Capital gains and income arising from nonrenewable resources*

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Abstract

Should capital gains be included in income arising from nonrenewable resources? In the present paper, I show that capital gains from nonrenewable resources can be divided into two terms: real price change effects and real interest rate change effects. By application of sectoral income theory developed by Asheim and Wei (2009), only the former term is part of real income of the resource and the latter term should not be included. This result is significant in the sense that all change in real resource wealth can be included as part of real income only if future real interest rates are assumed to be constant. Hotelling rule always implies that capital gains from nonrenewable resources coincide with real interests on resource wealth; net investment generated from the resource cancels out current cash flow from the resource; and real income comes from price change effects.

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1. Introduction

In the theory of resource accounting, the well-known Hartwick rule (Hartwick 1977) tells that Hotelling rent from the resource has to be reinvested in physical capital and other forms of reproducible capital in order to sustain consumption in a closed economy with a nonrenewable resource assuming a constant population and with no exogenous technological changes. Dixit et al. (1980) generalize the rule saying that consumption is sustainable only if net investment is nonnegative. Asheim (1986) and Hartwick (1995) then offer reformulated rule in an open-economy context by discussing who are responsible for the reinvestment, resource-consumers or resource-producers, and how much to reinvest by each of them.

Instead of focusing on net investment, Asheim (1996) focuses on capital gains arising from a nonrenewable resource in an open economy and argues that part of the capital gains have to be reinvested to keep national wealth constant due to decreasing interest rate over time. Vincent et al. (1997) argue that necessary amount of reinvestment should be the difference between Hotelling rent and the discounted sum of future terms-of-trade effects, where the latter is termed as capital gains and associated with the exogenous resource price change effects under the assumption of resource rent maximization. Contrast to Asheim (1996) and previous studies above, Vincent et al. (1997) assume constant interest rate over time and exogenous resource price for a small open economy as well as increasing production costs. All these studies aim to sustain consumption level by keeping national wealth constant rather than welfare maximization.

Recent studies have shown that constant national wealth is not a plausible welfare indicator (e.g., Sefton and Weale 2006). A plausible indicator of welfare for both closed and open economics is recently developed by Asheim and Wei (2009) following the theory of national accounting originated with Weitzman (1976) and developed further by Sefton and Weale

(2006). By the theory of sectoral income (Asheim and Wei 2009), I will derive the capital gains of a resource without knowing the exact information of quantity of the resource as “capital”.

The results in the present paper generalize the results of Asheim (1996) and Vincent et al. (1997) in the sense that both cases discussed by them become special cases of the general results presented here. In particular, The Hotelling rule applied by both Asheim (1996) and Vincent et al. (1997) is not necessary for the results in the present paper. In an open economy, perfect or imperfect, any capital gains equivalent to the decreasing interest rates have to be re-invested to keep non-decreasing welfare level. This is a generalization of the result in Asheim (1996). I also show that the necessary reinvestment to sustain consumption level involves three terms: net investment from the resource, price change effects, and interest rate change effect. This is a generalization of the main result in Vincent et al. (1997).

The paper is organized as follows. The next section will briefly present the two methods to calculate real resource income according to the theory of sectoral income. Section 3 first clarifies the meaning of capital gains arising from a resource and then show that capital gains from the resource can be divided into two terms related to real price change and real interest rate change respectively. Section 4 discusses several special cases and the final section offers concluding remark.

2. Theory of sectoral income

The theory of sectoral income provides two ways to calculate real income arising from a resource. One is the sum of present value of real interest on future cash flows generated from the resource, i.e.,

$$Y(t) = \int_t^\infty \frac{\pi(\tau)}{\pi(t)} R(\tau) \boldsymbol{P_x}(\tau) \boldsymbol{x}(\tau) d\tau \tag{1}$$

where π is discount factor, R real interest rate, $\boldsymbol{x}$ the vector of commodities used and produced during the resource extraction activities, and $\boldsymbol{P_x}$ the vector of real prices of $\boldsymbol{x}$. t and τ are points in time starting from initial time 0.

Another way to calculate real resource income is to sum up three terms: current cash flow, net investment, and price change effects,

$$Y(t) = \underbrace{\boldsymbol{P}_x(t)\boldsymbol{x}(t)}_{\text{current cash flow}} + \underbrace{\int_t^\infty \frac{\pi(\tau)}{\pi(t)} \boldsymbol{P}_x(\tau)\dot{\boldsymbol{x}}(\tau)d\tau}_{\text{net investment}} + \underbrace{\int_t^\infty \frac{\pi(\tau)}{\pi(t)} \dot{\boldsymbol{P}}_x(\tau)\boldsymbol{x}(\tau)d\tau}_{\text{price change effects}} \tag{2}$$

where a dot over a variable indicates first order derivative of the variable with respect to time.

3. Capital gains arising from nonrenewable resources

Real resource wealth is associated with the present value of future cash flows generated from the resource,

$$V(t) = \int_t^\infty \frac{\pi(\tau)}{\pi(t)} \boldsymbol{P}_x(\tau)\boldsymbol{x}(\tau)d\tau \tag{3}$$

For any time $t \geq 0$. Take the resource stock as kind of capital $S(t)$ at time t and denote its present value price by $q(t)$. Then resource wealth or the value of the resource as "capital" can be expressed by the product of S and q, i.e.

$$V(t) = q_t(t)S(t) \tag{4}$$

Following Dixit, Hammond et al. (1980) and Asheim (1996), capital gains from the resource is associated with change in wealth due to change in present value price of the resource alone, i.e.,

$$G(t) = \dot{q}_t(t)S(t) \tag{5}$$

Generally the resource stock is represented by the amount of the resource underground. However, this is not a plausible indicator of the resource stock as capital since the underground resource is generally not homogeneous. Marginal extraction costs are generally increasing with more extraction activities and net price of the resource is decreasing given the market price of the extracted resource. In addition, sometimes it is difficult to separate the value of resource stock and manmade capital. For example, with a deeper oil well, the value of the resource stock is changing. How much is the change? Definitely a resource stock combined with more manmade capital is more valuable but difficult to measure the difference. Hence, the present value price of the resource cannot be calculated by Eq.(4) and neither the capital gains by Eq.(5).

It is worth noting the difference between the present value price $q(t)$ and shadow price of the resource. Shadow price of the resource is formally defined as the marginal welfare change when one additional unit of the resource becomes available given other things being equal. Besides the shadow price of the resource, the present value price $q(t)$ may also involve time value of various reasons other than the resource alone. For example, the optimal extraction rate may not be achieved due to available technology and capacity; zero extraction rate is not an option in order for the producers to keep the business going on or take social responsibility; and the natural situation around the resource stock may imply additional extraction costs. Hence, the present value price $q(t)$ is the shadow price of the resource plus shadow price implied by the time value related to the resource (Wei 2011). Only in a theoretical case where time value is excluded can the present value price equal to the shadow price of the resource stock as capital. In empirical studies, it is a misinterpretation to associate the present value price $q(t)$ directly with the shadow price of the resource alone as done by Hamilton and Ruta (2009).

Given the fact of unknown $q(t)$ and $S(t)$, how could we calculate capital gains from the resource?

As shown by Wei (2011), net investment of the resource stock as capital can be calculated by

$$q_t(t)\dot{S}(t) = \int_t^{\infty} \frac{\pi(\tau)}{\pi(t)} \boldsymbol{P_x}(\tau)\dot{\boldsymbol{x}}(\tau)d\tau \tag{6}$$

On the other hand, the change in real resource wealth can be calculated by Eq. (3). Differentiating on both sides of Eq.(3), the change in real resource wealth,

$$\dot{V}(t) = -\boldsymbol{P_x}(t)\boldsymbol{x}(t) + R(t)V(t) \tag{7}$$

At the same time, the change in real resource wealth can be calculated by another method. We can rewrite Eq. (3) to be

$$V(t) = \int_0^{\infty} \frac{\pi(\tau+t)}{\pi(t)} \boldsymbol{P_x}(\tau+t)\boldsymbol{x}(\tau+t)d\tau \tag{8}$$

And differentiate on both sides of Eq.(8) with respect to time t

$$\begin{aligned} \dot{V}(t) = \int_t^{\infty} -R(\tau)\frac{\pi(\tau)}{\pi(t)} \boldsymbol{P_x}(\tau)\boldsymbol{x}(\tau)d\tau + R(t)V(t) \\ + \int_t^{\infty} \frac{\pi(\tau)}{\pi(t)} \dot{\boldsymbol{P}}_{\boldsymbol{x}}(\tau)\boldsymbol{x}(\tau)d\tau + \int_t^{\infty} \frac{\pi(\tau)}{\pi(t)} \boldsymbol{P_x}(\tau)\dot{\boldsymbol{x}}(\tau)d\tau \end{aligned} \tag{9}$$

where the first two terms on the right hand side of Eq.(9) is just the difference between real resource income and interests on real resource wealth.

Noticing that

$$\frac{d}{d\tau}\left[\frac{\pi(\tau)}{\pi(t)}V(\tau)\right] = \frac{\pi(\tau)}{\pi(t)}\left[\dot{V}(\tau) - R(\tau)V(\tau)\right] = -\frac{\pi(\tau)}{\pi(t)}\boldsymbol{P_x}(\tau)\boldsymbol{x}(\tau) \tag{10}$$

where the second equation is obtained by applying Eq.(**7**). Then real resource income can be expressed by two terms: interests on real resource wealth and a term of real interest rate change effects, i.e.,

$$\begin{aligned}\int_t^{\infty} R(\tau)\frac{\pi(\tau)}{\pi(t)}\boldsymbol{P_x}(\tau)\boldsymbol{x}(\tau)d\tau &= \int_t^{\infty} -R(\tau)\frac{d}{d\tau}\left[\frac{\pi(\tau)}{\pi(t)}V(\tau)\right]d\tau \\ &= -R(\tau)\frac{\pi(\tau)}{\pi(t)}V(\tau)\Big|_{\tau=t}^{\infty} + \int_t^{\infty}\dot{R}(\tau)\frac{\pi(\tau)}{\pi(t)}V(\tau)\,d\tau \\ &= \underbrace{R(t)V(t)}_{\text{interests on real wealth}} + \underbrace{\int_t^{\infty}\dot{R}(\tau)\frac{\pi(\tau)}{\pi(t)}V(\tau)\,d\tau}_{\text{interest rate change effects}}\end{aligned} \quad (11)$$

If real interest rates are assumed to be constant over time, then real resource income coincides with interests on real resource wealth. This result is somehow straightforward since constant real interest rates can be moved directly from the integral on the left hand side of Eq.(11). This holds even if an economy is imperfect.

Applying Eq.(11) to Eq.(9), the change in real resource wealth can be expressed by three terms:

$$\begin{aligned}\dot{V}(t) = &\underbrace{\int_t^{\infty} -\dot{R}(\tau)\frac{\pi(\tau)}{\pi(t)}V(\tau)d\tau}_{\text{interest rate change effects}} + \underbrace{\int_t^{\infty}\frac{\pi(\tau)}{\pi(t)}\dot{\boldsymbol{P}}_{\boldsymbol{x}}(\tau)\boldsymbol{x}(\tau)d\tau}_{\text{price change effects}} \\ &+ \underbrace{\int_t^{\infty}\frac{\pi(\tau)}{\pi(t)}\boldsymbol{P_x}(\tau)\dot{\boldsymbol{x}}(\tau)d\tau}_{\text{net investment}}\end{aligned} \quad (12)$$

which implies in order to keep constant real resource wealth over time, the reinvestment includes three terms: net investment from the resource, price change effects, and interest rate change effect.

By inserting Eq.(6) into Eq.(12) and noticing that

$$\dot{V}(t) = \dot{q}_t(t)S(t) + q_t(t)\dot{S}(t),$$

capital gains from the resource can be expressed by two terms: real interest rate change effects and real price change effects,

$$G(t) = \dot{q}_t(t)S(t) = \underbrace{\int_t^{\infty} -\dot{R}(\tau)\frac{\pi(\tau)}{\pi(t)}V(\tau)d\tau}_{\text{interest rate change effects}} + \underbrace{\int_t^{\infty} \frac{\pi(\tau)}{\pi(t)}\dot{\boldsymbol{P}}_{\boldsymbol{x}}(\tau)\boldsymbol{x}(\tau)d\tau}_{\text{price change effects}} \quad (13)$$

This is the main result of the paper.

By Eq.(13), it is true that part of capital gains from the resource have to cancel out the interest rate change effects if interest rate is not constant over time. If interest rate is decreasing over time, which is typical in a competitive closed economy, then capital gains is greater than price change effects. Asheim (1996) emphasizes the effects of decreasing interest rate and concludes part of capital gains cannot be used for consumption. Vincent, et al. (1997) then focus on real price change effects by assuming constant interest rate over time. In their case, capital gains come from price change effects alone. More will further be discussed in the Hotelling rule case of next section.

4. Special cases

To highlights the importance of the result of Eq.**(** 13 **)**, in this section we assume x only includes the extraction rate of the resource and price P_x becomes only the net price of the extraction rate. In addition, we also assume the resource is homogeneous such that the sum of x from now on equals current resource stock $S(t)$. Under these assumptions, the present value price of the resource stock equals real price of the extracted resource at current time, $q(t) = P_x(t)$.

<u>Case 1. Hotelling rule</u>

This case is widely discussed in the literature. Hotelling rule implies the change in real resource price is equal to real interest rate, i.e.,

$$\dot{P}_x(\tau) = -\frac{\dot{\pi}(\tau)}{\pi(\tau)} P_x(\tau) = R(\tau) P_x(\tau) \tag{14}$$

If Hotelling rule holds, then capital gains from the resource are real interests on resource wealth since $\dot{P}_x(t) = R(t) P_x(t)$ according to Eq.(14). That is to say,

$$\dot{q}(t)S(t) = \dot{P}_x(t)S(t) = R(t)P_x(t)S(t) = R(t)V(t) \tag{15}$$

no matter whether interest rate changes over time or not.

Furthermore, by inserting Eq.(14) into the term of price change effects on the right hand side of Eq.(13) we observe that price change effects equal exactly real resource income. This implies that net investment from the resource cancels out current cash flow from the resource according to the decomposition form of sectoral income Eq.(2). If current cash flow is taken as gross returns to the resource as an input in the economy, then net returns to the resource is zero after the net investment from the resource is deducted given the present value price at current time. This leads to an important result: net investment generated from the resource always cancels out current cash flow from the resource as long as Hotelling rule holds. Whether real interest rate is constant or not is irrelevant to this result. This is the so-called Hartwick's rule, i.e., all current cash flow (or resource rent) has to reinvest in reproducible capital for a closed economy. If the economy owning the resource is open, then the rule should be reformulated (Asheim 1986; Hartwick 1995). Asheim (1996) argues the resource owner has to reinvestment the interest rate change effects and consume the price change effects.

In addition to the Hotelling rule, Vincent, et al. (1997) also assume constant real interest rate over time. This implies capital gains from the resource are price change effects. Since net investment, the last term on the right hand side of Eq.(12) equals negative current cash flow, then the amount of reinvestment is represented by negative $\dot{V}(t)$ in order to keep total wealth constant. By Eq.(12), the result expressed by Eq.(8) in Vincent, et al. (1997) can be derived as current cash flow minus price change effects given constant real interest rate over time. In the case of Vincent, et al. (1997), Hotelling rule can hold together with exogenous resource price since an increasing cost function is introduced. If the extraction costs are assumed to be zero or constant, then Hotelling rule cannot hold together with exogenous resource price.

Case 2. Constant real interest rate

This assumption is widely adopted in resource accounting practice. If real interest rate is constant over time, then the first term on the right hand side of Eq.(13) disappears. Capital gains of the resource coincide with the real price change effects, which should be included in resource income. Vincent, et al. (1997) considers only the change in the price of the resource itself. In fact, real price change of any inputs during the resource extraction can disturb capital gains and further resource income.

This assumption also implies real income equals real interests on wealth as shown by Eq.(11).

Case 3. Constant real price of the resource

If real price of the resource is assumed to be constant in the future, then the second term disappears and capital gains from the resource become the real interest rate change effects alone. It makes no difference to ignore the capital gains in the calculation of resource income since the interest rate change effects are not part of real income arising from the resource.

In this case, the negative net investment is always smaller than current cash flow since present value prices are decreasing over time.

Case 4. Constant real interest rate and resource price

If both real interest rate and resource price are assumed to be constant, then the capital gains from the resource become zero and can be ignored from estimation of resource income. This case is widely adopted in empirical resource accounting (e.g., Atkinson and Hamilton 2007; Hamilton and Ruta 2009).

In this case, the calculation is simplified to a large extent. Real resource income coincides with interests on real resource wealth and net investment from the resource coincides with the change in real resource wealth. In addition, the negative net investment is always smaller than current cash flow and real resource income is always positive.

5. Concluding remark

In the present paper, I have applied theory of sectoral income to a nonrenewable resource to study whether capital gains from the resource can be included in resource income. I found out that only part of capital gains can be included in resource income, i.e., the real price change effects defined by the theory of sectoral income. The other part of capital gains is determined by change in future real interest rate and should not be included in resource income. The results are plausible for imperfect market and can be used directly to practical national accounting.

Several special cases are discussed. If Hotelling rule holds, capital gains from nonrenewable resources coincide with real interests on resource wealth and real resource income comes from price change effects alone. If real interest rate over time is assumed to be constant, then capital gains of a resource equal real price change effects alone. Furthermore, if real price of a resource is also constant over time, capital gains of the resource become zero and can be ignored from estimation of resource income.

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