Comparing Approaches to Valuing Sectoral Net Investments*

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Abstract

In the literature of comprehensive national accounts, national net investments are used to indicate dynamic welfare improvement in an economy. A well-known approach associates national net investments with the shadow value of change in stock of capital assets in an economy. Following this *capital stock approach*, sectoral net investments can be defined as the shadow value of change in stock of capital assets owned by a sector in an economy. An alternative approach is based on future commodity flows to a sector. This *commodity flow approach* associates sectoral net investments with the present value of changes in future commodity flows to a sector. In the present paper, I compare these two approaches and prove that they are coincide with each other only if the future commodity flows to the sector can be attributed to current stock of capital assets in the sector alone. In empirical studies, commodity flow approach can be a better alternative if the purpose is to estimate the change in welfare of a recipient of future cash flows.

Key words: comprehensive national accounting, genuine saving, income, non-renewable resource, sectoral net investments, sectoral wealth

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1 Introduction

How can we measure welfare improvement in an economy? The theory of comprehensive national accounts provides a concept of national net investments, which is defined as the value of change in stock of all types of capital assets in an economy[1]. In practical accounts, however, it is almost impossible to figure out all types of capital assets. What we can do is to value net investments related to a subset of capital assets in a consistent manner such that they can in principle sum up to obtain national net investments. If we assume any capital asset is owned by some agents and consider any owner of capital assets as a sector, then we demand proper approaches to valuing net investments of the sector.

A well-known approach is to associate *national* net investments with the shadow value of change in an economy's stock of capital assets (e.g. Dasgupta, 2009). Following this *capital stock approach*, sectoral net investments can be defined as the value of change in stock of capital assets owned by a sector evaluated at shadow prices. The shadow price of a capital asset represents marginal social welfare level generated by one additional unit of the capital asset. To apply this approach in practice, we generally need to measure the quantity of capital assets. If quality of a capital asset is changed to some extent, we may have to find out some way to reflect the quality change by the quantity change of the asset or consider it as another capital asset. Furthermore, it may also be a hard task to calculate the shadow price of a capital asset even for a sophisticated accountant (Arrow et al., 2003, for a discussion). In addition, capital assets in a sector may contribute not only to the welfare of the sector but also to the welfare of the other sectors. For example, the stock of capital assets owned by the rest of the world (RoW) has to be valued for an open economy since it also affects the welfare of the economy via international trade. In this sense, sectoral net investments by the capital stock approach may not be a plausible indicator for the change in feasible welfare of the sector. Hence, an alternative approach may be preferred to calculate the value of sectoral net investments.

An alternative approach is recently proposed by Asheim and Wei (2009) on the basis

[1] The term "net investments" here has been called "Genuine saving" after Hamilton (1994) and "comprehensive investment" in Section 4.5 of Dasgupta (2009).

of future commodity flows to a sector. This *commodity flow approach* associates sectoral net investments with the present value of changes in future commodity flows to a sector. In the present paper, I compare this approach with the capital stock approach and show that they coincide with each other only if the future commodity flows to the sector can be attributed to current stock of capital assets in the sector alone. The commodity flow approach can be a better alternative if the sector is defined as a recipient of future cash flows and necessary future information is available. By the commodity flow approach, I show that change in sectoral real wealth is not a plausible indicator for sectoral net investments if sectoral real wealth is defined as the present value of future real cash flows to the sector.

The paper is organized as follows. The next two sections introduce the capital stock approach and commodity flow approach respectively. Section 4 shows consistency between the two approaches. Section 5 compares the two approaches and applies the commodity flow approach to show that change in sectoral real wealth is not a plausible indicator for sectoral net investments. The final section offers concluding remarks.

2 Capital stock approach[2]

In a deterministic economy, let an l-dimensional vector $\mathbf{C}(s)$ represent all commodities for final consumption at any time $s \geq 0$, where each element of the vector represents the quantity of one commodity. For a given unidimensional utility flow $\{U(\mathbf{C}(s))\}_{s=0}^{\infty}$ over time, dynamic welfare at any time $t \geq 0$ is defined by the sum of discounted utilitarian,

$$W(t) = \int_t^{\infty} e^{-\rho(s-t)} U(\mathbf{C}(s))\, ds, \tag{1}$$

where ρ is a given constant utility discount rate. We assume the integral exists.

The status of the economy at any time $t \geq 0$ is defined by an m-dimensional vector of stock of all capital assets $\mathbf{K}(t)$. The stock of capital assets may change over time in order to satisfy future final consumption subject to certain constraints due to the scarcity of capital assets. These constraints include not only technological and ecological constraints,

[2]See Dasgupta (2009) for a recent review of this approach.

but also a wide range of institutional constraints such as property rights, market types, tax/subsidy rates, insurance, common property resources, and non-market arrangements for credits. Under all these constraints, which may change over time, the economy makes its actual decisions to utilize capital assets to generate commodities that are allocated for net investments and final consumption at any point in time. The net investments capture the changes in stock of capital assets over time. The evolution of the economy subject to changing constraints can be reflected by a *resource allocation mechanism* (RAM, as introduced by Dasgupta and Mäler, 2000; Dasgupta, 2001; Arrow et al., 2003), which is defined as a many-one mapping from any vector of stock of capital assets to an attainable path of future net investments and final consumption. If current stock of capital assets is $\mathbf{K(t)}$, a RAM can be denoted by a mapping

$$\alpha : (\mathbf{K}(t), t) \rightarrow \left(\dot{\mathbf{K}}(s), \mathbf{C}(s), s\right)_{s=t}^{\infty}.$$

Taking a RAM (α) as given, the dynamic welfare defined by (1) can be expressed by

$$W(\mathbf{K}(t), t) = \int_t^{\infty} e^{-\rho(s-t)} U(\mathbf{C}(\mathbf{K}(t), t, s))\, ds, \tag{2}$$

which is the *value function* at t. Assume the value function is differentiable. Differentiating $W(\mathbf{K}(t), t)$ with respect to t yields

$$\dot{W}(t) = \nabla W(\mathbf{K}(t), t)\, [\dot{\mathbf{K}}(t), 1]', \tag{3}$$

where the $(m+1)$-dimensional vector $\nabla W(\mathbf{K}(t), t)$ represents the vector of marginal welfare with respect to the stock of capital assets and time itself at t, i.e.

$$\nabla W(\mathbf{K}(t), t) = \left(\frac{\partial W}{\partial k_1}(\mathbf{K}(t), t), \frac{\partial W}{\partial k_2}(\mathbf{K}(t), t), ..., \frac{\partial W}{\partial k_m}(\mathbf{K}(t), t), \frac{\partial W}{\partial t}(\mathbf{K}(t), t)\right).$$

Define an $(m+1)$-dimensional vector of shadow prices of capital assets and time itself at t^3,

$$\mathbf{q}(t) = \nabla W(\mathbf{K}(t), t),$$

and national net investments,

$$I(t) = \mathbf{q}(t)[\dot{\mathbf{K}}(t), 1]'$$

which is the shadow value of change in the stock of capital assets and time itself. The shadow value of time itself has also been called "value of passage of time" and taken as one part of net investments in the literature (for a discussion, see Asheim, 2003, p. 124-5).

Substituting $q(t)$ and $I(t)$ into Eq. (3) gives us

$$\dot{W}(t) = I(t) = \mathbf{q}(t)[\dot{\mathbf{K}}(t), 1]', \tag{4}$$

which shows that instantaneous change in dynamic welfare can be measured by national net investments. Notice that the net investments are calculated at *current* shadow prices of capital assets and time itself.

This definition of net investments can also be used in a sector as long as shadow prices represent marginal dynamic welfare. Assume the economy is divided into n sectors. If a sector i owns a vector of stock of capital assets $\mathbf{K}_i(t)$ at time t, then net investments of the sector i can be calculated by

$$I_i^s(t) = \mathbf{q}(t)[\dot{\mathbf{K}}_i(t), \gamma_i(t)]', \tag{5}$$

where

$$\sum_{i=1}^{n} \dot{\mathbf{K}}_i(t) = \dot{\mathbf{K}}(t)$$

and γ_i is the share of shadow value of time itself that can be attributed to the sector and satisfies the condition,

$$\sum_{i=1}^{n} \gamma_i(t) = 1.$$

[3] The marginal welfare with respect to time itself can be interpreted as shadow price of time if the time itself is regarded as a capital asset.

The sectoral net investments can sum up to national net investments to indicate dynamic welfare improvement,

$$\sum_{i=1}^{n} I_i^s(t) = \sum_{i=1}^{n} \mathbf{q}(t) \left[\dot{\mathbf{K}}_i(t), \gamma_i(t)\right]' = \dot{W}(t). \tag{6}$$

In the present paper, this way to calculate sectoral net investments is called the *capital stock approach.*

3 Commodity flow approach[4]

As shown in the previous section, sectoral net investments calculated by the capital stock approach are derived from transforming the dynamic welfare function (1) into a function of current stock of capital assets and time itself (2). On the contrary, by directly differentiating on both sides of the welfare function (1), Asheim and Wei (2009) derive an alternative approach to valuing sectoral net investments, called the *commodity flow approach* in the present paper.

Since dynamic welfare defined by Eq. (1) can, by letting $\tau = s - t$, be rewritten as

$$W(t) = \int_0^\infty e^{-\rho\tau} U(\mathbf{C}(\tau + t))\, d\tau,$$

we can differentiate on both sides of the above equation with respect to t to obtain

$$\dot{W}(t) = \int_0^\infty e^{-\rho\tau} \dot{U}(\mathbf{C}(\tau + t))\, d\tau = \int_t^\infty e^{-\rho(s-t)} \nabla U(\mathbf{C}(s))\, \dot{\mathbf{C}}(s)\, ds, \tag{7}$$

where $\nabla U(\mathbf{C}(s))$ represents the l-dimensional vector of marginal utility with respect to consumption at a future time $s \geq t$, i.e.

$$\nabla U(\mathbf{C}(s)) = \left(\frac{\partial U}{\partial c_1}(\mathbf{C}(s)), \frac{\partial U}{\partial c_2}(\mathbf{C}(s)), ..., \frac{\partial U}{\partial c_l}(\mathbf{C}(s))\right).$$

Define an l-dimensional vector of present value prices of commodities given time t,

[4] See Asheim and Wei (2009) for details of this approach.

$\{\mathbf{p}_c(s|t)\}_{s=0}^{\infty}$ for all $s \geq 0$, satisfying,

$$\mathbf{p}_c(s|t) = e^{-\rho(s-t)} \nabla U(\mathbf{C}(s)),$$

which implies all the present value prices $\mathbf{p}_c(s|t)$ can be rewritten as $\mathbf{p}_c(s|0)$ multiplied with a constant $e^{\rho t}$, i.e.,

$$\mathbf{p}_c(s|t) = e^{\rho t} \mathbf{p}_c(s|0) \tag{8}$$

By substituting $\mathbf{p}_c(s|t)$ into Eq. (7), we obtain that for a given smooth path of future consumption flows $\{\mathbf{C}(s)\}_{s=t}^{\infty}$,

$$\dot{W}(t) = \int_t^{\infty} \mathbf{p}_c(s|t) \dot{\mathbf{C}}(s) \, ds, \tag{9}$$

which shows that instantaneous change in dynamic welfare is represented by the present value of changes in future consumption flows.

To express the change in welfare $\dot{W}$ in terms of real prices at current time $t \geq 0$ instead of at time 0, we consider the Divisia consumer price index $(\pi(s))_{s=0}^{\infty}$ defined by $\pi(0) = 1$ and

$$\frac{\dot{\pi}(s)}{\pi(s)} = \frac{\dot{\mathbf{p}}_c(s|0) \mathbf{C}(s)}{\mathbf{p}_c(s|0) \mathbf{C}(s)},$$

for all $s \geq 0$. Then real prices of commodities $\{\mathbf{P}_c(s)\}_{s=0}^{\infty}$ can be expressed by

$$\mathbf{P}_c(s) = \mathbf{p}_c(s|0) / \pi(s) = e^{-\rho t} \mathbf{p}_c(s|t) / \pi(s)$$

for all $s \geq 0$, where the second equation is a result by using (8). The definition has to coincide with the fact that real prices at time t equal present value prices at time t given time t, i.e., $\mathbf{P}_c(t) = \mathbf{p}_c(t|t)$, which implies $e^{-\rho t} = \pi(t)$ by the second equation. Then real prices $\{\mathbf{P}_c(s)\}_{s=0}^{\infty}$ can always be written as

$$\mathbf{P}_c(s) = \pi(t) \mathbf{p}_c(s|t) / \pi(s) \tag{10}$$

as long as $t \geq 0$. By directly substituting (10) to (9), the change in welfare at time t can be rewritten as

$$\dot{W}(t) = \int_t^\infty \frac{\pi(s)}{\pi(t)} \mathbf{P}_c(s) \dot{\mathbf{C}}(s) \, ds. \tag{11}$$

To show how sectoral net investments can be calculated by the *commodity flow approach*, we assume there are $r \geq l$ types of commodities in the economy. Define an augmented r-dimensional vector $\mathbf{X}$ to include all commodities, with $\mathbf{C}$ as the first part and a zero $(r-l)$-dimensional vector $\mathbf{O}$ as the second part of the vector. We have

$$\mathbf{X}(s) = [\mathbf{C}(s), \mathbf{O}]' \text{ and } \dot{X}(s) = [\dot{\mathbf{C}}(s), \mathbf{O}]',$$

where the corresponding element in the vector $\mathbf{O}$ is always zero since these commodities are not directly used for final consumption. However, real price of these commodities in $\mathbf{O}$ may differ from zero since these commodities may be necessary to produce commodities for final consumption. Let $\mathbf{P}_x$ be the r-dimensional vector of real prices of all commodities,

$$\mathbf{P}_x(s) = [\mathbf{P}_c(s), \mathbf{P}_o(s)]$$

for all time $s \geq 0$. By the above two definitions, we have

$$\mathbf{P}_x(s) \dot{\mathbf{X}}(s) = \mathbf{P}_c(s) \dot{\mathbf{C}}(s). \tag{12}$$

Commodity flows always move from one sector to another. Hence, use of one commodity is always equal to supply of the commodity at any point in time. If the final consumption vector $\mathbf{C}$ is considered as commodity use by some sectors, then the same amount of commodity must be supplied by some other sectors in the economy. Denote all commodity flows to a sector, excluding flows of commodities used for final consumption, by a r-dimensional vector $\mathbf{X}_i$ (including all types of commodities), where commodity use is denoted with negative sign and supply with positive sign. Hence, $\mathbf{P}_x(s)\mathbf{X}_i(s)$ represent cash flows to the sector. Still assume the economy is divided into n sectors. We have

$$\mathbf{X}(s) = \sum_{i=1}^{n} \mathbf{X}_i(s)$$

and

$$\dot{\mathbf{X}}(s)=\sum_{i=1}^{n}\dot{\mathbf{X}}_i(s) \tag{13}$$

for all $s \geq 0$. Substituting Eq. (13) into Eq. (12) yields

$$\mathbf{P}_c\left(s\right)\dot{\mathbf{C}}\left(s\right)=\sum_{i=1}^{n}\mathbf{P}_x\left(s\right)\dot{\mathbf{X}}_i(s), \tag{14}$$

which shows that at each point in time, the real value of changes in final consumption equals the sum of real value of changes in sectoral commodity flows. Substituting Eq. (14) into Eq. (11) yields

$$\dot{W}\left(t\right)=\sum_{i=1}^{n}\int_t^{\infty}\frac{\pi\left(s\right)}{\pi\left(t\right)}\mathbf{P}_x\left(s\right)\dot{\mathbf{X}}_i(s)ds, \tag{15}$$

which shows that instantaneous change in dynamic welfare can be measured by the sum of sectoral net investments calculated by the *commodity flow approach*,

$$I_i^f\left(t\right)=\int_t^{\infty}\frac{\pi\left(s\right)}{\pi\left(t\right)}\mathbf{P}_x\left(s\right)\dot{\mathbf{X}}_i(s)ds, \tag{16}$$

which is the present value of changes in future commodity flows to a sector. Notice that the changes in future commodity flows at a point in time are evaluated at the real prices at that point. Also notice that as shown by Asheim and Wei (2009), the derivation in this section also holds even if the discount rates (ρ) over time are not constant.

By the commodity flow approach, any part of an economy where cash flows may occur in the future, explicitly or implicitly, can be considered as a sector even though the capital assets belonging to the sector may not be well defined. For example, an individual person, who owns human capital, can be considered as a sector even though the human capital cannot be well measured. Technology is also an example since it may improve over time to generate more future cash flows given other things being equal. In some cases, the sector as a recipient of the cash flows may not own any capital assets and all the cash flows are generated by capital assets by the other sectors in the economy. An example is an individual whose nationality entitles the person to receive pensions from the government even if the person never pays taxes to the government.

4 Consistency between approaches

As shown in the previous two sections, sectoral net investments by both approaches can sum up to indicate change in dynamic welfare. By (4), (5), (15), and (16), we have

Proposition 1 *Sectoral net investments by both approaches can sum up to national net investments indicating change in dynamic welfare,*

$$\dot{W}(t) = \sum_{i=1}^{n} \mathbf{q}(t) [\dot{\mathbf{K}}_i(t), \gamma_i(t)]' = \sum_{i=1}^{n} \int_{s=t}^{\infty} \frac{\pi(s)}{\pi(t)} \mathbf{P}_x(s) \dot{\mathbf{X}}_i(s) ds.$$

Hence, sectoral net investments by both approaches can be interpreted as contributions of the sector to change in dynamic welfare. However, sectoral net investments by the capital stock approach is probably different from the commodity flow approach since future commodity flows to a sector may depend not only on the capital stock in the sector but also on capital stocks in the other sectors. For example, even if current capital stock owned by the sector of oil extraction is given, the future commodity flows to the sector are also affected by market demand for oil determined by current capital stocks in other sectors. How can we show the consistency between approaches at a sectoral level?

In the capital stock approach, the status of a sector in a deterministic economy is defined by its stock of current capital assets $\mathbf{K}_i(t)$ given a RAM in the economy. On the other hand, in the commodity flow approach, the status of a sector is defined by the future commodity flows to the sector, $(\mathbf{X}_i(s))_{s=t}^{\infty}$. The current capital stock in sector i ($\mathbf{K}_i(t)$) may contribute to future commodity flows of many sectors due to the interdependence between sectors. Denote future commodity flows that can be attributed to $(\mathbf{K}_i(t), t)$ alone by $(\mathbf{X}_j(K_i(t), t, s))_{s=t,j=1}^{\infty,n}$. Then the cash flows to a sector is always the sum of the contributions to the sector of all kinds of capital assets in the economy, i.e.,

$$\sum_{i=1}^{m} (\mathbf{X}_j(K_i(t), t, s))_{s=t,j=1}^{\infty,n} = (\mathbf{X}_i(s))_{s=t}^{\infty}. \tag{17}$$

Hence, a given RAM in the economy also defines a many-one mapping from $\mathbf{K}_i(t)$ to $(\mathbf{X}_j(K_i(t), s))_{s=t,j=1}^{\infty,n}$ at the sectoral level. This implies that a change in $\mathbf{K}_i(t)$ can be mapped into a change in $(\mathbf{X}_j(K_i(t), t, s))_{s=t,j=1}^{\infty,n}$ in the deterministic economy. As shown in the previous two sections, both shadow prices of $\mathbf{K}_i(t)$ and real prices of $(\mathbf{X}_i(s))_{s=t}^{\infty}$ in this context are derived from the dynamic welfare function for the whole economy. Since shadow price of a capital asset are marginal dynamic welfare generated by one additional unit capital asset, the shadow price can also be expressed by the sum of the present value of future marginal utility generated through changes in final consumption flows caused by the one additional unit capital asset. Hence, for the purpose of calculating national net investments, we have

Proposition 2 *Sectoral net investments by the capital stock approach can be expressed by the commodity flow approach*

$$I_i^s(t) = \mathbf{q}(t)[\dot{\mathbf{K}}_i(t), \gamma_i(t)]' = \int_{s=t}^{\infty} \sum_{j=1}^{n} \frac{\pi(s)}{\pi(t)} \mathbf{P}_x(s) \dot{\mathbf{X}}_j(K_i(t), s) ds = \widetilde{I}_i^f(t).$$

only if the future commodity flows to the sector are re-defined as all the future commodity flows that can be attributed to current stock of capital assets in the sector, i.e., (17) holds.

A direct corollary from Proposition 2 is[5]

Corollary 1 *Sectoral net investments by both approaches coincide with each other,*

$$I_i^s(t) = \mathbf{q}(t)[\dot{\mathbf{K}}_i(t), \gamma_i(t)]' = \int_{s=t}^{\infty} e^{-\rho(s-t)} \mathbf{P}_x(s) \dot{\mathbf{X}}_i(s) ds = I_i^f(t),$$

only if all the future commodity flows to the sector can be attributed to current capital stock in the sector,

$$(\mathbf{X}_i(s))_{s=t}^{\infty} = (\mathbf{X}_i(K_i(t), t, s))_{s=t}^{\infty}.$$

Example: a sector in a competitive economy

In a competitive economy, real prices of commodities can be defined as market prices at any point in time since every sector in the economy assumes market prices as given and the dynamic welfare of the economy is maximized. This simplifies the analysis of price determination. Consider any given sector in a competitive economy. At each point in

[5] Notice that Corollary (1) is invoked by the Eq. (6) in Wei (2012).

time, the sector can utilize a stock of capital assets for generation of future consumption flows. In principle, these capital assets should include everything that can bring cash flows to the sector in the future, which implies Corollary (1) holds. As mentioned at the beginning of the paper, it may be difficult to have a complete list of these capital assets and determine their stocks. Future commodity flows and real prices of these flows also may not be available. For simplicity of the analysis, they are presumed to be known in this example.

At each point in time, the sector faces constraints related to four types of variables: commodity flows to the sector, sectoral stock of capital assets, changes in capital assets, and pure time-related variables like exogenous technical progress. The constraints may be one or many. Assume the commodity flows to the sector at time $t \geq 0$ is denoted by an n-dimensional vector $\mathbf{x}(t)$[6]. Let a vector $\mathbf{P}_x(t)$ denote real (or market) prices of $\mathbf{x}(t)$. All the prices are taken as given by the sector since the market is competitive. Let $\mathbf{k(t)}$ denote the m-dimensional vector of stock of capital assets in the sector at t, and $\dot{\mathbf{k}}(\mathbf{t})$ the m-dimensional vector of instantaneous change in $\mathbf{k(t)}$. All the possible constraints faced by the sector at any time $\tau \geq t$ can be expressed by

$$\mathbf{f}\left(\mathbf{x}(\tau), \mathbf{k}(\tau), \dot{\mathbf{k}}(\tau), \tau\right) \geq 0, \tag{18}$$

where $\mathbf{f}$ is an r-dimensional vector of functions ($1 \leq r < \infty$), so that the inequality $\mathbf{f}(\cdot) \geq 0$ represents a group of r inequalities

$$f_k\left(\mathbf{x}(\tau), \mathbf{k}(\tau), \dot{\mathbf{k}}(\tau), \tau\right) \geq 0, \qquad k = 1, ..., r.$$

Suppose $\mathbf{f}$ is a convex function and the boundary $\mathbf{f}(\cdot) = 0$ is smooth and differentiable with respect to all the arguments.

Given a stock of capital assets in the sector at current time t, the competitive sector

[6] In this subsection, I write sectoral vectors in small letters and ignore the subscript i, which indicates the sector i, for notational simplicity.

aims to maximize the present value of future cash flows subject to the Ineq. (18),

$$\begin{aligned} &\max_{\mathbf{x}} \textstyle\int_t^\infty \frac{\pi(s)}{\pi(t)} \mathbf{P}_x(\tau)\,\mathbf{x}(\tau)\,d\tau \\ s.t.\ \ &\mathbf{f}\left(\mathbf{x}(\tau), \mathbf{k}(\boldsymbol{\tau}), \dot{\mathbf{k}}(\boldsymbol{\tau}), \tau\right) \geq 0, \\ &\mathbf{k}(t) \ \text{ is given,} \end{aligned} \tag{19}$$

By Corollary (1), we can claim that sectoral net investments by both approaches coincide with each other,

$$I_i(t) = \boldsymbol{\lambda}(t)\,[\dot{\mathbf{k}}_i(t), 1] = \int_t^\infty \frac{\pi(s)}{\pi(t)} P_x(s)\,\dot{\mathbf{x}}(s) ds,$$

where $\boldsymbol{\lambda}$ is the shadow prices of stock of capital assets in the sector and time itself at t[7]. A standard derivation process of the maximum problem is given in the Appendix of this paper.

Stationary technology. If at the beginning, we assume $\mathbf{f}$ is time-invariant, i.e. stationary technology is assumed,

$$\mathbf{f}\left(\mathbf{x}(\tau), \mathbf{k}(\tau), \dot{\mathbf{k}}(\tau)\right) \geq 0,$$

then the shadow value of time itself disappears and the sectoral net investment can be simplified as

$$\int_t^\infty \frac{\pi(s)}{\pi(t)} \mathbf{P}_x(\tau)\,\dot{\mathbf{x}}(\tau)\,d\tau = \boldsymbol{\lambda}(t)\,\dot{\mathbf{k}}(t). \tag{20}$$

Noticing that the market is competitive by assumption, this implies that shadow prices of capital assets coincide with their market prices $\mathbf{P}_k(t) = \boldsymbol{\lambda}(t)$, which gives us[8],

$$\int_t^\infty \frac{\pi(s)}{\pi(t)} \mathbf{P}_x(\tau)\,\dot{\mathbf{x}}(\tau)\,d\tau = \mathbf{P}_k(t)\,\dot{\mathbf{k}}(t). \tag{21}$$

[7] The shadow price of time itself $\boldsymbol{\lambda}$ here can be scaled up/down by $\gamma_i(t)$ to yield the same expression by the capital stock approach. However, $\gamma_i(t)$ is generally not known.

[8] In this case, we can apply a simple method following the proof of Theorem 1 in Dixit, Hammond and Hoel (1980), where they proved a similar equation for an economy.

5 Differences between approaches

In this section, I compare the two approaches and focus on differences between approaches. I summarize the main differences on assumptions and information demand between approaches in Table 1.

(Insert Table 1 here.)

The key difference between the two approaches is the definition of a sector. The capital stock approach associates a sector with an owner of certain capital assets at current time while the commodity flow approach associates a sector with a recipient of future cash flows. In a competitive market, an owner of capital assets is the same as a recipient of future cash flows generated by his/her capital assets and hence the sectoral net investments by both approaches coincide with each other. However, in an imperfect market such as externalities, monopolistic competition, distortionary taxation, or redistribution of income for the purpose of social security, the consistency between approaches probably does not hold any more, i.e., the cash flows generated by a stock of capital assets may not be obtained fully by the capital owner. Hence, we may prefer one approach to the other depending on our purpose. If the purpose is to estimate contributions of a stock of capital assets to dynamic welfare, the capital stock approach may be preferred. On the contrary, if the purpose is to estimate change in dynamic welfare of a sector who receives future cash flows, the commodity flow approach may be preferred. In this sense, the capital stock approach is production-based and the commodity flow approach is consumption-based. If an open-economy is taken as a sector, then net investments of the economy by the capital stock approach indicate the change in contributions of the economy to dynamic welfare not only of the open economy, but also of the other part of the global economy. On the contrary, the net investments by the commodity flow approach indicate the change in dynamic welfare of the economy, where the welfare may be generated by the economy or the other part of the global economy. The advantage of the commodity flow approach becomes more obvious if the purpose is to estimate the change in dynamic welfare of a pensioner who has never paid any taxes to the government.

The assumption on differentiability is essentially the same for both approaches. By definition, differentiable dynamic welfare implies the condition of differentiable utility. If utility is differentiable, the dynamic welfare is differentiable.

The information demand differs between approaches. The capital stock approach indirectly demands a given RAM that is not necessary for commodity flow approach. In order to calculate sectoral net investments, The capital stock approach demands changes in stocks of capital assets and shadow prices of all types of capital assets in the sector (including shadow price of time, if applicable) just at *current time*. On the contrary, commodity flow approach demands *future* information on a path of commodity flows, real prices of commodities, and discount factors. Hence, which one is preferred may also depend on availability of information. If a path of future commodity flows to a sector, real prices of commodities, and discount factors are given (or assumed), then sectoral net investments can straightforwardly be calculated by the commodity flow approach.

It is worth noticing that sectoral net investments by both approaches indicate the change in dynamic welfare of a sector excluding capital gains even though capital gains may also contribute to the change in the welfare of a sector itself as shown by Wei (2012). Below the subtle issue is highlighted by an example to calculate the change in sectoral real wealth.

Example: Change in sectoral real wealth

Define sectoral real wealth by the present value of future cash flows to the sector,

$$V_i(t) = \int_t^{\infty} e^{-\rho(s-t)} \mathbf{P}_x(s)\, \mathbf{X}_i(s)\, ds. \tag{22}$$

Hence, the change in real wealth of the sector can be expressed by

$$\begin{aligned} \dot{V}_i(t) &= \frac{d}{dt}\left[\int_t^{\infty} e^{-\rho(s-t)} \mathbf{P}_x(s)\, \mathbf{X}_i(s)\, ds\right] = \frac{d}{dt}\left[\int_0^{\infty} e^{-\rho\tau} \mathbf{P}_x(\tau+t)\, \mathbf{X}_i(\tau+t)\, d\tau\right] \\ &= \underbrace{\int_t^{\infty} e^{-\rho(s-t)} \mathbf{P}_x(s)\dot{\mathbf{X}}_i(s) ds}_{\text{sectoral net investments}} + \underbrace{\int_t^{\infty} e^{-\rho(s-t)} \dot{\mathbf{P}}_x(s)\mathbf{X}_i(s) ds}_{\text{price change effects}}, \end{aligned} \tag{23}$$

where the first term is sectoral net investments by the commodity flow approach and the second term is interpreted as price change effects (following the interpretation in Asheim and Wei, 2009). The price change effects may be considerable if real prices are changing over time. If so, the change in sectoral real wealth is not a plausible approximation of sectoral net investments by commodity flow approach. In addition, if discount rates over time are not constant, there will appear another term of interest rate effects on the right hand side of Eq.(23). Hence, by the commodity flow approach, we show that the change in sectoral real wealth is equal to sectoral net investment only in cases of constant real prices and constant discount rates in the future. Otherwise, the change in sectoral real wealth does not have welfare significance since it differs from sectoral net investments.

6 Concluding remarks

In the paper, I compared two approaches to valuing sectoral net investments. I showed that the commodity flow approach coincides with capital stock approach only if the future cash flows generated by the sectoral capital assets are fully obtained by the owner of the sectoral capital assets. Which approach is preferred in practice depends on purposes and availability of information. The commodity flow approach may be preferred if the purpose is to estimate the change in welfare of a sector as a recipient of future cash flows and information on future variables is available. Otherwise, the capital stock approach may be preferred if the purpose is to estimate the change in welfare of a sector as an owner of certain capital assets and information on capital assets is available.

Noticing that sectoral net investments here are calculated for net investments to indicate dynamic welfare at the national level. Hence, it can be interpreted as contributions of the sector to national dynamic welfare or national net investments. However, it may not be a plausible indicator for sectoral dynamic welfare since sectoral dynamic welfare may differ from the national one. Hence, if our purpose is to indicate dynamic welfare improvement of a sector alone, another sectoral indicator may be preferred[9].

Appendix

The appendix offers a standard derivation process of the maximum problem (19). The Hamil-

[9] A candidate could be sectoral income proposed by Asheim and Wei (2009).

tonian of the problem is

$$H = \frac{\pi(\tau)}{\pi(t)} \mathbf{P}_x(\tau)\,\mathbf{x}(\tau) + \boldsymbol{\lambda}(\tau)\,\dot{\mathbf{k}}(\tau)\,,$$

and the Lagrangian is

$$L = H + \sum_{i=1}^{r} \boldsymbol{\mu}_i(\tau)\, f_i\left(\mathbf{x}(\tau), \mathbf{k}(\tau), \dot{\mathbf{k}}(\tau), \tau\right),$$

where $\boldsymbol{\lambda} := (\lambda_1, \lambda_2, ..., \lambda_m)$ is the vector of shadow prices of the state variable $\mathbf{k}$ and $\boldsymbol{\mu} := (\mu_1, \mu_2, ..., \mu_r)$ the vector of shadow prices of the constraints $\mathbf{f}$. The necessary conditions for an optimum, among others, include

$$\frac{\partial L}{\partial x_j} \; : \; \frac{\pi(\tau)}{\pi(t)} P_{x_j}(\tau) = -\sum_{i=1}^{r} \mu_i(\tau) \frac{\partial f_i}{\partial x_j} \quad j = 1, ..., n \tag{24}$$

$$\frac{\partial L}{\partial \dot{k}_j} \; : \; \lambda_j(\tau) = -\sum_{i=1}^{r} \mu_i(\tau) \frac{\partial f_i}{\partial \dot{k}_j} \quad j = 1, ..., m \tag{25}$$

$$\frac{\partial L}{\partial k_j} \; : \; \dot{\lambda}_j(\tau) = -\sum_{i=1}^{r} \mu_i(\tau) \frac{\partial f_i}{\partial k_j} \quad j = 1, ..., m \tag{26}$$

$$\mu_i(\tau) \;\geq\; 0,\; \mu_i(\tau)\, f_i(\tau) = 0 \quad i = 1, ..., r\,. \tag{27}$$

Eq. 24 shows that total shadow value of marginal production with respect to. each commodity is equated to the present value price of the commodity. Eq. 25 implies that shadow price of each capital asset is determined by total shadow value of marginal production with respect to. the change in the capital asset. The arbitrage condition 26 means that total marginal value of each capital asset is equated to the instantaneous change of shadow price of the capital over time. The last expression 27 is a group of complementary slackness conditions.

Since the market prices $\mathbf{P}_x \geq 0$ and shadow prices $\boldsymbol{\lambda}, \boldsymbol{\mu} \geq 0$, then by 24 and 25, the derivatives of the function f exhibit properties as follows,

$$\sum_{i=1}^{r} \mu_i(\tau) \frac{\partial f_i}{\partial x_j} \;\leq\; 0, \tag{28}$$

$$\text{and} \sum_{i=1}^{r} \mu_i(\tau) \frac{\partial f_i}{\partial \dot{k}_j} \;\leq\; 0. \tag{29}$$

Given other things being equal, Inequality 28 shows if more commodities are available to go out of the sector, i.e., more output from the sector, then the production approaches much closer to the boundary of the constraints $\mathbf{f}(\cdot) = 0$ as a whole; If one more unit of input enters the sector, i.e. its corresponding element of $\mathbf{x}$ is one unit smaller, then the constraints are relaxed to allow more output to produce. By Inequality 29, the change in stock of capital assets $\dot{\mathbf{k}}$ can be thought of as a kind of output of the sector, which is not sold at current time. The value of $\dot{\mathbf{k}}$ will be realized in the future production and contribute to future cash flow to the sector.

By Eq. 26, if shadow price of a capital asset is decreasing all the time, i.e. $\dot{\boldsymbol{\lambda}} \leq 0$, then as a whole, the constraint is relaxed with more available capital assets, $\sum_{i=1}^{r} \mu_i(\tau) \frac{\partial f_i}{\partial K_j} \geq 0$. By (27), we can always have

$$\sum_{i=1}^{r} \mu_i(\tau) \frac{df_i(\tau)}{d\tau} = 0 \tag{30}$$

at all continuity points of μ_i since $f_i(\tau) = 0$ always holds for any continuity point where $\mu_i(\tau) \neq 0$. By the definition of $\mathbf{f}(\cdot)$ in (18), we have

$$\frac{df_i}{d\tau} = \sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j} \dot{x}_j(\tau) + \sum_{j=1}^{m} \frac{\partial f_i}{\partial k_j} \dot{k}_j(\tau) + \sum_{j=1}^{m} \frac{\partial f_i}{\partial \dot{k}_j} \frac{d\dot{k}_j}{d\tau}(\tau) + \frac{\partial f_i}{\partial \tau} \qquad i = 1, ..., r. \tag{31}$$

We can substitute (31) into (30) to obtain

$$\begin{aligned} &\sum_{i=1}^{r} \sum_{j=1}^{n} \mu_i(\tau) \frac{\partial f_i}{\partial x_j} \dot{x}_j(\tau) + \sum_{i=1}^{r} \sum_{j=1}^{m} \mu_i(\tau) \frac{\partial f_i}{\partial k_j} \dot{k}_j(\tau) \\ &+ \sum_{i=1}^{r} \sum_{j=1}^{m} \mu_i(\tau) \frac{\partial f_i}{\partial \dot{k}_j} \frac{d\dot{k}_j}{d\tau}(\tau) + \sum_{i=1}^{r} \mu_i(\tau) \frac{\partial f_i}{\partial \tau} = 0. \end{aligned} \tag{32}$$

By applying (24), (25), (26) and (32), we can express the present value of changes in commodity flows at each point in time by

$$
\begin{aligned}
& \frac{\pi(\tau)}{\pi(t)}\sum_{j=1}^{n}P_{x_j}(\tau)\,\dot{x}_j(\tau) && (33)\\
= & -\sum_{i=1}^{r}\sum_{j=1}^{n}\mu_i(\tau)\,\frac{\partial f_i}{\partial x_j}\dot{x}_j(\tau) \qquad \text{by (24)}\\
= & \sum_{i=1}^{r}\sum_{j=1}^{m}\mu_i(\tau)\,\frac{\partial f_i}{\partial k_j}\dot{k}_j(\tau)+\sum_{i=1}^{r}\sum_{j=1}^{m}\mu_i(\tau)\,\frac{\partial f_i}{\partial \dot{k}_j}\frac{d\dot{k}_j}{d\tau}(\tau)+\sum_{i=1}^{r}\mu_i(\tau)\,\frac{\partial f_i}{\partial \tau} \qquad \text{by (32)}\\
= & -\sum_{j=1}^{m}\dot{\lambda}_j(\tau)\,\dot{k}_j(\tau)-\sum_{j=1}^{m}\lambda_j(\tau)\,\frac{d\dot{k}_j}{d\tau}(\tau)+\sum_{i=1}^{r}\mu_i(\tau)\,\frac{\partial f_i}{\partial \tau} \qquad \text{by (25) and (26)}\\
= & -\sum_{i=1}^{m}\frac{d\left[\lambda_i(\tau)\,\dot{k}_i(\tau)\right]}{d\tau}+\sum_{i=1}^{r}\mu_i(\tau)\,\frac{\partial f_i}{\partial \tau} \qquad \text{by integration by parts.}
\end{aligned}
$$

Then integration on both sides of (33) yields the sectoral net investment by the commodity flow approach

$$
\begin{aligned}
\int_t^\infty \frac{\pi(\tau)}{\pi(t)}\mathbf{P}_x(\tau)\,\dot{\mathbf{x}}(\tau)\,d\tau &= \int_t^\infty \frac{\pi(\tau)}{\pi(t)}\sum_{j=1}^{n}P_{x_j}(\tau)\,\dot{x}_j(\tau)\,d\tau && (34)\\
&= -\int_t^\infty\sum_{i=1}^{m}\frac{d\left[\lambda_i(\tau)\,\dot{k}_i(\tau)\right]}{d\tau}d\tau+\int_t^\infty\sum_{i=1}^{r}\mu_i(\tau)\,\frac{\partial f_i}{\partial \tau}d\tau\\
&= \sum_{i=1}^{m}\lambda_i(t)\,\dot{k}_i(t)+\int_t^\infty\sum_{i=1}^{r}\mu_i(\tau)\,\frac{\partial f_i}{\partial \tau}d\tau\\
&= \underbrace{\boldsymbol{\lambda}(t)\,\dot{\mathbf{k}}(t)}_{\text{shadow value of change in stock of capital assets}}+\underbrace{\int_t^\infty\boldsymbol{\mu}(\tau)\,\frac{\partial \mathbf{f}}{\partial \tau}d\tau}_{\text{shadow value of time itself}}
\end{aligned}
$$

given the transversality conditions $\boldsymbol{\lambda}(\tau)\,\dot{\mathbf{k}}(\tau)\rightarrow 0$ and $\mu(\tau)\,\frac{\partial \mathbf{f}}{\partial \tau}\rightarrow 0$ as $\tau\rightarrow\infty$. The last equation in (34) shows the same result in Proposition 2, i.e., sectoral net investment by the commodity flow approach coincide with the one calculated by capital stock approach.

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	Capital stock approach	Commodity flow approach
Definition of a sector	An owner of certain capital assets	A recipient of future cash flows
Assumption	Differentiable dynamic welfare	Differentiable utility
Indirect info. demand	RAM invoked	RAM not necessary
Direct info. demand 1	Changes in stock of capital assets	A path of future commodity flows
Direct info. demand 2	Shadow price of capital assets	Future real prices of commodities and discount factors
Direct info. demand 3	Shadow price of time	No need for shadow price of time

Table 1. Differences between approaches to valuing sectoral net investments