimal for firms and society. But in our case there is an externality problem. The externality is represented by the difference between the true marginal pollution damage and the price paid by the firms \((T_\iota(x) - \hat{\iota})\) or \((J_\iota(v) - H_\iota(v))\), under a taxes and a standards regime respectively.

When the externality problem is present, society pays a portion of the total marginal cost of waste, the social marginal cost of waste. Define \(SC(x) = (T(x) - tx)\) and \(SC(v) = J(v) - H(v)\) as the portion of the waste cost left to society to be paid under a taxes and a standards regime respectively. Before technological progress occurs, the marginal social cost of each type of waste is the same, because the policy instruments have been set to equate those marginal costs. After technological progress, both marginal social costs change, but they change differently. In a taxes regime, only the level of treated waste is increased. In a standards regime, the level of illegal waste increases. The change in the marginal social cost that society end up paying depends on: i) how the marginal social damage \(J_\iota(v)\) and treatment cost change \(T_\iota(x)\), ii) how the marginal private cost faced by the firm changes, \(H_\iota(v)\) and \(t\), and iii) the implications that these changes have on the rate at which the amount of waste increases. These elements can be represented by:

\[
\frac{\partial^2 SC}{\partial M^2} T - \frac{\partial^2 SC}{\partial M^2} s = (J_\omega(v) - H_\omega(v)) \left[ \frac{\partial v}{\partial M} s \right]^2
\]

\[+ (T_\iota(x) - \hat{\iota}) \frac{\partial^2 x}{\partial M^2} T - (J_\iota(v) - H_\iota(v)) \frac{\partial^2 v}{\partial M^2} s.\]  

The first term in brackets shows the change of the total marginal damage of illegal waste. The second term shows the difference between waste level rates of increase under each
regime when technological progress takes place. Not only the amount of the increase in waste differs between each regime but the rate at which it increases also differs. The sign of this expression depends on the shape of the different functions. If (17) is positive, a taxes regime shows a larger welfare rate of increase than a standards regime. If (17) is negative, the difference in the welfare rate of increase (13) is unknown. There is always an advantage for the taxes regime, represented by (16), the output and thus the value of output produced under a taxes regime is larger than under a standards regime. The difference is eliminated if the rate of increase of the social cost of waste (15) is large enough.

There are two important points to study in comparing these policy regimes: First, how the marginal damage and treatment cost function change with technological progress under each policy; and second, the differences between the rate at which the amount of waste increase under each policy. Notice that the differences between the increase in the marginal cost function of each type of waste, and the differences between the increase in the quantity of waste are crucial to decide the sign of this expression. If we assume that treatment shows constants returns to scale \( (T_{st} = 0) \), it is clear that a regime where treatment is possible is more attractive since the marginal social cost of pollution does not increase per unit of waste.\(^7\) The larger the difference \((J_{w}(v) - T_{st}(x))\), the better it will be to use a policy regime that encourages the treatment of pollution. Such is the case of the taxes regime, in contrast to the standards regime.

The differences between the rate at which the level of waste increases depends on how the marginal private cost of waste to the firm \((H_{s}(v) and t)\) increases in each regime. Under a treatment regime, the firm faces a tax rate \( t \) per unit of waste that is constant, and under a standards regime the firm is facing an expected penalty \( H_{s}(v) \), that increases at an increasing rate with waste. So as \( H_{w} > 0 \) then \( \frac{\partial x}{\partial M_{T}} > \frac{\partial v}{\partial M_{T}} \). The social damage is assumed to increase faster in a standards regime than in a taxes regime \( J_{w}(v) > T_{st}(x) \), but the increase in the illegal waste level is smaller than the increase of treated waste in a taxes regime. The difference between the rate of increase of marginal social damage of pollution will have to be balanced out with the rate at which the quantity of waste increases under each regime. Note that these two facts are independent: we could have the same increase in the level of waste under both regime if \( t_{s}(x) = H_{w}(v) \) for \( x \) and \( v \).\(^8\) The decision of how much will the waste increase is made by the firms, since price is the relevant factor. The increase in social damage is given by the damage and treatment functions that are exogenous to both the agency
7 CONCLUSIONS

Several lessons can be learned from this analysis, we group them in two different sections. First, the lessons that emerge from the comparison of the two policy regimes. Second, the conclusions that are obtained from the presence of technological progress in the model.

If firms can violate the pollution control regulations and give rise to externalities, the optimal tax rate if lower than the optimal tax rate in a world, where firms cannot violate regulations. There is a trade-off between legal an illegal waste. Treated and illegal waste play the same role in the production process. If the agency wants to decrease the level of illegal waste used in the production process, it should decrease the price of the treated waste.

A taxes regime presents the firm with a larger possibility of choice, it allows the firm to use treated waste once technological progress has occurred. In a standards regime, the firm has no choice, if technological progress makes waste more profitable, the only type of waste that firm can increase is the illegal one.

It has been shown that, without externalities, the welfare level increase if technological progress takes place, but that the welfare can even decrease if technological progress takes place in an environment with externalities. Technological change reduces the marginal cost of producing. Given a output price, a profit maximizing firms will increase the output level. As waste is a byproduct of the production process, technological progress ends up increasing the amount of waste in the production process. In the presence of externalities firms will choose their profit maximization allocation taking into account the prices that they pay for waste, not the social cost that they cause. The cost of waste left without paying due to the externality can be high enough to eliminate the positive effects that the increase in output has in the welfare level. So, in the presence of externalities, there is no guarantee that an increase in technological progress increases the welfare level. Thus, in the case of output augmenting technological progress, if the elasticity of waste with respect to technological progress is smaller than the proportion that firm revenue represents \((py)\) in the amount of waste treatment cost subsidized by society \(s(x+y)\), the welfare level will tend to increase with technological progress. If the revenue that the firm obtains for the production of output (and waste) is large, the more likely it will be that technological progress will increase welfare.

In summary, there is no policy regime that is optimal for all situations. the optimal policy regime will depend on the characteristics of the social damage, treatment function and production function. But several conclusions can be drawn from the comparison of these two
policy regimes.

A taxes regime seems to allow for much more flexibility than a standards regime. It allows to assign different types of prices to the same type of waste depending on its uses. Also with taxes regime we will have a measure, in terms of relative prices, of the advantage of using waste instead of other costly input in the production process.

Our major interest in analyzing these results is to be able to outline some policy recommendations. We analyze under which circumstances is more likely that one regime be better than another. To know when and which policy regime gives the rise to larger increases in welfare, we have to take into account the differences between the rate of increase of the social damage, and the treatment cost function. We also need to examine the differences between the rates at which the level of waste increases.

The faster the rate of increase of the damage function $J(v)$ the more convenient it will be to implement a taxes regime that encourages the treatment of pollution. Also, if treatment cost does not present constant but decreasing returns to scale $T_{st}(x) > 0$, the faster the rate of increase of the treatment cost function and the less convenient a taxes regime will be. In principle, the implementation of a taxes regime presents some advantages over a standards regime as long as $J_{vw}(v) > T_{st}(c)$.

The larger the rate of increase of the expected penalty function, the less benefits will be obtained from implementing a taxes regime. Under a standard regime the larger the rate of increase of the expected penalty $H(v)$ the smaller will be the increase in the illegal amount of waste. Thus, given a constant tax rate, the larger the rate of increase of the expected penalty function, the larger the difference between increases in treated and illegal waste due to technological progress $(\frac{\partial T}{\partial M} > \frac{\partial v}{\partial M})$. Thus, the larger the increase in the quantity treated under a taxes regime the larger the increase in treated cost.

The last point to discuss is the role played by the characteristics of the production function in the generation of waste. The increase in the quantity of waste due to technological progress depends on the characteristics of the production process. At the initial point both types of waste have the same price, but as soon as technological progress takes place, the firm under a taxes regime increases the demand of treated waste, whose price is assumed constant, and the firm under a standards regime increases the demand of illegal waste the price of which increases with the waste amount. The difference between waste increases under each regime will depend not only on the rate of increase of the expected
penalty function but also on the responsiveness of the demand of waste to changes in the price of waste. The larger increase in the quantity of waste in a treatment will not only obey to the fact that a constant tax rate allows a larger increase but also to the production function. The faster the price elasticity of demand decreases with prices, the larger the difference in the increase in waste between a taxes regime and a standards regime. Thus, for production processes where waste level is really elastic with respect to price, it will be more convenient to use expected penalty or tax functions that increase at an increasing rate with the level of waste.

These results can be generalized to the waste reducing case. The major difference between output augmenting technological progress and a waste reducing one is that, in the first case, the waste level necessarily increases as technological progress takes place, but in the case of waste reducing the waste level can either increase or decrease. The conclusions will be the same if we look at what happens with the waste level, in the case where the waste level increase, if the waste level decreases the welfare level will certainly increase, (see equations (36) and (37). In the comparison between a taxes and a standards regime, the points that are crucial in deciding which of the two regimes is better will be the same whether the level of waste increases or decreases. The decision, thus, will depend on the rates of increase of the treatment cost and damage functions and on the rate of increase of both legal and illegal waste.
Footnotes

1. The same result can be expressed as a subsidy rate $\hat{s}$:

$$\hat{s} = (T_s(x)-t)[J_s(v)-T_s] \frac{(f_{11} f_{22} - f_{12}^2)}{f_{11} H_w}.$$  

(10a)

2. If we define cost as $C = f(y N(x+v))$, this condition (33) also can be expressed as:

$$\frac{\partial^2 C}{\partial y^2} \geq \left| \frac{(x+v)}{\partial (x+v)} \left[ \frac{\partial^2 C}{\partial (x+v)^2} - \frac{\partial^2 C}{\partial (x+v) \partial y} \right] \right|$$

If and only if $\frac{\partial x}{\partial N} T > 0$ and $\frac{\partial y}{\partial N} s > 0$

3. It would be negative as long as we assume that there is an externality, either $J_s(v) > H_s(v')$ or $(T_s(x) > t)$. See appendix 1.

4. Welfare will increase if an only if the increase in the profit level due to technological progress is larger than the increase in the cost that society pays for waste.

$$\frac{\partial \pi}{\partial M} = \frac{py}{M} > 0,$$  and

$$\frac{\partial SC}{\partial M} T = (T_s(x)-t) \frac{\partial x}{\partial M} T$$

where $SC = (T(x)-tx)$. Thus,

$$\frac{\partial W}{\partial M} > 0 \text{ if and only if } \frac{\partial \pi}{\partial M} > \frac{\partial SC}{\partial M}$$
The same can be shown for the standards regime case and for the waste reducing technological progress cases.

5. It can also be shown that if instead of having a constant tax rate, we have \( t_{wr}(x) \neq 0 \), the differences in elasticity would depend on the differences between the rates of increase \( H_{w0} - t_{wr} \).

6. It also can be shown that if instead of having a constant tax rate, we have \( t_{wr}(x) > 0 \), the differences in elasticity would depend on the differences between the rates of increase \( H_{w0} - t_{wr} \). Not only is the slope of the expected penalty function.

7. If it is the case that \( t = H_{d}(v) \) and \( t_{wr}(x) = H_{w}(v) \) for all \( x,v \), the firm’s first order conditions and comparative static results will be the same. The only difference that would arise will be in the welfare function, and always the regime that encourages the lower marginal social cost method will be cheaper for society. Since, we assumed that \( J_{e}(v) > T_{f}(x) \), then a taxes regime will be better (see appendix 2).

8. An optimal expected penalty function \( H(v) \) would be such that for each unit of waste, the marginal cost of enforcing the law equals the marginal benefit of enforcing it.
Appendix 1

To see more clearly the role played by externalities, we compare those results with the ones obtained in the case that no externality is present. The crucial difference is that firms will be facing the real cost of pollution. The firm problem may be expressed as:

$$\max_{y,x,v} \pi = py - f\left(\frac{y}{M},(x+v)\right) - T(x) - J(v).$$

under a treatment regime, and

$$\max_{y,v} \pi = py - f\left(\frac{y}{M},(x+v)\right) - J(v) - \text{Fix Cost.}$$

under a penalty regime. Notice that now the firm faces the real cost of waste. $J(v)$ and $T(x)$.

The agency maximizes the same the welfare function:

$$W = S(y) - f\left(\frac{y}{M},(x+v)\right) - T(x) - J(v).$$

The firm’s FOC, and agency’s welfare maximization conditions require;

$$p = w_1 f_1\left(\frac{y}{M},(x+v)\right) \frac{1}{M}$$

$$T_x(x) = -w_2 f_2\left(\frac{y}{M},(x+v)\right)$$

$$J_v(v) = -w_2 f_2\left(\frac{y}{M},(x+v)\right).$$

The first difference appears when we look at the welfare change due to technological progress.

**Lemma:** If there are no externalities an increase in the output augmenting technological progress increases the welfare level and the increase is the same under both regimes.
\[
\frac{\partial W}{\partial M_T} = \frac{\partial W}{\partial M_s} = f_1 \left( \frac{M}{M} \right)^{x+y} \frac{M}{M^2} = p \frac{M}{M} > 0.
\]

So that under both regimes an improvement in output augmenting technological progress increases welfare. The second derivative will tell us the difference between the rate of increase under both.

Under treatment regime the welfare rate of increase turns out to be;

\[
\frac{\partial^2 W}{\partial M_T^2} = \frac{1}{M^2} \left[ \frac{f_{22}}{f_{11} f_{22}} \right] > 0.
\]

and under a standards, it is;

\[
\frac{\partial^2 W}{\partial M_P^2} = \frac{1}{M^2} \left[ \frac{(f_{22} + J_{w} f_1)}{f_{11} f_{22}} \right] > 0.
\]

Thus, since

\[
\frac{\partial^2 W}{\partial M_T^2} - \frac{\partial^2 W}{\partial M_P^2} = \frac{J_{w} f_1^2 f_{21}}{M^2 (f_{11} f_{22} - f_{21}) f_{11} f_{22} - (f_{21} + J_{w} f_{11})}.
\]

then:

\[
\frac{\partial^2 W}{\partial M_T^2} > \frac{\partial^2 W}{\partial M_P^2} s.
\]

This implies that welfare under taxes regime increases at a faster rate than under a standards regime.

Therefore it is the presence of externalities that makes the welfare comparison ambiguous. Without externalities, when the firms are paying the cost of the waste that they generate, a taxes regime shows a large rate of increase in the welfare function than a standards regime. The basic difference is that under a taxes regime the real price of the waste is lower, allowing the firms to use more waste and to produce more output. The difference between the rate of increase of welfare coincides with the difference in output increase. The difference represent the extra amount of output that can be produced under a treatment regime.
Appendix 2: Both types of waste have the same cost for the firms.

If the firm that faces a treatment possibility is facing the same type of cost that a firm that is facing an illegal type of waste, the FOC, and comparative static results will be the same under both regime, and the only differences would arise in the welfare functions.

The welfare function in a treatment regime can be stated as:

\[ W_T = S(y) - f\left(\frac{y}{M}\right)x + v - T(x), \]

and in a standards regime:

\[ W_S = S(y) - f\left(\frac{y}{M}\right)x + v - J(v), \]

An increase in technological progress will generate an increase in welfare that respectively can be expressed as:

\[ \frac{\partial W_T}{\partial M} = \frac{py}{M} + (T_x(x) - \hat{t}) \frac{f_{y21}}{(f_{1y22}f_{21})}, \]

\[ \frac{\partial W_T}{\partial M} = \frac{py}{M} + (J_s(v) - H_s(v)) \frac{f_{y21}}{(f_{1y22}f_{21})}, \]

If by assumption \( t = H_s(v) \) and \( t_{xx} = H_{sv} = 0 \), then we will have:

\[ \frac{\partial W_T}{\partial M_T} - \frac{\partial W_S}{\partial M_s} = (J_s(v) - T_x(x)) \frac{-f_1f_{21}}{M(f_{1y22}f_{21})}. \]

The crucial difference will be between the two marginal cost functions \( (J_s(v) - T_x(x)) \). Always the regime with a lower social cost will be cheaper. The situation and the result will be the same if \( t_{xx} = H_{sv} \) even though they both are different from zero.
Appendix 3: Standards Policy Perfectly Enforced

Let's look now at which are the consequences of technological progress in a regime with a perfectly enforced standard. The problem of the firm will be reduced to:

\[ \text{MAX } \pi = py - f \left( \frac{y}{M} x + v \right). \]

The comparative results will be:

\[ \frac{\partial y}{\partial M} = \frac{y}{M} + \frac{f_1}{f_{11}}. \]
\[ \frac{\partial x}{\partial M} = 0, \]
\[ \frac{\partial y}{\partial M} = 0. \]

The welfare level will increase with technological progress at an increasing rate, thus:

\[ \frac{\partial W}{\partial M} = \frac{py}{M} > 0, \]

and

\[ \frac{\partial^2 W}{\partial M^2} = \frac{p}{M} \frac{f_1}{f_{11}} > 0. \]

But this does not guaranty the new allocation, after technological progress is efficient, a better allocation would have been attained if an increase in the level of waste would have been allowed. In the case that the firm would have been allowed to increase the waste level paying the treatment cost of the waste the increase in welfare, and the rate of increase will be, respectively:

\[ \frac{\partial W}{\partial M} = \frac{py}{M} \]
\[
\frac{\partial^2 W}{\partial M^2} = \frac{p}{M} \left[ \frac{f_1 f_{22}}{(f_1 f_{22} - f_{21})^2} \right].
\]

It can be shown that:

\[

\frac{\partial^2 W}{\partial M^2_x} < \frac{\partial^2 W}{\partial M^2_r}.
\]

So the welfare level under strict quantity or technological standards will increase at a lower rate than in a regime where it is possible to increase the waste level. If the firms are faced by the real cost of treatment a change in technological progress will increase the waste level whenever it is profitable for the firm to do so. As there is no externality problem the firm will increase the level of waste also when it is profitable for society.
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