



# Market and policy risk under different renewable electricity support schemes



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## ABSTRACT

Worldwide, renewable electricity projects are granted production support to ensure competitiveness. Depending on the design of these support schemes, the cash inflows to investment projects will be more or less exposed to fluctuations in electricity and/or subsidy prices. Furthermore, as renewable electricity technologies mature, there is a possibility that the current support scheme will be terminated or revised in ways that make it less generous or more in line with market mechanism.

Using a real options approach, we examine how investors in power projects respond to such market and policy risks. We show that: (1) due to price diversification, the differences in market risk between support schemes like tradeable green certificates, feed-in premiums and feed-in tariffs are less than commonly believed; (2) the prospects of termination will slow down investments if it is retroactively applied, but speed up investments if it is not; and, (3) this policy uncertainty may add a substantial risk to investments, especially in the first case where investors expect future curtailment of subsidies to affect new and old installations alike. We conclude the paper by discussing the division of risk between investor and government.

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## 1. Introduction

At present, many renewable electricity projects are granted production support to ensure competitiveness. These support schemes can be either quantity-driven (the government sets the quantity of new renewable electricity production and lets the market determine the subsidy level) or price-driven (the government sets the subsidy level and lets the market determine the quantity). An example of a quantity-driven scheme is a quota system, in which green certificates are issued to producers in proportion to the volume of renewable electricity generated and traded to satisfy a quota for renewable electricity. Other common terms for the same concept are “renewable portfolio standard” and “renewables obligation”. A feed-in scheme is an example of a price-driven scheme, and it can be implemented as either a tariff that replaces the electricity price or as a price premium paid on top of this price. As of 2013, 71 countries had implemented price-driven

support schemes and 24 countries had implemented quantity-driven schemes.<sup>1</sup>

Depending on the design of these support schemes, the cash inflows to investment projects will be more or less exposed to fluctuations in electricity and/or subsidy prices. In addition to this market risk is the risk that the policy will change in the future. As renewable electricity technologies mature, governments may eventually want to terminate these support schemes or revise them in ways that make them less generous. The prospect of revised renewable electricity support schemes in the EU post 2020 may serve as an example. Most EU member states support the production of electricity using renewable energy sources by offering fixed feed-in tariffs for a given number of years. Because these feed-in tariffs have systematically exceeded the marginal costs of renewable electricity production, in 2012 the tariffs for new plants were cut significantly (e.g., Germany) or removed (e.g., Spain). Moreover, Spain, Belgium, the Czech Republic, Bulgaria and Greece have recently enacted retroactive adjustments to their feed-in tariffs,

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<sup>1</sup> Source: REN 21 Renewable Energy Policy Network for the 21st Century, GSR Policy Table. [<http://www.ren21.net/RenewablePolicy/GSRPolicyTable.aspx>, 16th of February 2014.].

thereby reducing the profitability of already installed plants [7]. Furthermore, a greater influx of intermittent renewable electricity funded by fixed feed-in tariffs challenges the functioning of power markets. In a communication on the internal energy market published in November 2012, the EU Commission suggests that the support schemes are revised to better reflect market mechanisms.

We examine how such market and policy uncertainties affect investment decisions in the renewable electricity sector. The benchmark case is a situation in which investors expect the current support scheme to stay the same indefinitely. We assume that investors receive an electricity price and a subsidy payment for each unit of electricity produced. We allow for different combinations of deterministic and stochastic, geometric Brownian motion diffusion processes. The resulting models can be used to evaluate support schemes of tradable green certificates (both prices are stochastic), feed-in premiums (a stochastic electricity price and a deterministic subsidy payment) and feed-in tariffs (only a deterministic subsidy payment). We further assume that at some random point in time, the subsidy payment will be terminated, and that investors either expect or do not expect that this decision will be retroactively applied. This is modeled by including a Poisson jump process.

We formulate the investment decision as a real option problem in which the option to delay an irreversible investment decision has a value [8]. Our optimization problems are solved analytically using dynamic programming. The essence of this method is to compare the value of immediate investment with the expected value of delaying the investment decision. In our case, finding the optimal timing of an investment implies identifying the sum of the electricity price and the subsidy payment—the threshold revenue—that defines the border between the continuation region (in which the optimal decision is to wait) and the stopping region (in which the optimal decision is to invest). Uncertainty will affect the value of the option to wait and therefore this threshold.

Taking the perspective of an energy firm, real options theory has been used to derive the optimal investment and operative decisions under uncertain policy conditions. Most studies aim at correctly modeling the market-driven sources of uncertainty under specific policy schemes, like the carbon price process under the EU emission trading scheme (e.g. Refs. [10,12,15,19,26,29–31]). Some studies acknowledge that policy uncertainty could be modeled more drastically. This can be done by including stochastic jumps in the prices of policy instruments reflecting sudden changes in the policy target (e.g. Refs. [28,11]), or by modeling the risk that a scheme will be introduced (e.g. Ref. [16]), or that an existing scheme will be replaced (e.g., Ref. [4] and Ref. [23]) or simply removed (e.g. Refs. [2,3,24]). analyze policy uncertainty from a different perspective. They examine the uncertainties arising from public support for renewable energy and show how these uncertainties generate real regulatory options, not in the hands of the project's promoter, that reduce the net present value of the project. Finally, a few studies have used project-level data to test whether energy firms time their decisions as predicted by real options models under uncertain policy conditions [16,25]. These empirical studies find that uncertain policy and regulatory conditions significantly affect the pattern of development in the electric power industry.

The nearest papers apparently to ours are Boomsma et al. [4] and Ref. [2]. Boomsma et al. [4] examine investment timing and capacity choice under uncertainty in capital costs, electricity price and subsidy payments under different renewable electricity support schemes, and the possibility of a change from one support scheme to another. Using simulations they find that feed-in tariffs encourage earlier investments than feed-in premiums and green certificates [2]. derive the investment timing for a renewable energy facility with price and quantity uncertainty, where there might be a subsidy proportional to the quantity of production. Including

the possibility that the subsidy is retroactively terminated, they conclude that a subsidy, even one having an unexpected withdrawal, will hasten investment compared to a situation with no subsidy. Like Boomsma et al. [4] we allow for more than one stochastic price process in order to realistically model the support schemes in use. We extend their analysis by allowing for correlation in prices to better investigate the risk of green certificates under different assumptions of price dependencies. In order to more clearly convey how individual price and policy uncertainties are related to the threshold revenue, we choose to derive the solution analytically following an approach developed in Ref. [1] and applied in Ref. [2]. Like [2] we examine the prospects of scheme termination; but we reach a somewhat different conclusion than Ref. [2] because we compare and contrast situations where investors believe this decision will be retroactively applied or not.

Real options studies that have derived analytical solutions for cases with two, possibly correlated, geometric Brownian motion diffusion processes include the classical reference by Ref. [20]. They examine the perpetual American option to pay a stochastic cost  $I$  against a project of stochastic value  $S$ . The option value function is homogenous of degree one and thus the investment rule is simplified to wait until  $S/I$  reaches a constant threshold value [1]. extend this model to a two dimensional real options problem where the option value function is not homogenous of degree one and, as a consequence, it is not possible to reduce the dimensionality down to one. More specifically, they examine the perpetual American option to pay a constant cost  $I$  against the net cash flow  $S-K$  where both cash flows follow, possibly correlated, geometric Brownian motion processes. They develop an implicit representation of the investment boundary as the solution to a set of  $n$  simultaneous equations in  $n+1$  unknown variables and parameters. By fixing one of the random variables, say  $S$ , they derive a threshold value for the other random variable  $K$  as a function of the first. We use their approach to examine a similar problem; to pay a fixed cost  $I$  against the sum of two, possibly correlated, price processes  $S+K$ . We show that the optimal threshold provides a non-linear relation between these two random variables.

Merton, (1976) [22] was the first to construct an option pricing formula where the value of the underlying asset is generated by a mixture of both jump and diffusion processes. Later real option studies have applied Merton's jump-diffusion model to processes involving sudden death, birth and change of the value of the underlying asset (e.g. Refs. [2,5,8,27]). Our study builds upon Ref. [8] who examine the prospects of an introduction or termination of an investment tax credit. In contrast to Ref. [8]; we assume that policy change is permanent; that is, once the support scheme is terminated, it is never altered. This is the same set-up as in Refs. [2,27]. However, by assuming that investors either expect or do not expect that these changes will be retroactively applied, we show that including jump mechanisms may increase but also decrease the value of waiting.

Our choice of price processes results in a threshold revenue with important characteristics. In cases where both prices are random, such as tradable green certificates, the optimal threshold revenue is a convex function of the observed electricity (subsidy) price. Consequently, as long as the electricity and subsidy prices are not perfectly correlated, part of their individual risks will be diminished through diversification when they are combined. One may argue that the electricity and certificate prices are negatively correlated (see Refs. [13,17]), in which case the gains from risk diversification may be substantial. It follows that the market risk and therefore the threshold revenue may be higher but also lower under a quantity-driven scheme as compared with a price-driven scheme. By including a Poisson jump process, we add further characteristics to the optimal investment threshold. The prospects of termination

will raise the threshold revenue and thus slow down investments if it is retroactively applied, but lower the threshold revenue and thus speed up investments if it is not. Thus, policy uncertainty may add a substantial risk to investments, especially in the first case where investors expect future curtailment of subsidies to affect new and old installations alike.

In the next section we show how uncertainty is modelled. In the third and fourth sections we derive the threshold revenue analytically for different support schemes and different uncertainty assumptions. In the fifth section we illustrate these analytical solutions with a numerical example for a wind power project. We discuss policy implications in the final section.

## 2. Market and policy uncertainty

We distinguish between two types of uncertainty: market uncertainty and policy uncertainty. Market uncertainty relates to electricity prices and subsidy payments, and evolves continuously over time. In contrast, policy uncertainty, such as future termination of the current support scheme, occurs at discrete points in time.

When modeling market uncertainty, we let electricity prices  $(S_t)_{t \geq 0}$  and subsidy payments  $(K_t)_{t \geq 0}$  follow geometric Brownian motion processes such that

$$dS_t = \mu_S S_t dt + \sigma_S S_t dz_{S_t}, \tag{1}$$

$$dK_t = \mu_K K_t dt + \sigma_K K_t dz_{K_t}. \tag{2}$$

Here,  $\mu_S, \mu_K$  and  $\sigma_S, \sigma_K$  are constants that represent the trends and volatilities of prices and payments, respectively, and  $dz_{S_t}, dz_{K_t}$  are standard Brownian motions with  $\mathbb{E}[dz_{S_t} dz_{K_t}] = \rho dt$ . Hence, current values of the stochastic processes are known, whereas future values are log-normally distributed with means, variances and covariance that grow linearly with time. In the following presentation, we may occasionally refer to subsidy payments as subsidy prices.

This definition of market uncertainty covers various market designs. The subsidy may be paid out either as a substitute for the electricity price ( $S_t \equiv 0$ ) or in addition to this price ( $S_t > 0$ ). We will specifically focus on risk exposure related to both processes, and so our real options problem is bivariate. We will also derive solutions for the cases where the producer faces variations in one price (univariate real options problem) or in none of the prices (deterministic problem).

**Remark 1.** A support scheme for which  $\sigma_K = 0$  is usually referred to as feed-in tariff ( $S_t \equiv 0$ ) or a feed-in premium ( $S_t > 0$ ). A scheme with  $\sigma_S > 0$  and  $\sigma_K > 0$  and  $S_t > 0$  and  $K_t > 0$  can be viewed as a certificate trading scheme, in which certificate prices represent the subsidy payments.

We model policy uncertainty as a Markov process  $(\delta_t)_{t \geq 0}$  with states  $\{0, 1\}$  such that

$$\delta_t = \begin{cases} 1, & \text{if a policy change has occurred in the time interval } [0, t), \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

with  $\delta_0 = 0$ . The jump-intensities of the Markov process are denoted by  $\lambda_{ij}$ , where we assume that

$$\lambda_{ij} = \begin{cases} \lambda, & \text{if } i = 0, j = 1, \\ 0, & \text{if } i = 1, j = 0, \end{cases} \tag{4}$$

for constant  $\lambda > 0$ . Roughly speaking,  $d\delta_t = 1$ ; that is, a change of policy occurs during a short time interval  $dt$ , with probability  $\lambda dt$ .<sup>2</sup> Furthermore, if  $\delta_{t-} = 0$  ( $t-$  denotes the left-hand limit of  $t$ ), then  $\delta_t = 0$  with probability  $1 - \lambda dt$ , and if  $\delta_{t-} = 1$ , then  $\delta_t = 1$  with probability 1, such that 1 is an absorbing state. We assume that policy change is independent of the evolution of electricity prices and subsidy payments.

**Remark 2.** We motivate policy change by advances in technology. In particular, governments may eventually decide to terminate the current support scheme as renewable electricity technologies become increasingly mature. For this reason, we assume that once the support scheme has been terminated, it is never re-introduced.

## 3. Support scheme with an infinite lifespan

We start by valuing an operating renewable electricity project that is entitled to a deterministic or stochastic subsidy throughout the lifetime of the project.

The project lifetime is finite and denoted by  $T$ . We assume constant expected production.<sup>3</sup> If profit scales with production, this implies we can value a single unit of production. We consider a price-taking producer, whose instantaneous per unit revenue is given by the sum of the electricity price and a subsidy payment. If operating costs are constant, these can be incorporated into the investment costs, and to simplify the presentation, we therefore disregard these costs. We denote the required rate of return of the project by  $r$ <sup>4</sup> and assume that  $r > \mu_S$  and  $r > \mu_K$ . Without these assumptions, investment may never occur.

We denote the value of the project by  $V(S, K)$ , which is a function of the current electricity price  $S$  and subsidy payment  $K$ . This value is the expected present value of future revenues over the project's lifetime, that is,

$$V(S, K) = \mathbb{E} \left[ \int_0^T e^{-rt} (S_t + K_t) dt \mid S_0 = S, K_0 = K \right] := r_S S + r_K K, \tag{5}$$

where we define

$$r_S = \int_0^T e^{-(r-\mu_S)t} dt = \frac{1 - e^{-(r-\mu_S)T}}{r - \mu_S}, \quad r_K = \int_0^T e^{-(r-\mu_K)t} dt = \frac{1 - e^{-(r-\mu_K)T}}{r - \mu_K}, \tag{6}$$

see Appendix B.1 for the derivation. In spite of the correlation between electricity prices and subsidy payments, the project value is additive. Also, note that the result continues to hold if  $\mu_K = 0, \sigma_K = 0$  and/or  $S_t \equiv 0$ . Finally, we let investment costs be constant and denote these by  $I$ , such that the net present value of the project is  $V(S, K) - I$ .

<sup>3</sup> For many renewable energy sources, production is completely determined by the weather conditions and may be highly varying on hourly, daily and seasonal time scales. Variations are usually smaller on a yearly time scale, which justifies the assumption of constant expected production.

<sup>4</sup> Many asset pricing papers apply risk-neutral valuation, assuming market completeness. Under this assumption, the discount rate reflects the required rate of return on projects with similar risk. However, electricity markets may be far from complete due lack of suitable hedging instruments for volume risk, policy risk etc. For this reason, we assume an exogenously given discount rate.

<sup>2</sup> The probability that a jump occurs during a short time interval  $dt$  is in fact  $\lambda dt + o(dt)$ , where  $o(dt)$  is a term of order less than  $dt$ . In accordance with real options theory, we ignore  $o(dt)$ .

The value of the investment option  $W(S,K)$  is also a function of the state variables  $S$  and  $K$ . Assuming that our option to invest has an infinite lifespan, we obtain a time-homogeneous value process and time does not need to be a state variable. The investment option value is the expected net present value of the project at the optimal time of investment. Because the underlying stochastic processes are Markovian, the value function satisfies the Bellman equation

$$W(S,K) = \max \left\{ V(S,K) - I, \frac{1}{1+rdt} \mathbb{E}[W(S+dS, K+dK)|S,K] \right\}. \quad (7)$$

According to the Bellman equation, at any point in time, the investor decides whether to invest in the project or continue to delay investment. By applying Itô's lemma (see Ref. [14]) to expand the expectation, and rearranging terms, we arrive at the following second order homogenous partial differential equation (PDE), which holds when continuation is optimal

$$\frac{1}{2} \left( \sigma_S^2 S^2 \frac{\partial^2 W}{\partial S^2} + \sigma_K^2 K^2 \frac{\partial^2 W}{\partial K^2} + 2\sigma_S \sigma_K \rho SK \frac{\partial^2 W}{\partial S \partial K} \right) + \mu_S S \frac{\partial W}{\partial S} + \mu_K K \frac{\partial W}{\partial K} - rW = 0. \quad (8)$$

Intuitively, this PDE requires that the expected rate of return on investment equals the discount rate. It is subject to conditions at the boundary at which investment becomes optimal.

To solve the PDE, we assume a generic solution of the form

$$W(S,K) = \beta S^{\alpha_S} K^{\alpha_K}, \quad (9)$$

in which  $S > 0$  and  $K > 0$ .<sup>5</sup>

**Remark 3.** If the value function had been homogenous of degree one, the solution could be shown to have this form (see Ref. [20]).<sup>6</sup> In our case, however, the value of the project is *not* homogenous of degree one, and therefore our bivariate real options problem cannot be reduced to a univariate real options problem. Instead we follow a quasi-analytical approach developed by Ref. [1] for an equivalent problem to ours, and applied in Ref. [2].

For the above expression to be a solution,  $\alpha_S$  and  $\alpha_K$  must satisfy the equation  $\mathcal{L}(\alpha_S, \alpha_K) = 0$ , where

$$\mathcal{L}(\alpha_S, \alpha_K) = \frac{1}{2} \left( \sigma_S^2 \alpha_S (\alpha_S - 1) + \sigma_K^2 \alpha_K (\alpha_K - 1) + 2\sigma_S \sigma_K \rho \alpha_S \alpha_K \right) + \mu_S \alpha_S + \mu_K \alpha_K - r. \quad (10)$$

This is the equation of an ellipse that is present in all four quadrants of the plane.<sup>7</sup> For the investment value to be increasing

<sup>5</sup> This solution covers a situation with two stochastic prices following a geometric Brownian Motion. For support schemes with only one stochastic price (for instance a feed-in premium scheme in which  $\sigma_K = 0$ ) or no stochastic price (for instance a feed-in tariff scheme in which  $S = 0$  and  $\sigma_K = 0$ ), the optimization problem degenerates to a lower dimension real option problem. The solutions to these problems are shown at the end of Section 3. The numerical illustrations in Section 5 show that when  $K(S)$  approach zero, the resulting threshold revenue for two stochastic prices  $S$  and  $K$  approach the same value as for one stochastic price  $S$ . Equivalently, when  $S$  approach zero, the resulting threshold revenue for one stochastic price  $S$  approach the same value as for no stochastic price.

<sup>6</sup> [20] show that if the value function is homogenous of degree one, then one can let  $W(S,K) = Sw(s), s = K/S$  for some function  $w(\cdot)$  and reduce the PDE for the two-factor problem to a one-factor PDE.

<sup>7</sup> Note that for  $\alpha_S = 0$  or  $\alpha_K = 0$ , the equation reduces to the quadratic equation of the standard one-factor PDE from real options analysis. This equation is known to have a positive and a negative root (see Ref. [8]).

in both prices, we restrict attention to solutions in the first quadrant; that is, we assume that  $\alpha_S \geq 0$  and  $\alpha_K \geq 0$ .

When investment is optimal, the option value equals the net present value of the project  $W(S,K) = r_S S + r_K K - I$ . Hence, the boundary conditions are

$$\beta S^{\alpha_S} K^{\alpha_K} = r_S S + r_K K - I, \quad \beta \alpha_S S^{\alpha_S - 1} K^{\alpha_K} = r_S, \quad \beta \alpha_K S^{\alpha_S} K^{\alpha_K - 1} = r_K, \quad (11)$$

where  $S$  and  $K$  must be the prices at which investment is optimal (i.e. threshold prices). The first equality is the value matching condition and the second and third equalities are the so-called smooth pasting conditions (see Ref. [8]). With  $\mathcal{L}(\alpha_S, \alpha_K) = 0$ , we obtain four equalities in the five unknowns  $(\alpha_S, \alpha_K, \beta, S, K)$ , and so the threshold prices define a one-dimensional subset of  $S \times K$ -space. By fixing  $S$  and manipulating the boundary conditions, we arrive at the following result.

**Proposition 1.** Assume that  $\mu_S \neq 0 \vee \sigma_S > 0$  and  $\mu_K \neq 0 \vee \sigma_K > 0$ . Given  $S > 0$ , the optimal time  $t$  to invest is the first time  $K_t \geq K^*(S)$ , where  $K^*(S)$  is determined by the following equalities

$$S = \frac{\alpha_S}{\alpha_S + \alpha_K - 1} \cdot \frac{I}{r_S}, \quad K^*(S) = \frac{\alpha_K}{\alpha_S + \alpha_K - 1} \cdot \frac{I}{r_K}, \quad (12)$$

$$\mathcal{L}(\alpha_S, \alpha_K) = 0, \quad \alpha_S, \alpha_K \geq 0.$$

**Remark 4.** Note that we may fix either  $S$  or  $K$  such that the optimal time  $t$  to invest is the first time  $K_t \geq K^*(K)$  or  $S_t \geq S^*(K)$ . In most cases, we present the threshold as  $K^*(S)$ , reflecting the required subsidy payment to trigger investment. In the case of a fixed subsidy payment, however, we present the threshold as  $S^*(K)$  following the standard of real options analysis, in which the threshold relates to the stochastic process. From a policy perspective, one may be interested in the total revenue required to trigger investment. Thus, for a given electricity price  $S$ , we present  $S + K^*(S)$  when we obtain the thresholds numerically in Section 5.

When investment is optimal, we find that

$$\beta = \frac{1}{S^{\alpha_S} K^*(S)^{\alpha_K}} \cdot \frac{I}{\alpha_S + \alpha_K - 1} = \frac{(\alpha_S + \alpha_K - 1)^{\alpha_S + \alpha_K - 1}}{\alpha_S^{\alpha_S} \alpha_K^{\alpha_K}} \cdot \frac{r_S^{\alpha_S} r_K^{\alpha_K}}{I^{\alpha_S + \alpha_K - 1}}, \quad (13)$$

with  $\beta > 0$ .

For  $\alpha_K = 0$ , note the similarity of the result in Proposition 1 to a univariate real options problem of the type in Ref. [8]. Thus, the above result for two uncertainty factors is the natural extension of the one-factor problem.

Because there is a value of delaying investment, the expected present value of future revenue revenues must exceed investment costs to trigger investment in a case with two uncertainty factors.

### 3.1. Corollary 1

$$r_S S + r_K K^*(S) = \frac{\alpha_S + \alpha_K}{\alpha_S + \alpha_K - 1} \cdot I > I. \quad (14)$$

See Appendix C.1 for the proof.

In Proposition 1, the investment threshold  $K^*(S)$  depends on  $S$  through the equation  $\mathcal{L}(\alpha_S, \alpha_K) = 0$ . It can, however, be stated in another way. For a given price  $S$ , let

$$\eta(S) = \frac{I - r_S S}{r_S S}. \quad (15)$$



Then, the optimal time  $t$  to invest is the first time  $K_t \geq K^*(S)$ , where

$$K^*(S) = \frac{\alpha\eta(S) + 1}{\alpha(\eta(S) + 1)} \cdot \frac{I}{r_K}, \quad \mathcal{C}(\alpha, \alpha\eta(S) + 1) = 0, \quad \alpha, \alpha\eta(S) + 1 \geq 0. \quad (16)$$

This is instructive in the sense that for a given electricity price  $S$ , we can isolate the subsidy payment required to trigger investment.

For completeness, we provide the solutions to the univariate real options problem that arise with a fixed price premium and to the deterministic problem with a fixed tariff.

**Fixed feed-in premium** When subsidies are paid out as a constant price premium  $K$ , i.e.  $\mu_K = \sigma_K = 0$  and  $S_t > 0$ , the optimal time  $t$  to invest is the first time  $S_t \geq S^*(K)$ , where

$$S^*(K) = \frac{\alpha_S}{\alpha_S - 1} \cdot \frac{I - r_K K}{r_S}, \quad \mathcal{C}_S(\alpha_S) = 0, \quad (17)$$

with  $\mathcal{C}_S(\alpha_S) := \mathcal{C}(\alpha_S, 0)$  and we assume that  $r_K K < I$ ; that is, the subsidy payment is not by itself sufficient to justify investment.

**Fixed feed-in tariff** When subsidies are paid out as a constant tariff  $K$  (i.e.  $\mu_K = \sigma_K = 0$  and  $S_t = 0$ ) immediate investment is optimal if and only if

$$r_K K \geq I, \quad (18)$$

which is also known as the net present value (NPV) rule.

#### 4. Termination of the support scheme

We now analyze the prospect that at a random point in time the current support scheme may be terminated. We distinguish between two cases. The investor either believes that government may decide not to enter into new contracts but will commit to existing ones (i.e., there is no retroactive termination of subsidy payments), or it may decide neither to enter into new contracts nor to commit to existing ones (i.e., subsidy payments are terminated retroactively).

In valuing the project and investment, we consider two regimes: Regime 0, in which termination has not yet occurred, and regime 1, in which the support scheme has already been terminated. Accordingly, we denote the project value in regimes 0 and 1 by  $V_0(S, K)$  and  $V_1(S, K)$ , respectively, and likewise for the value of the investment option.

Again, the value of an operating project is the expected present value of future revenues. In regime 0, revenues from electricity prices continue throughout the project lifetime, but subsidy payments may be terminated at a random point in time. In regime 1, revenues stem from electricity prices only. Hence,

$$V_0(S, K) = \mathbb{E} \left[ \int_0^T e^{-rt} (S_t + K_t \mathbf{1}_{\{\delta_t=0\}}) dt \mid S_0 = S, K_0 = K, \delta_0 = 0 \right] : \\ = r_S S + r_{K0}(\lambda) K, \quad (19)$$

$$V_1(S) = \mathbb{E} \left[ \int_0^T e^{-rt} S_t dt \mid S_0 = S, \delta_0 = 1 \right] = r_S S, \quad (20)$$

where  $r_S$  is defined as above and

$$r_{K0}(\lambda) = \int_0^T e^{-(r+\lambda-\mu_K)t} dt = \frac{1 - e^{-(r+\lambda-\mu_K)T}}{r + \lambda - \mu_K}, \quad (21)$$

see Appendix B.2 for the derivation. Without retroactive termination, we use the compounding factor  $r_{K0}(\lambda)$ , where  $\lambda=0$ ; whereas with retroactive termination, we use  $r_{K0}(\lambda)$ , where  $\lambda>0$ . Consequently, the project value in regime 0 will be lower if investors expect the termination to be retroactively applied than if investors expect the termination decision to only affect future installations.

The value of the investment option must satisfy the following Bellman equations:

$$W_0(S, K) = \max \left\{ V_0(S, K) - I, \frac{1 - \lambda dt}{1 + r dt} \mathbb{E}[W_0(S + dS, K + dK) \mid S, K] \right. \\ \left. + \frac{\lambda dt}{1 + r dt} \mathbb{E}[W_1(S + dS) \mid S] \right\}, \quad (22)$$

$$W_1(S) = \max \left\{ V_1(S) - I, \frac{1}{1 + r dt} \mathbb{E}[W_1(S + dS) \mid S] \right\}. \quad (23)$$

Whether in regime 0 or 1, at any point in time, the investor decides whether to invest in the project or continue to delay investment. In regime 0, new investment is entitled to subsidies, and the investment value depends on both electricity prices and subsidy payments. If continuation is optimal, the probability that the support scheme will be terminated during a short time interval  $dt$  is  $\lambda dt$ , whereas the probability that it will not is  $1 - \lambda dt$ . In regime 1, new investment is no longer entitled to subsidies and future revenues stem from electricity prices only.

By applying Itô's lemma and rearranging terms, we obtain the following system of second order PDEs, which holds when continuation is optimal

$$\frac{1}{2} \left( \sigma_S^2 S^2 \frac{\partial^2 W_0}{\partial S^2} + \sigma_K^2 K^2 \frac{\partial^2 W_0}{\partial K^2} + 2\sigma_S \sigma_K \rho SK \frac{\partial^2 W_0}{\partial S \partial K} \right) \\ + \mu_S S \frac{\partial W_0}{\partial S} + \mu_K K \frac{\partial W_0}{\partial K} - \lambda(W_0 - W_1) - rW_0 = 0, \quad (24)$$

$$\frac{1}{2} \sigma_S^2 S^2 \frac{\partial^2 W_1}{\partial S^2} + \mu_S S \frac{\partial W_1}{\partial S} - rW_1 = 0. \quad (25)$$

In regime 0, the expected rate of return on the investment decreases because of the risk of termination, which produces an additional term in the PDE. Hence, for a given discount rate, a higher expected rate of return is required to off-set termination risk.

To derive a solution to this system of PDEs, we define the equations

$$\mathcal{C}_0(\alpha_S, \alpha_K) = \frac{1}{2} \left( \sigma_S^2 \alpha_S (\alpha_S - 1) + \sigma_K^2 \alpha_K (\alpha_K - 1) + 2\sigma_S \sigma_K \rho \alpha_S \alpha_K \right) \\ + \mu_S \alpha_S + \mu_K \alpha_K - (r + \lambda), \quad (26)$$

$$\mathcal{C}_1(\alpha_S) = \frac{1}{2} \sigma_S^2 \alpha_S (\alpha_S - 1) + \mu_S \alpha_S - r. \quad (27)$$

The derivations can be found in Appendix D.1 and provide the following result:

**Proposition 2.** Assume that  $\lambda > 0$ ,  $\mu_S \neq 0 \vee \sigma_S > 0$  and  $\mu_K \neq 0 \vee \sigma_K > 0$ . Then,

1. The optimal time  $t_1$  to invest under regime 1 is the first time  $S_{t_1} \geq S_1$ , where<sup>8</sup>

$$S_1 = \frac{\alpha_{S1}}{\alpha_{S1} - 1} \cdot \frac{I}{r_S}, \quad \mathcal{C}_1(\alpha_{S1}) = 0, \quad \alpha_{S1} \geq 0. \quad (28)$$

2. Moreover, given  $S > 0$ , the optimal time  $t_0$  to invest under regime 0 is the first time  $K_{t_0} \geq K_0(S)$ , where  $K_0(S)$  is determined by the following equalities

$$S = \frac{\alpha_{S0}}{\alpha_{S0} + \alpha_{K0} - 1} \cdot \frac{I}{r_S} \cdot \left( 1 + \frac{\alpha_{S1}(\alpha_{K0} - 1) + \alpha_{S0}}{\alpha_{S0}(\alpha_{S1} - 1)} \cdot \left(\frac{S}{S_1}\right)^{\alpha_{S1}} \right), \quad (29)$$

$$K_0(S) = \frac{\alpha_{K0}}{\alpha_{S0} + \alpha_{K0} - 1} \cdot \frac{I}{r_{K0}(\lambda)} \cdot \left( 1 - \left(\frac{S}{S_1}\right)^{\alpha_{S1}} \right), \quad (30)$$

$\mathcal{C}_0(\alpha_{S0}, \alpha_{K0}) = 0, \quad \alpha_{S0}, \alpha_{K0} \geq 0.$

Under regime 1, the support scheme has been terminated, and the problem is a univariate real options problem. For  $\alpha_{K0} = 0$ , the problem likewise reduces to a univariate one under regime 0.

As above, to trigger investment, the expected net present value of future revenues must exceed investment costs, and hence at the boundary we have.

#### 4.1. Corollary 2

Assume that  $S \leq S_1$ . Then,

$$r_S S + r_{K0}(\lambda) K_0(S) > I. \quad (31)$$

The proof can be found in [Appendix C.2](#).

As above, we state an alternative formulation of [Proposition 2](#). For a given  $S > 0$ , let

$$\eta(S) = \frac{(I - r_S S)(\alpha_{S1} - 1)S_1^{\alpha_{S1}} + IS^{\alpha_{S1}}}{r_S S(\alpha_{S1} - 1)S_1^{\alpha_{S1}} - \alpha_{S1}IS^{\alpha_{S1}}}. \quad (32)$$

Then, the optimal time  $t$  to invest is the first time  $K_t \geq K_0(S)$ , where we can isolate  $K_0(S)$  such that

$$K_0(S) = \frac{\alpha\eta(S) + 1}{\alpha(\eta(S) + 1)} \cdot \frac{I}{r_{K0}(\lambda)} \cdot \left( 1 - \left(\frac{S}{S_1}\right)^{\alpha_{S1}} \right), \quad (33)$$

$$\mathcal{C}_0(\alpha, \alpha\eta(S) + 1) = 0, \quad \alpha, \alpha\eta(S) + 1 \geq 0. \quad (34)$$

**Fixed feed-in premium** As above, when  $\mu_K = \sigma_K = 0$  and  $S_t > 0$ , the optimal time  $t$  to invest under regime 1 is the first time  $S_t \geq S_0(K)$ , where

$$S_0(K) = \frac{\alpha_{S0}}{\alpha_{S0} - 1} \cdot \frac{I - r_{K0}(\lambda)K}{r_S} + \frac{\alpha_{S0} - \alpha_{S1}}{(\alpha_{S0} - 1)(\alpha_{S1} - 1)} \cdot \frac{I}{r_S} \cdot \left(\frac{S_0(K)}{S_1}\right)^{\alpha_{S1}}, \quad (35)$$

$\mathcal{C}_{S0}(\alpha_{S0}) = 0,$

where  $\mathcal{C}_{S0}(\alpha_K) := \mathcal{C}_0(0, \alpha_K)$  and assuming  $r_{K0}(\lambda)K < I$ .

Assuming the subsidy payment alone is not sufficient to justify investment, we can show the existence of the investment threshold

$S_0(K)$  and compare it with the threshold under a support scheme with an infinite lifespan  $S^*(K)$  and that under the risk of termination  $S_1$ :

#### 4.2. Corollary 3

Assume that  $\mu_K = \sigma_K = 0$  and  $r_{K0}(\lambda)K < I$ . Then.

1. Eq. (35) has a unique root in  $(0, S_1)$ .
2. If  $r_{K0}(\lambda) := r_{K0}(0)$ , then  $S_0(K) < S^*(K) < S_1$ .

The proof can be found in [Appendix C.3](#).

We see that, under a fixed price premium without retroactive termination upon investment, the risk of termination always reduces the required electricity price to trigger investment, and therefore speeds up the investment rate. We show numerically that this is also the case under a certificate trading scheme without retroactive termination. Under a subsidy scheme with retroactive termination risk, our numerical analysis likewise shows that the prospect of termination in most cases reduces the threshold and speeds up investment, but there are cases in which investment slows down.

**Fixed feed-in tariff** When  $\mu_K = \sigma_K = 0$  and  $S_t = 0$ , immediate investment is optimal if and only if

$$r_{K0}(\lambda)K \geq I, \quad (36)$$

which is again the NPV rule.

### 5. Numerical solutions

In this section we obtain the threshold revenues numerically for a wind power project. The thresholds are obtained for given values of the electricity price  $S$  using Propositions 1 and 2 (green certificates), Eqs. (4) and (6) (feed-in premiums) and Eqs. (5) and (7) (feed-in tariffs). The benchmark is a support scheme that investors believe will never be altered. We then examine how investors change their behavior if they suspect that the current support scheme will be terminated at a future unknown point in time. More specifically, we examine under what circumstances such changes in expectations will decrease (increase) the threshold revenue required to invest relative to the benchmark case and thus increase (decrease) the investment rate. Our examinations are conducted assuming that investors either believe the changes will be applied retroactively or they do not.

#### 5.1. Case study

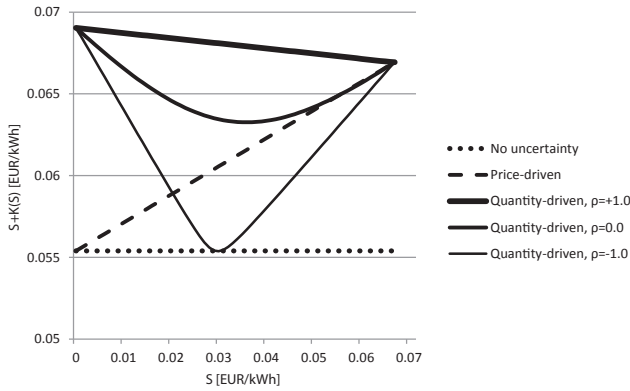
The parameter values used in the numerical illustrations below are presented in [Table 1](#). The project life and investment cost of a wind power installation are set equal to  $T = 20$  years and  $I = 0.7$  EUR/kWh. These numbers are representative for a typical 2 MW wind turbine under average wind speeds in Europe [9]; pages 9–10).<sup>9</sup> The investment cost estimate reflects both investment costs (approximately 75% of total costs) and operation and maintenance costs (approximately 25% of total costs) and is derived using a risk-adjusted nominal discount rate of 7.5%. We use real values in our optimization problem. Thus, when we let  $I$  be fixed over time, we implicitly assume that the nominal investment cost grows with the general price level. The risk adjusted real discount

<sup>8</sup> The quadratic equation  $\mathcal{C}_1(\alpha_{S1}) = 0$  has the root  $\alpha_{S1} = 1/2 - \mu_S/\sigma_S^2 + \sqrt{(1/2 - \mu_S/\sigma_S^2)^2 + 2r/\sigma_S^2}$ , with  $\alpha_{S1} > 1$ .

<sup>9</sup> A more recent study by Ref. [21] of the potential for and cost of onshore windpower installations in Germany confirms concludes that total generation costs are 5–15 EUR/kWh which is equal to an upfront cost of 0.5–1.5 EUR/kWh.

**Table 1**  
Parameter values.

Parameter		Benchmark value	Sensitivity values
$I$	Investment cost	0.7 EUR/kWh	
$T$	Project life	20 years	
$r$	Discount rate	0.05	
$\mu_S$	Trend parameter electricity prices	0	
$\mu_K$	Trend parameter subsidy prices	0	+/- 0.025.
$\sigma_S$	Volatility parameter electricity prices	0.00 or 0.06	0.16
$\sigma_K$	Volatility parameter subsidy prices	0.00 or 0.07	0.16
$\rho$	Correlation coefficient	0	+/- 1.
$\lambda$	Policy uncertainty parameter	0 or 0.1	0.2 and 0.05



**Fig. 1.** Threshold revenue functions for different support schemes assuming there is no policy risk ( $\lambda=0$ ). No uncertainty is illustrated for  $\sigma_K=\sigma_S=0$  (e.g. feed-in tariff). Price-driven support schemes are illustrated for  $\sigma_K=0$  and  $\sigma_S=0.06$  (e.g. feed-in premium). Quantity-driven support schemes are illustrated for  $\sigma_S=0.06$  and  $\sigma_K=0.07$  and different values of  $\rho$  (e.g. tradable green certificates). Real prices are fixed:  $\mu_K=\mu_S=0$ .

rate is set to  $r=5.0\%$ , reflecting an inflation rate of  $2.5\%$ . We use  $\mu_S=\mu_K=0$  in the price processes in Eqs. (1) and (2), which implies that these prices likewise grow with the general price level.

We use data from the period with tradable green certificates in Sweden and Norway to estimate the parameter values for  $\sigma_S$ ,  $\sigma_K$  and  $\rho$ . The electricity price volatility,  $\sigma_S=0.06$ , and the subsidy price volatility,  $\sigma_K=0.07$ , are estimated by the annual standard deviation of the log returns implied by average weekly prices of three-year forward contracts traded at NASDAQ OMX and Svenska Kraftme-kling, respectively, for the period 1 January 2005 to 30 April 2015. The corresponding correlation coefficient is estimated to 0.04, but for convenience we set the benchmark value equal to zero. We also include numerical illustrations using other parameter values. For example, we illustrate the impact of higher volatilities by setting  $\sigma_S=0.16$ . This electricity volatility parameter is estimated by Ref. [16] in a similar fashion as above, but for an earlier period: 2001–2010.

In the simulations where policy uncertainty is introduced, we assume that  $\lambda=0.1$ , implying that investors expect the regime shift to happen in  $1/0.1=10$  years. We also vary this parameter value, to investigate its relative impact on the threshold revenue.

Our model setup assumes that at any point in time  $t$  operating plants receive the same subsidy payment  $K_t$  given in Eq. (2), irrespective of when they were installed. Although this is a realistic feature for tradable green certificates, it is in conflict with feed-in tariff/premium schemes in which the tariff/premium in effect for new plants is adjusted downward over time to reflect an expected decrease in long-run marginal costs as technology matures. However, our setup is suitable for situations in which investors expect the downward shift in the real feed-in tariff/premium in effect for new plants to be equal to the expected fall in long-run marginal

costs, that is,  $E(dK)=E(dI)/r_K$ . In that case, neither the decline in real investment costs nor the corresponding decline in the real tariff/premiums in effect for new plants has to be modeled explicitly because we will get the same results by modeling tariffs/premiums and investment costs as fixed in real values.<sup>10</sup>

In the remaining subsections, the behavior of the investors is as follows. Investors observe the current prices  $S$  and  $K$ . They then calculate the threshold revenue as a function of the current electricity price,  $S+K(S)$ , using the appropriate threshold function derived in Sections 3 and 4. Finally, they decide to invest if the current revenue exceeds the threshold revenue:  $S+K \geq S+K(S)$ .

5.2. The benchmark: support scheme with an infinite lifespan

Assuming investors expect the current support scheme never to be altered ( $\lambda=0$ ), the threshold revenue function  $S+K(S)$  is given in Section 3. We first consider the situation in which there is no growth in the project value ( $\mu_K=\mu_S=0$ ). Thus, the value of waiting, and thereby the level of threshold revenue, will depend only on project value uncertainty. Fig. 1 illustrates the threshold revenue function for three different support schemes: feed-in tariff, feed-in premium and tradable green certificates. For  $\sigma_K=\sigma_S=0$ , the threshold is given by the net present value rule and is fixed to 0.0554 EUR/kWh. This is also the threshold for the fixed feed-in tariff, in which case  $S=0$  and  $K=I/r_K$ , as stated at the end of Section 3. For  $\sigma_K=0$  and  $\sigma_S=0.07$ , the threshold revenue is given at the end of Section 3; it increases linearly with the electricity price, starting at 0.0554 EUR/kWh for  $S=0$  and ending at 0.0690 EUR/kWh for  $K(S)=0$ . This is the threshold revenue function for a fixed feed-in premium. For  $\sigma_S=0.07$  and  $\sigma_K=0.06$ , the threshold revenue function is given in Proposition 1 and is applicable for tradable green certificates. We show the threshold revenues for  $\rho=-1$ , 0 or +1. For  $\rho=0$ , the threshold revenue is convex in the electricity price and reaches a minimum at 0.0634 EUR/kWh.

The threshold revenue functions for the three support schemes can be understood by examining the variance of the relative change in project value at time  $t$  given by

$$\text{Var}\left(\frac{dV_t}{V_t}\right) = w_t^2 \sigma_S^2 dt + (1-w_t)^2 \sigma_K^2 dt + 2w_t(1-w_t)\rho\sigma_K\sigma_S dt, \tag{37}$$

in which  $w_t = r_S S_t / (r_S S_t + r_K K_t)$  (Appendix E). It follows that combining two stochastic price processes, part of the risk is diversified away as long as the two price processes are not perfectly correlated. If two stochastic price processes are perfectly negatively

<sup>10</sup> Under feed-in schemes, a plant will receive the tariff/premium in effect in the year of installation, and it is either kept fixed for the entire period or it is fully or partly adjusted each year for inflation. For an example of details on feed-in tariffs, see Ref. [6].

correlated, i.e.  $\rho = -1$ , all project risk can be diversified away for a given combination of  $S_t$  and  $K_t$ . If two stochastic price processes are perfectly positively correlated, i.e.  $\rho = +1$ , the total risk is the weighted average of the standard deviations of  $S_t$  and  $K_t$ . For all other correlation values, part of total project risk can be diversified away when combining two stochastic processes.

Based on historical data, we have estimated a correlation between weekly electricity and certificate prices for the Swedish-Norwegian market close to zero (see Table 1). However, based on theory, there are reasons to expect that these prices are negatively correlated over time, implying high gains from diversification. In particular, to fulfill the renewable electricity target, the sum of prices should ideally sum to the long-run marginal cost of the marginal producer. Thus, an exogenous shock in the electricity price should be counterbalanced by an adjustment of the certificate price in the opposite direction (see Refs. [13,17]). For negatively correlated prices, investors may therefore in fact require a strictly lower threshold revenue under tradable green certificates as compared with feed-in premiums. This will be the case when the subsidy constitutes a relatively small share of threshold revenues and/or the correlation is negative. The feed-in tariff will always be the least risky support scheme for the investor.

Thus far we have assumed that electricity and subsidy payments remain fixed in real value. We will now consider the impact of real increases and decreases in subsidy payments under a quantity-driven scheme such as tradable green certificates. In this scheme the subsidy payment  $K_t$  is determined in a market and reflects the long-run cost of the marginal producer at each point in time  $t$ . Investors could expect a decrease in  $K_t$  over time because the technology of the marginal producer mature, or they could expect an increase in  $K_t$  over time because society has to put increasingly more expensive technologies into use to meet long-term renewable electricity targets.

Including a non-zero trend of subsidy payments complicates the analysis. On one hand, an increase in  $\mu_K$  will always increase the net present value of immediate investment  $V-I$  because it increases the project value  $V$  in Eq. (5) through its impact on the discount factor  $r_K$ . On the other, an increase in  $\mu_K$  may increase or decrease the value of waiting through its effect on both the growth rate and the variance of the return on project value, the latter being affected by a change in the weight of electricity prices,  $w$ . Thus, the net impact on threshold revenue of an increase in  $\mu_K$  will depend on the parameter values and the current electricity price,  $S$ .

Fig. 2 gives the threshold revenue functions for a quantity-driven support scheme for different subsidy payment trends,  $\mu_K$ . For simplicity, we set  $\sigma_K = \sigma_S = 0.07$  in this example. A positive trend in subsidy payments ( $\mu_K = +0.025$ ) results in a higher threshold revenue as compared with a zero trend in subsidy payments. This is a result of two effects. First, a growth in subsidy payments leads to a growth in project value  $V$  in Eq. (5), which ignoring uncertainty, increases the value of waiting relative to the value of immediate investment for all  $S$ .<sup>11</sup> This first effect increases the threshold  $K(S)$  for all  $S$ . Second, a positive trend in subsidy payments affects the variance of the return on project value in Eq. (37) because it reduces the weight of electricity prices,  $w$ . In our example, where  $\sigma_K = \sigma_S$ , the total project risk may be higher or lower in the positive trend case as compared with the zero trend case, depending on the current electricity price  $S$ , and so the second effect may increase or decrease the threshold  $K(S)$  depending on  $S$ .

A negative trend in subsidy payments ( $\mu_K = -0.025$ ) surprisingly also results in a higher threshold revenue as compared with a

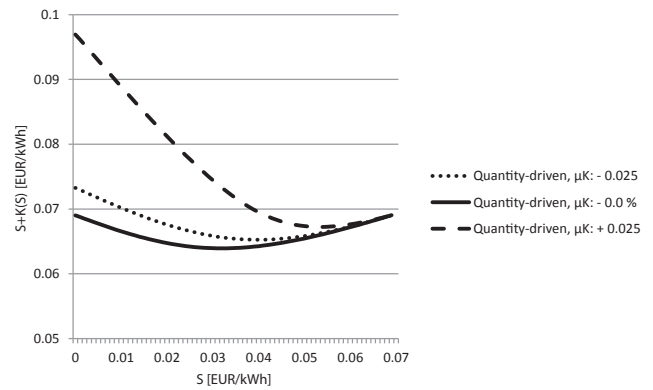


Fig. 2. Threshold revenue functions for a quantity-driven support scheme for different subsidy payment trends,  $\mu_K < r$ , assuming there is no policy risk ( $\lambda = 0$ ). Other parameters are set at:  $\mu_S = 0$ ,  $\sigma_K = \sigma_S = 0.07$  and  $\rho = 0$ .

zero trend in subsidy payments. Ignoring uncertainty, a negative trend in subsidy payments decreases the net present value of immediate investment relative to the value of waiting. Recall that in the zero-trend zero-volatility case ( $\mu_K = \mu_S = \sigma_K = \sigma_S = 0$ ) the value of waiting is zero and the investor should invest if the net present value is greater than or equal to zero at a threshold revenue of 0.054 EUR/kWh. When a negative trend in subsidy payments is introduced, the value of waiting is still zero but investor requires a higher threshold revenue to keep the project value equal to investment costs. Introducing uncertainty, a negative trend in subsidy payments affects the variance of the return on project value in a similar fashion as described for the positive trend. In this specific case, the sum of these two effects results in an increase in the threshold revenue for all electricity prices.

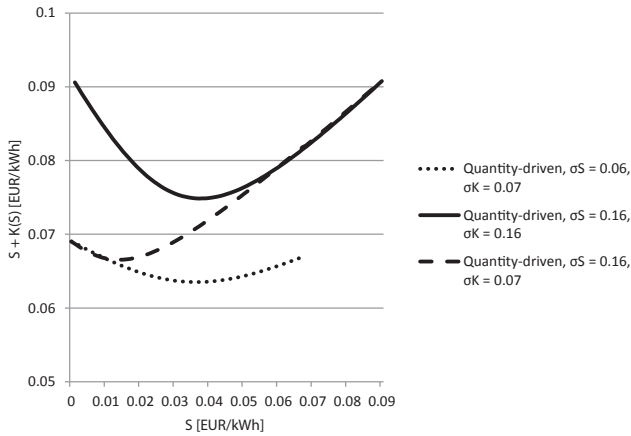
Finally, in Fig. 3, we consider how different combinations of electricity and subsidy price volatilities affect the threshold revenue for a quantity-driven support scheme. In particular, we show how much higher the threshold revenues would be if investors expect the electricity price volatility to be  $\sigma_S = 0.16$ , that is, the parameter value estimated by Ref. [16] for the period 2001–2010. An increase in either  $\sigma_S$  or  $\sigma_K$  will increase the threshold revenue for all values of  $S$ , all else being equal. If  $\sigma_S > \sigma_K$ , the threshold revenue will be higher for high range electricity prices because, in this case, subsidies account for a relatively smaller share of revenues. We also show the threshold revenues if investors expect both volatility parameters to increase to  $\sigma_S = \sigma_K = 0.16$ . If investors expect such high volatility levels in the future, the threshold revenues will be 20–30% higher (1–2 EUR/kWh higher) than in the benchmark case where  $\sigma_S = 0.06$  and  $\sigma_K = 0.07$ .

### 5.3. Termination of the support scheme

First, we consider the situation in which investors believe that such a termination will not be applied retroactively. In this case the prospect of termination tends to reduce threshold revenues and thus speed up the rate of investment as investors will seek to lock in future subsidies. The threshold revenue function given in Proposition 2 is illustrated in Fig. 4 for a quantity-driven support scheme such as tradable green certificates. The decrease in threshold revenues will be particularly strong when electricity prices are low because the investor is then dependent on the relatively higher share provided by subsidies. For the parameter values chosen, the threshold revenue may decrease by as much as 10%. However, if the investor believes that electricity prices and subsidy payments are negatively correlated, the impact of termination is much more moderate, especially for medium range

<sup>11</sup> For an examination of the value of waiting in the deterministic case with a positive project value growth, see Refs. [8]; pages 138–139.

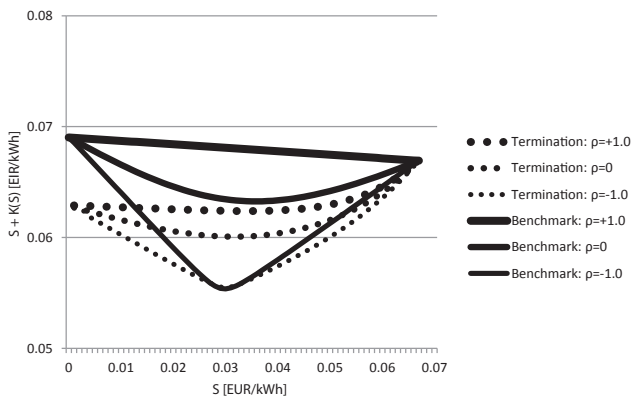




**Fig. 3.** Threshold revenue functions for a quantity-driven support scheme for different combinations of volatility parameters,  $\sigma_S$  and  $\sigma_K$ , assuming there is no policy risk ( $\lambda=0$ ). Other parameters are set at:  $\mu_S=\mu_K=0$  and  $\rho=0$ .

electricity prices. With no expected growth in project value and part or all of the project risk is diversified away, the investment rule will be close to the traditional net present value rule in which an investor invests if  $V-I \geq 0$ . Consequently, the risk of termination may be less of an issue for investors under a green certificate scheme if they expect a termination will only affect installations made after the scheme is terminated.

We next consider the situation in which investors believe that a future termination of the current support scheme will be applied retroactively. In this case, the prospect of termination results in a substantial increase in threshold revenues and thus slows down investments because investors cannot lock in future subsidies by investing immediately. Fig. 5 illustrates this situation for tradable green certificates. The increase in threshold revenues is greater when electricity prices are in the low to medium range. For example, for an observed electricity price  $S=0.03$  EUR/kWh and assuming  $\rho=0$ , the threshold revenue increases by 42% from 0.0634 EUR/kWh to 0.0900 EUR/kWh. Furthermore, the impact of a retroactively applied termination has the same impact on investors whether the prices are positively or negatively correlated. For example, for an observed electricity price  $S=0.03$  EUR/kWh and assuming  $\rho=-1$ , the threshold revenue increases by 46% from 0.0554 EUR/kWh to 0.0809 EUR/kWh. Obviously, the risk of termination of a quantity-driven support scheme is always a



**Fig. 4.** The effects of the prospect of termination on threshold revenue assuming the support scheme is quantity-driven and there is no retroactive arrangement. Termination is expected in 10 years ( $\lambda=0.1$ ), and the benchmark is no policy uncertainty ( $\lambda=0$ ). Other parameters are set at  $\mu_K=\mu_S=0$  and  $\sigma_S=0.06$  and  $\sigma_K=0.07$ .

serious issue for investors if it is expected to be retroactively applied.

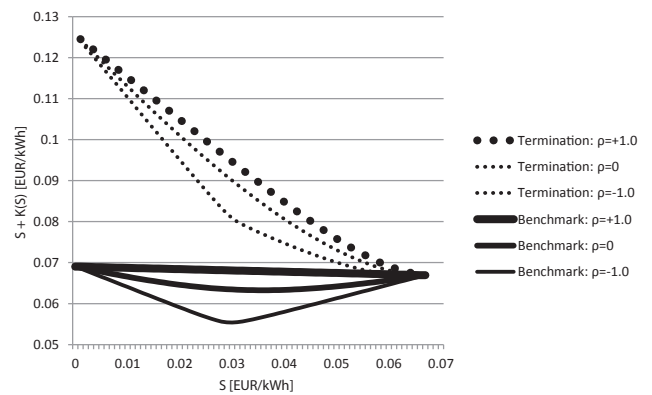
Fig. 6 shows the effects of the prospect of termination on a price-driven support scheme (e.g., feed-in tariff/premium). With no retroactive arrangement, the prospect of termination always decreases threshold revenue, as stated in Corollary 3. However, in this case, the impact is negligible. For example, for an observed electricity price  $S=0.03$  EUR/kWh, the threshold revenue decreases by 3% from 0.0607 EUR/kWh to 0.0586 EUR/kWh. With a retroactive arrangement, the prospect of termination raises the threshold revenue substantially and thus decreases the investment rate. For example, for an observed electricity price  $S=0.03$  EUR/kWh, the threshold revenue increases by 44% from 0.0607 EUR/kWh to 0.0862 EUR/kWh.

Finally we consider how different values of  $\lambda$  affect threshold revenues for a quantity-driven scheme assuming that termination is either retroactively applied or it is not (Fig. 7). If investors assume the termination will not be applied retroactively, the threshold revenue will be only 1–2 % lower when the change is expected to happen five ( $\lambda=0.2$ ) as compared with 10 ( $\lambda=0.1$ ) years into the future. If investors believe the termination will be applied retroactively, however, the threshold revenue will much higher when the change is expected to happen in the near future. For example, for an observed electricity price  $S=0.03$  EUR/kWh, the threshold revenue increases by 36% from 0.0900 EUR/kWh to 0.1224 EUR/kWh when the change is expected to happen five ( $\lambda=0.2$ ) as compared with 10 ( $\lambda=0.1$ ) years into the future.

## 6. Concluding remarks

This paper examines how the market risk inherent in different renewable electricity support schemes and the policy risk that these schemes will eventually be terminated affect investor behavior. The aim is to provide policymakers with a better understanding of the consequences of different policy actions. In this section, we take the perspective of society and ask how risk should optimally be divided between investors and the government.

Risk is not inherently bad. Market risk (i.e. fluctuating electricity and subsidy prices) reflects uncertainties related to supply and demand. Exposed to such risk, investors will make investment and operational decisions that contribute to a better functioning market. Furthermore, in a well-functioning market, companies can hedge against unsystematic risk and increase their expected rate of



**Fig. 5.** The effects of the prospect of termination on threshold revenue assuming the support scheme is quantity-driven and there is a retroactive arrangement. Termination is expected in 10 years ( $\lambda=0.1$ ), and the benchmark is no policy uncertainty ( $\lambda=0$ ). Other parameters are set at  $\mu_K=\mu_S=0$  and  $\sigma_S=0.06$  and  $\sigma_K=0.07$ .

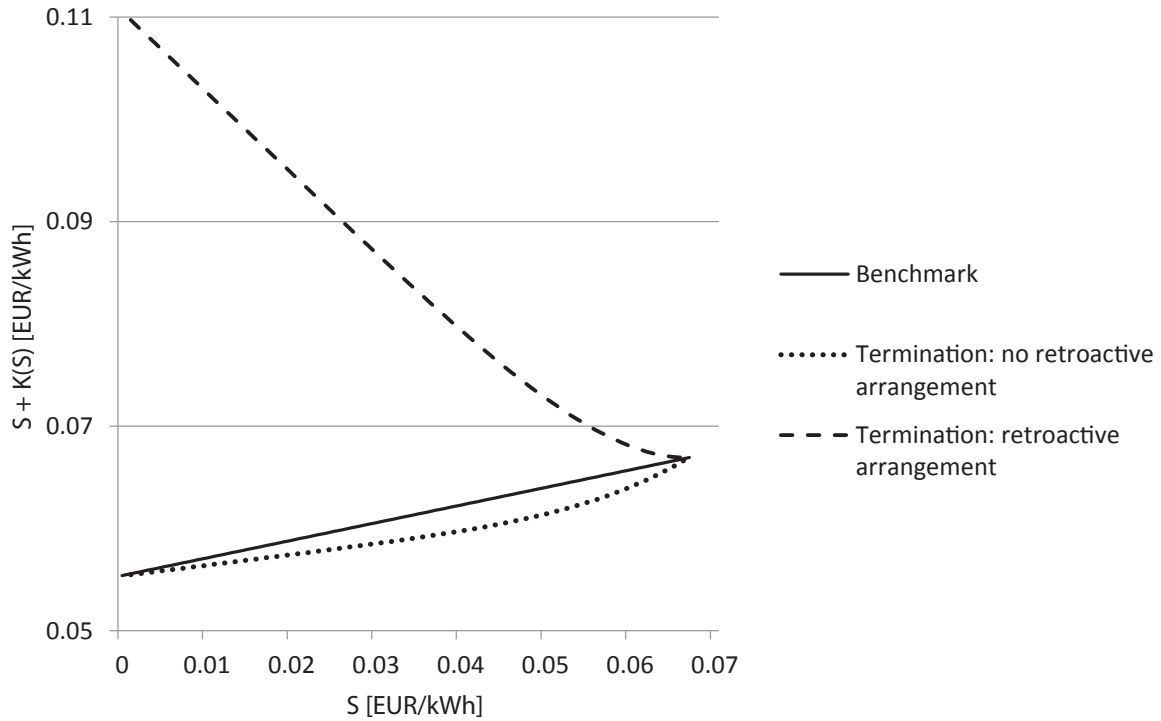


Fig. 6. The effects of the prospect of termination on threshold revenue assuming the support scheme is price-driven and there is a retroactive arrangement or there is not. Termination is expected in 10 years ( $\lambda=0.1$ ), and the benchmark is no policy uncertainty ( $\lambda=0$ ). Other parameters are set at  $\mu_K=\mu_S=0$  and  $\sigma_K=0$  and  $\sigma_S=0.07$ .

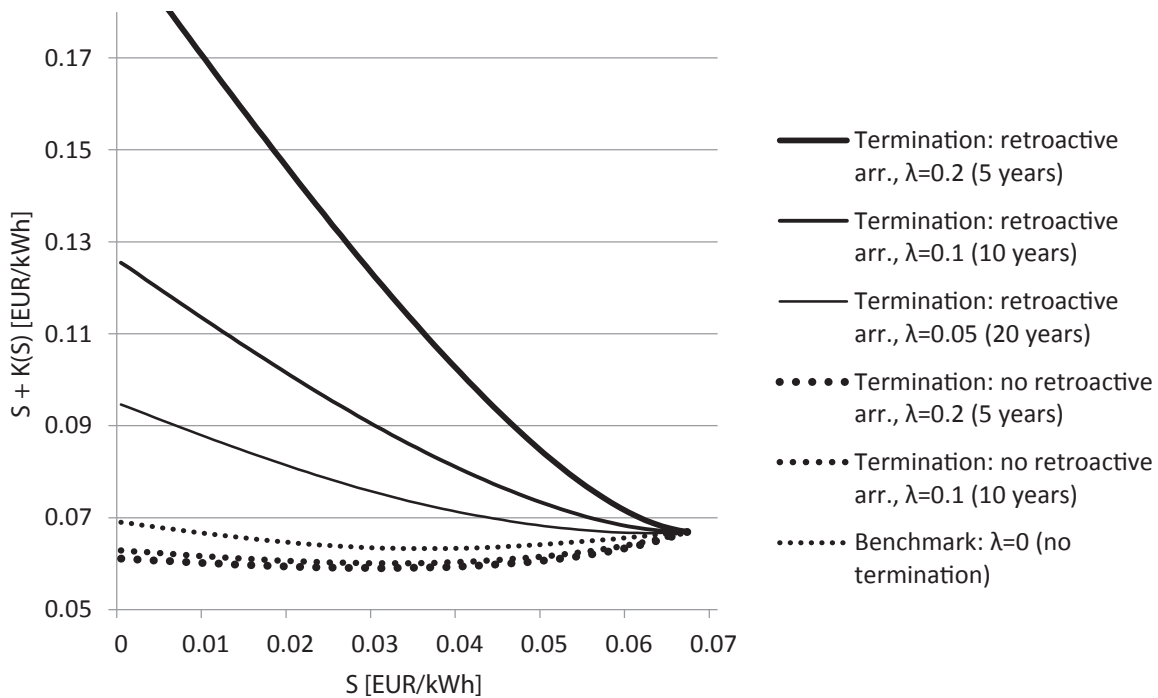


Fig. 7. The effects of different values of  $\lambda$  on threshold revenue for various termination scenarios for a quantity-driven scheme. Other parameters are set at  $\mu_K=\mu_S=0$ ,  $\sigma_S=0.06$  and  $\sigma_K=0.07$  and  $\rho=0$ .

return on investments by increasing their amount of systematic risk.

Policy risk (i.e. the possibility that the current scheme will be revised or terminated) reflects the ability of policymakers to flexibly adapt to a changing environment. With a flexible policy, policymakers

can respond to improved information on the science of climate change, the cost and benefits of renewable electricity technologies, political decisions and trends in other countries, the impact of an increased share of intermittent, renewable energy sources on the power market, and the need to ensure continued political support.

One way to address the problem of uncertainty is to ask: Who is best positioned to cope with these risks, the investor or the government? Renewable energy investors' needs for long-term stability need to be weighed against the benefits of policy flexibility and a well-functioning power market. Our analysis provides a basis for discussing this problem.

We argue that investors should not be completely shielded from market risk. That is, governments should consider replacing fixed feed-in tariffs with more market-based renewable electricity support schemes such as tradable green certificates. Our analysis has shown that, as a result of risk diversification, investors' risk may be less than commonly expected under a tradable green certificate scheme. In fact, even in the case where electricity and certificate prices are uncorrelated, a significant part of investor risk will be eliminated. In addition, although investor risk will be higher under a green certificate scheme than it would be under a fixed feed-in tariff scheme, the difference may be small. In fact, compared with a fixed feed-in premium scheme, investor risk may sometimes even be lower under a green certificate scheme.

We do, however, argue that investors should be shielded against some policy risk. Political debate on scheme termination will increase the threshold revenue and thus slow down investments if investors believe the decision will be retroactively applied, and vice versa if they do not. The termination effect can be substantial, especially in the first case where investors expect future curtailment of subsidies to affect new and old installations alike. The circumstances under which retroactive termination would be a likely policy action are complex and difficult to define. Such policy behavior may be more likely under a fixed feed-in tariff scheme because the tariff is fixed for a long time period, and deviations between the tariff and the actual cost of power production will most likely result in an investment level that deviates substantially from the policy target. Although the purpose of a scheme termination is usually to slow down

investment, scheme termination will in fact speed up investment if it is not retroactively applied. If electricity and certificate prices are positively correlated, this effect can be substantial under a green certificate scheme.

An interesting example that combines both the speed up and slow down incentives of termination is the Swedish-Norwegian market for green certificates. The scheme was implemented in 2012 and the last plant to be entitled to green certificates in Norway must have started operations by the end of 2020. Thus, in the beginning of this period, investors may speed up their rates of investment to lock in a future stream of subsidy revenues (termination, no retroactive effects). But, as they approach the end of the period, they slow down their rates of investment because of the increasing risk of missing the operational deadline (termination, retroactive effects).

To sum up, policy risk reflects the ability of policymakers to flexibly adapt to a changing environment. In some circumstances, policymakers should refrain from using this flexibility as it can result in overly large variations in investment levels. Governments that do not show restraint will have to pay a high social cost to achieve their renewable electricity targets in the future.

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### Appendix A. Nomenclature

Variable(s)	Definition
<b>Section 2:</b>	<b>Market and policy uncertainty</b>
$S_t, K_t$	Electricity price and subsidy price, respectively, at time $t$ .
$\mu_S, \mu_K$	Trend parameters for the prices.
$\sigma_S, \sigma_K$	Volatility parameters for the prices.
$dZ_{S_t}, dZ_{K_t}$	Brownian motion processes for the two prices.
$\rho$	Correlation coefficient for the two Brownian motion processes
$\delta_t$	Markov process and equal to 1 if a policy change has occurred in the time interval $[0, t]$ , 0 otherwise. $\delta_0 = 0$ .
$\lambda_{ij}$	Jump intensities of the Markov process, equal to either $\lambda$ or 0.
$\lambda dt$	The constant probability that a jump occurs during a short time interval $dt$ .
<b>Section 3:</b>	<b>Support scheme with an infinite lifespan</b>
$T$	Project lifetime.
$r$	Discount rate for future revenue streams. Exogenously given.
$r_S, r_K$	Compounding factors used to translate current prices into present value.
$V$	Present value of the project. The project produces one unit of electricity.
$I$	Investment cost per unit of production. Assumed constant.
$V - I$	Net present value of the project. The project produces one unit of electricity.
$W$	Value of the option to invest.
$\beta, \alpha_S, \alpha_K$	Parameters in the option value equation.
$\mathcal{C}$	Quadratic function. $\mathcal{C} = 0$ is a condition for optimum.
$S_-(K), K_-(S)$	The threshold electricity price as a function of the observed subsidy price, and vice versa.
<b>Section 4:</b>	<b>Termination of the support scheme</b>
$V_0, W_0, r_{K0}$	Subscript 0 refers to regime 0, that is, the termination of the support scheme has not yet occurred.
$V_1, W_1$	Subscript 1 refers to regime 1, that is, the support scheme has been terminated.
$r_{K0}(\lambda)$	Compounding factor for subsidy prices in regime 0. Without retroactive termination we set $\lambda=0$ , whereas with retroactive termination we set $\lambda>0$ . The compounding factor for electricity prices is the same in both regimes, and is the same as derived in Section 3: $r_S$ .
$S_0(K), K_0(S), S_1(K)$	Threshold prices in regime 0 and 1.
$\alpha_S0, \alpha_K0, \alpha_S1$	Parameters in the option value equation in regime 0 and 1.
<b>Section 5:</b>	<b>Numerical solutions</b>
$Var(dV_t/V_t)$	Variance of the relative change in project value at time $t$ .
$w$	Fraction of the project value stemming from future sale of electricity. $1-w$ is the fraction of the project value stemming from future subsidies.

**Appendix B. The value of the project**

*B.1 Support scheme with an infinite lifespan*

With infinite life of the support scheme, the project value is

$$\begin{aligned}
 V(S, K) &= \mathbb{E} \left[ \int_0^T e^{-rt} (S_t + K_t) dt \mid S_0 = S, K_0 = K \right] \\
 &= \int_0^T e^{-rt} \mathbb{E}[S_t | S_0 = S] dt + \int_0^T e^{-rt} \mathbb{E}[K_t | K_0 = K] dt \\
 &= S \int_0^T e^{-(r-\mu_S)t} dt + K \int_0^T e^{-(r-\mu_K)t} dt := r_S S + r_K K. \quad (38)
 \end{aligned}$$

The first equality is based on the observations that conditional on the initial electricity price the current price is independent of the initial subsidy payment, and vice versa.

*B.2 Termination of the support scheme*

Without retroactive termination upon investment, the project value is unaffected by termination risk. With risk of retroactive termination, by the same arguments as above,  $V_1(S) := r_S S$ . Moreover,

$$\begin{aligned}
 V_0(S, K) &= \mathbb{E} \left[ \int_0^T e^{-rt} (S_t + K_t 1_{\{\delta_t=0\}}) dt \mid S_0 = S, K_0 = K \right] \\
 &= \int_0^T e^{-rt} \mathbb{E}[S_t | S_0 = S] dt + \int_0^T e^{-rt} \mathbb{E}[K_t | K_0 = K] \mathbb{P}(\delta_t = 0) dt \\
 &= S \int_0^T e^{-(r-\mu_S)t} dt + K \int_0^T e^{-(r+\lambda-\mu_K)t} dt := r_S S + r_{K0} K. \quad (39)
 \end{aligned}$$

Here, we further use that termination risk is independent of the subsidy payment. The second equality holds since the time to termination is exponentially distributed.

**Appendix C. The value of waiting**

*C.1 Proof of corollary 1*

Note that

$$r_S S + r_K K_*(S) = \frac{\alpha_S + \alpha_K}{\alpha_S + \alpha_K - 1} \cdot I. \quad (40)$$

For the expression on the left hand side to be larger than  $I$ , it is sufficient to show that  $\alpha_S + \alpha_K > 1$ . To see this, recall that when  $\alpha_S = 0$ , it is well known from the univariate real options problem that the positive root of the quadratic equation satisfies  $\alpha_K > 1$ , and vice versa when  $\alpha_K = 0$  then  $\alpha_S > 1$ . Hence, the ellipse defined by  $\mathcal{C}(\alpha_S, \alpha_K) = 0$  must always be above the line  $\alpha_S + \alpha_K = 1$  in the first quadrant of the plane.

*C.2 Proof of corollary 2*

Now

$$\begin{aligned}
 r_S S + r_{K0}(\lambda) K_0(S) &= \frac{\alpha_{S0} + \alpha_{K0}}{\alpha_{S0} + \alpha_{K0} - 1} \cdot I \\
 &\quad + \frac{\alpha_{S0} + \alpha_{K0} - \alpha_{S1}}{(\alpha_{S0} + \alpha_{K0} - 1)(\alpha_{S1} - 1)} \cdot I \cdot \left(\frac{S}{S_1}\right)^{\alpha_{S1}}. \quad (41)
 \end{aligned}$$

By the same arguments as above,  $\alpha_{S0} + \alpha_{K0} > 1$  and  $\alpha_{S1} > 1$ . If  $\alpha_{S0} + \alpha_{K0} \geq \alpha_{S1}$ , then

$$r_S S + r_{K0}(\lambda) K_0(S) \geq \frac{\alpha_{S0} + \alpha_{K0}}{\alpha_{S0} + \alpha_{K0} - 1} \cdot I, \quad (42)$$

and the left hand side is greater than  $I$ . If  $\alpha_{S0} + \alpha_{K0} < \alpha_{S1}$ , then

$$\begin{aligned}
 r_S S + r_{K0}(\lambda) K_0(S) &> \frac{\alpha_{S0} + \alpha_{K0}}{\alpha_{S0} + \alpha_{K0} - 1} \cdot I + \frac{\alpha_{S0} + \alpha_{K0} - \alpha_{S1}}{(\alpha_{S0} + \alpha_{K0} - 1)(\alpha_{S1} - 1)} \cdot I \\
 I &= \frac{\alpha_{S1}}{\alpha_{S1} - 1} \cdot I, \quad (43)
 \end{aligned}$$

and again we obtain the desired.

*C.3 Proof of corollary 3*

1. We follow the lines of [27]. Let

$$f(S) = S - \frac{\alpha_{S0}}{\alpha_{S0} - 1} \cdot \frac{I - r_{K0}(\lambda)K}{r_S} - \frac{\alpha_{S0} - \alpha_{S1}}{(\alpha_{S0} - 1)(\alpha_{S1} - 1)} \cdot \frac{I}{r_S} \cdot \left(\frac{S}{S_1}\right)^{\alpha_{S1}}. \quad (44)$$

Then,

$$f''(S) = -\frac{(\alpha_{S0} - \alpha_{S1})\alpha_{S1}}{(\alpha_{S0} - 1)} \cdot \frac{I}{r_S} \cdot \left(\frac{S}{S_1}\right)^{\alpha_{S1}} \cdot \frac{1}{S^2}, \quad (45)$$

and since  $\alpha_{S0} > \alpha_{S1}$ ,  $f''(S) \leq 0$ , that is,  $f$  is concave on  $[0, \infty)$ . Furthermore,  $f(0) < 0$ , and

$$f(S_1) = \frac{\alpha_{S0}}{\alpha_{S0} - 1} \cdot \frac{r_{K0}(\lambda)K}{r_S} \quad (46)$$

which shows that  $f(S_1) > 0$ . Hence,  $f$  must have a unique root in  $(0, S_1)$ . We denote this root by  $S_0(K)$ .

2. Clearly,  $S^*(K) < S_1$ . Since,

$$f(S_*(K)) = \frac{\alpha_{S0} - \alpha_{S1}}{(\alpha_{S0} - 1)(\alpha_{S1} - 1)} \cdot \left( \frac{I - r_K(0)K}{r_S} - \frac{I}{r_S} \cdot \left(\frac{I - r_K(0)K}{I}\right)^{\alpha_{S1}} \right), \quad (47)$$

we have that  $f(S_*(K)) > 0$ . Hence,  $S^*(K) > S_0(K)$ .

**Appendix D. Solving the PDEs**

*D.1 Termination of the support scheme*

We repeat the system of second order PDEs, which holds when continuation is optimal



$$\frac{1}{2} \left( \sigma_S^2 S^2 \frac{\partial^2 W_0}{\partial S^2} + \sigma_K^2 K^2 \frac{\partial^2 W_0}{\partial K^2} + 2\sigma_S \sigma_K \rho SK \frac{\partial^2 W_0}{\partial S \partial K} \right) + \mu_S S \frac{\partial W_0}{\partial S} + \mu_K K \frac{\partial W_0}{\partial K} - \lambda(W_0 - W_1) - rW_0 = 0, \tag{48}$$

$$\frac{1}{2} \sigma_S^2 S^2 \frac{\partial^2 W_1}{\partial S^2} + \mu_S S \frac{\partial W_1}{\partial S} - rW_1 = 0. \tag{49}$$

Note that the PDE under regime 0 is inhomogenous.<sup>12</sup> We assume the generic solutions to the homogenous parts of the PDEs

$$W_0(S, K) = \beta_0 S^{\alpha_{S0}} K^{\alpha_{K0}}, \quad W_1(S) = \beta_1 S^{\alpha_{S1}}. \tag{50}$$

Then  $\alpha_{S0}, \alpha_{K0}$  and  $\alpha_{S1}$  must satisfy the equations  $\mathcal{C}_0(\alpha_{S0}, \alpha_{K0}) = 0$  and  $\mathcal{C}_1(\alpha_{S1}) = 0$ , where

$$\mathcal{C}_0(\alpha_S, \alpha_K) = \frac{1}{2} \left( \sigma_S^2 \alpha_S (\alpha_S - 1) + \sigma_K^2 \alpha_K (\alpha_K - 1) + 2\sigma_S \sigma_K \rho \alpha_S \alpha_K \right) + \mu_S \alpha_S + \mu_K \alpha_K - (r + \lambda), \tag{51}$$

$$\mathcal{C}_1(\alpha_S) = \frac{1}{2} \sigma_S^2 \alpha_S (\alpha_S - 1) + \mu_S \alpha_S - r. \tag{52}$$

For the same reasons as before, we restrict attention to  $\alpha_{S0} \geq 0, \alpha_{K0} \geq 0$  and  $\alpha_{S1} \geq 0$ .

We proceed by considering the boundaries at which investment becomes optimal. When continuation is optimal under both regimes 0 and 1, the solution to the system of equations (one which is inhomogenous) is<sup>13</sup>  $W_0(S, K) = \beta_{00} S^{\alpha_{S0}} K^{\alpha_{K0}} + \beta_{10} S^{\alpha_{S1}}$ ,  $W_1(S) = \beta_{10} S^{\alpha_{S1}}$ . The two different terms of the solution in regime 0 reflect the option value in the two cases, when there is risk of termination and when the support scheme has already been terminated. When investment is optimal under regime 0 but not under regime 1, we have that  $W_0(S, K) = r_S S + r_K K - I$ ,  $W_1(S) = \beta_{11} S^{\alpha_{S1}}$ . Note that it is never optimal to invest under regime 1 but not under regime 0. Finally, when investment is optimal under both regimes,  $W_0(S, K) = r_S S + r_K K - I$ ,  $W_1(S) = r_S S - I$ . The value matching conditions for investment in regimes 0 and 1 are then

$$\beta_{00} S_0^{\alpha_{S0}} K_0^{\alpha_{K0}} + \beta_{10} S_0^{\alpha_{S1}} = r_S S_0 + r_K K_0 - I, \quad \beta_{10} S_0^{\alpha_{S1}} = \beta_{11} S_0^{\alpha_{S1}}, \quad \beta_{11} S_1^{\alpha_{S1}} = r_S S_1 - I, \tag{53}$$

and corresponding smooth pasting conditions apply. With  $\mathcal{C}_0(\alpha_{S0}, \alpha_{K0}) = 0$  and  $\mathcal{C}_1(\alpha_{S1}) = 0$ , we have nine equations in the ten unknowns  $(\alpha_{S0}, \alpha_{K0}, \alpha_{S1}, \beta_{00}, \beta_{10}, \beta_{11}, S_0, K_0, S_1, K_1)$ . By manipulation of these conditions, the investment thresholds under regime 0 again define a one-dimensional subset in  $S_0 \times K_0$ -space. The thresholds under both regimes 0 and 1 are given in Proposition 2.

At the investment threshold, we have

$$\beta_{00} = \frac{1}{S^{\alpha_{S0}} K_0^{\alpha_{K0}} (S)^{\alpha_{K0}}} \cdot \frac{I}{\alpha_{S0} + \alpha_{K0} - 1} \cdot \left( 1 - \left( \frac{S}{S_1} \right)^{\alpha_{S1}} \right), \tag{54}$$

<sup>12</sup> A homogenous PDE only includes terms that involve the (partial) derivatives of the unknown function. Otherwise, the PDE is referred to as inhomogenous.

<sup>13</sup> We denote by  $\beta_{00}$  and  $\beta_{10}$  the indeterminates in the solutions under regimes 0 and 1 when investment is not optimal under either regime, hence the zero. Similarly,  $\beta_{11}$  denotes the indeterminate in the solution for regime 1 when investment is optimal under regime 0.

$$\beta_{10} = \beta_{11} = \frac{1}{S_1^{\alpha_{S1}}} \cdot \frac{I}{\alpha_{S1} - 1} = \frac{(\alpha_{S1} - 1)^{\alpha_{S1} - 1}}{\alpha_{S1}^{\alpha_{S1}}} \cdot \frac{r_S^{\alpha_{S1}}}{I^{\alpha_{S1} - 1}}. \tag{55}$$

### Appendix E. The variance of relative change in project value

Applying Ito's Lemma to the value of a project  $V(S_t, K_t) = r_S S_t + r_K K_t$  yields

$$dV_t = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS_t + \frac{\partial V}{\partial K} dK_t + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} dS_t^2 + \frac{1}{2} \frac{\partial^2 V}{\partial K^2} dK_t^2 + \frac{\partial^2 V}{\partial S \partial K} dS_t dK_t = r_S dS_t + r_K dK_t. \tag{56}$$

Substituting for  $dS_t$  and  $dK_t$  and dividing by  $V_t$  this can be written as

$$\frac{dV_t}{V_t} = w_t \mu_S dt + w_t \sigma_S dz_{S_t} + (1 - w_t) \mu_K dt + (1 - w_t) \sigma_K dz_{K_t}, \tag{57}$$

in which  $w_t = r_S S_t / (r_S S_t + r_K K_t)$ . The variance of the relative change in project value is then

$$Var \left( \frac{dV_t}{V_t} \right) = w_t^2 \sigma_S^2 dt + (1 - w_t)^2 \sigma_K^2 dt + 2w_t(1 - w_t) \rho \sigma_S \sigma_K dt, \tag{58}$$

which is equivalent to the portfolio variance of the returns on two assets derived by Ref. [18]. For  $\rho=1$ , the variance simplifies to  $Var(dV_t/V_t) = [\omega_t \sigma_S \sqrt{dt} + (1 - \omega_t) \sigma_K \sqrt{dt}]^2$  and the standard deviation is consequently equal to the weighted average of  $\sigma_S \sqrt{dt}$  and  $\sigma_K \sqrt{dt}$ . For  $\rho < 1$ , the standard deviation of the relative change in project value is reduced due to diversification.

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