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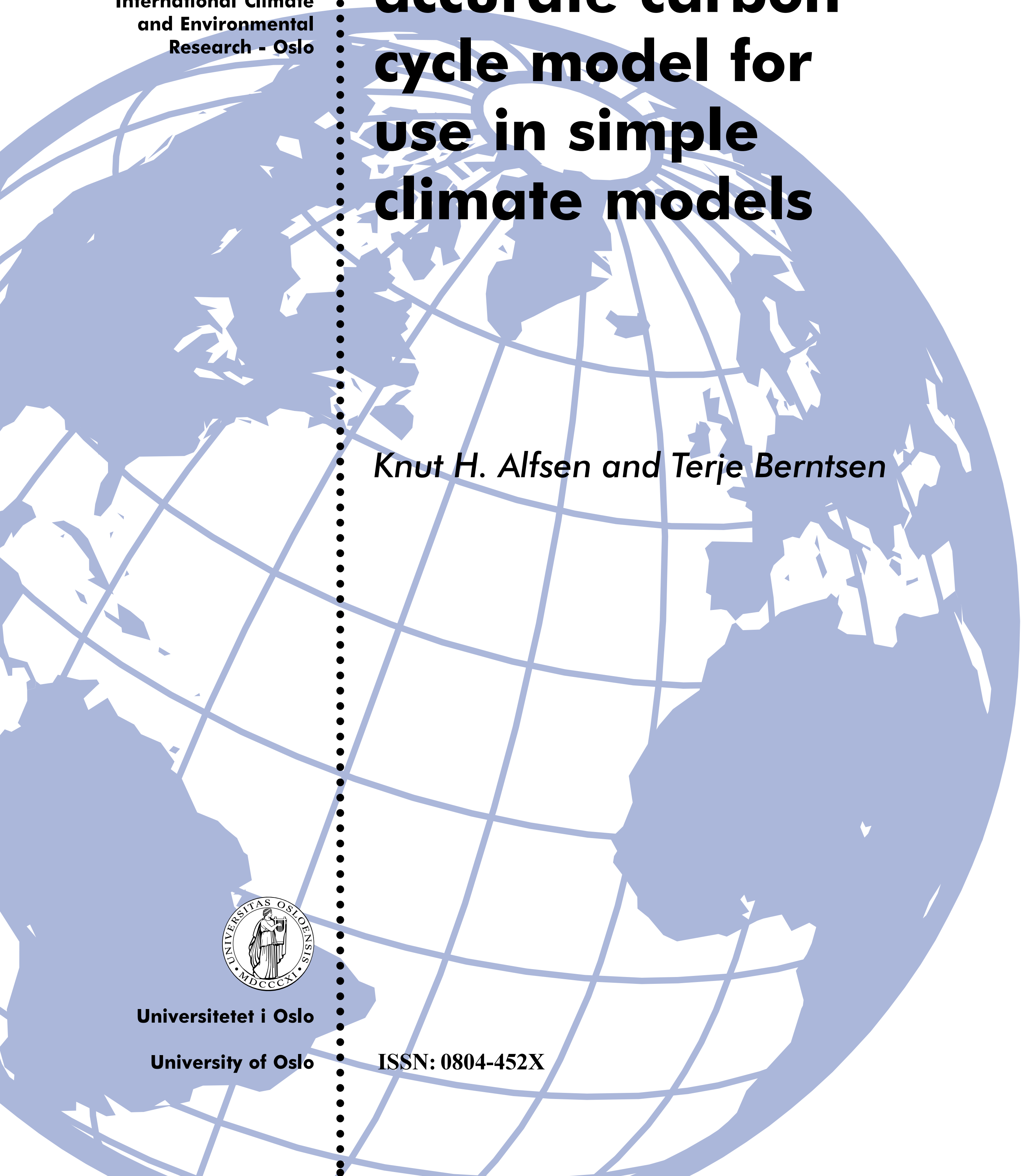
An efficient and accurate carbon cycle model for use in simple climate models



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Abstract

This publication describe a simple, but accurate, model for calculating CO_2 concentration levels in the atmosphere. The model is based on extracts from Joos, F., M. Bruno, R. Fink, U. Siegenthaler and T.F. Stocker (1996): An efficient and accurate representation of complex oceanic and biospheric models of antropogenic carbon uptake, Tellus 48B, 397-417.

Contents

1 Introduction

The aim of this note is to document a simple model of the atmospheric CO_2 concentration based on exogenous input of anthropogenic emission of CO_2 and taking air-sea exchange and biospheric responses into account.

The approach described by Joos et al (1996) is based on the application of a mixed layer pulse response function. The advantage of using a mixed layer pulse response function instead of an atmosphere pulse response function (Siegentaler and Oeschger, 1978; Oeschger and Heimann, 1983; Meier-Reimer and Hasselmann, 1987; Sarmiento et al., 1992) is that it is then possible to represent the non-linear effects of seawater chemistry. As long as the CO_2 increase in the atmosphere is below approximately 50% of the pre-industrial level, the CO_2 system behaves in a linear way and it is possible to represent the effects of anthropogenic emissions on the atmospheric concentrations by an atmospheric pulse response function. For CO_2 perturbations beyond this level the non-linear effects of the seawater chemistry becomes important, and it thus becomes necessary to apply a mixed layer pulse response function to obtain accurate results.

The approach described in this paper includes changes in CO_2 uptake and release by terrestrial vegetation by CO_2 fertilization, but does not take into account possible feedback mechanisms of climate change on the carbon cycle. Possible feedbacks include changes in CO_2 solubility due to sea surface temperature (SST) changes and changes in vertical mixing by reduced deepwater formation in the North Atlantic. Joos et al. (1999a) have estimated the marine part of this to be of minor importance (approximately 4%) up to 2100, increasing to about 20% in 2500.

Since damage caused by climate change could imply high costs, an accurate representation of the carbon cycle is very important in models that are to be used to estimate the costs of climate and evaluate different mitigation and/or adaptation strategies (Joos et al., 1999b). The carbon cycle model described in this paper is included in a simple model for scenario studies of changes in global climate (Fuglestvedt and Berntsen, 1999).

2 Air-sea interface modelling

The following expressions form the basis for the modelling of air-sea exchange of CO_2.

- The content of inorganic carbon in the ocean's mixing layer:

$$\delta\Sigma CO_2(t) \equiv \Sigma CO_2(t) - \Sigma CO_2(t_0) = \frac{c}{h}\int_{t_0}^{t} f_{as}(t')r_s(t-t')dt' \tag{1}$$

where h is the height of the mixing layer (typically 75 m) and c is a conversion factor translating the air-sea flux from $[ppm \cdot m^{-2} yr^{-1}]$ to $[\mu mol \cdot m^{-2} yr^{-1}]$ and sea water from $[m^3]$ to $[kg]$. We will apply the value $c = 1.722 \cdot 10^{17} \frac{\mu mol \cdot m^3}{ppm \cdot kg}$. The variable $\Sigma CO_2(t)$ is the content of inorganic carbon in the mixing layer, f_{as} is the transfer (flux) of CO_2 from air to the sea and r_s is the pulse response function of the *mixing layer*.

- The pulse response function is given by two expressions:

$0 < t < 2$ year:

$$\begin{aligned} r_s(t) &= 0.12935 + 0.21898 e^{-\frac{t}{0.034569}} + 0.17003 e^{-\frac{t}{0.26936}} \\ &\quad + 0.24071 e^{-\frac{t}{0.96083}} + 0.24093 e^{-\frac{t}{4.9792}} \end{aligned}$$

Mixing layer response function for $t < 2$ yr

and for $t \geq 2$ year:

$$\begin{aligned} r_s(t) &= 0.022936 + 0.24278 e^{-\frac{t}{1.2679}} + 0.13963 e^{-\frac{t}{5.2528}} + 0.089318 e^{-\frac{t}{18.601}} \\ &\quad + 0.037820 e^{-\frac{t}{68.736}} + 0.035549 e^{-\frac{t}{232.3}} \end{aligned}$$

Mixing layer response function for $t > 2$ years

- Carbon budget for the atmosphere:

The perturbation of the carbon concentration in the atmosphere from pre-industrial level (278 ppm) can be obtained from the conservation law equating the change in atmospheric concentration with the difference between (net) emissions and sea uptake:

$$\frac{d}{dt}\delta pCO_{2,a}(t) = e(t) - A_{oc} f_{as}(t) \tag{2}$$

where the unit is $[ppm \cdot yr^{-1}]$. Note that 1 $ppm = 2.123 \times 10^{15} gC = 7.779 \times 10^{15} gCO_{2.}$ The variable $pCO_{2,a}(t)$ is the partial pressure of CO_2 in the atmosphere, $e(t)$ is antropogenic emissions of CO_2 available for sea uptake (see below) and A_{oc} is the area of the ocean surface ($A_{oc} = 3.62 \cdot 10^{14} m^2$).

- Transfer rate air-sea:

The transfer rate between the atmosphere and the sea (mixing layer) depends on the difference in partial pressures:

$$f_{as}(t) = k_g \left[\delta pCO_{2,a}(t) - \delta pCO_{2,s}(t)\right] \tag{3}$$

where k_g is the gas exchange coefficient ($= \frac{1}{9.06 * A_{oc}} yr^{-1} m^{-2}$).

- Partial CO_2 pressure in the mixing layer:

The partial CO_2 pressure in the mixing layer is a complex function of the carbon content in the layer and sea temperature T :

$$\delta pCO_{2,s}(t) = F(\delta \Sigma CO_2(t), T) \tag{4}$$

An usual value for T is $T = 18.2°C$. Two functional forms of F are given. The first is a reasonable approximation to the true functional form when $0 < \delta pCO_{2,s}(t) < 200$ ppm and $0°C < T < 30°C$:

$$\begin{aligned} F(x,T) &= \frac{\left(1.7561 - 31.618 \cdot 10^{-3} T + 444.4 \cdot 10^{-6} T^2\right) x}{1 - (4.096 \cdot 10^{-3} - 77.086 \cdot 10^{-6} T + 0.61 \cdot 10^{-6} T^2)\, x} \\ &= \frac{1.3279x}{1 - 2.8951 \times 10^{-3} x} \text{ when T=18.2 } {}^oC \end{aligned}$$

First version of $F(x,T)$ with $T = 18.2\ ^0C$

The second version covers the range $0 < \delta pCO_{2,s}(t) < 1320$ ppm, but is rather narrow with respect to temperature: $17.7°C < T < 18.3°C$:

$$\begin{aligned} F(x,T) &= \left(1.5568 - 13.993 \cdot 10^{-3}T\right)x + (7.4706 - .20207T)\,10^{-3}x^2 \\ &\quad - (12.748 - 1.2015T)\,10^{-6}x^3 + (244.91 - 12.639T)\,10^{-9}x^4 \\ &\quad - (0.15468 - 0.015326T)\,10^{-9}x^5 \end{aligned}$$

which, with the chosen temperature T, can be simplified to

$$\begin{aligned} F(x,T &= 18.2) = 1.3021x + 3.7929 \times 10^{-3}x^2 + 9.1193 \times 10^{-6}x^3 \\ &+ 1.488 \times 10^{-8}x^4 + 1.2425 \times 10^{-10}x^5 \end{aligned}$$

The second version of $F(x,T = 18.2)$

In our model we employ the second version of F with $T = 18.2°C$.

2.1 Structure of the problem

The above equations 1-4 represent enough constraints to determine the four variables $\delta pCO_{2,a}$, $\delta pCO_{2,s}$, $\delta \Sigma CO_2$ and $f_{as.}$ Employing eq. 3 to eliminate f_{as} results in equations in the three variable $\delta pCO_{2,a}$, $\delta pCO_{2,s}$ and $\delta \Sigma CO_2$.

For simplicity I will denote these variable x, y and z, respectively. We also introduce the variable $s(t) = x(t) - y(t)$ which is proportional to the air-sea carbon flux. The equations can then be expressed as follows (denoting the parameters k_g by k and $A_{oc}\, by\; A$)

$$x(t) - x(0) = x(t) = \int_0^t [e(t') - Aks(t')]dt', \tag{5}$$

$$z(t) = k\frac{c}{h}\int_0^t s(t')r_s(t-t')dt', \tag{6}$$

$$y(t) = F(z(t)), \tag{7}$$

$$s(t) = x(t) - y(t)$$

3 Biosphere decay response function

Emission of relevance for atmosphere-ocean couplings may be written

$$e(t) = e_{anthropogenic}(t) - f_{fer}(t) \tag{8}$$

where f_{fer} is the biospheric sink due to additional net primary production (δf_{npp}) and decay (δf_{decay}) from CO_2 fertilization:

$$f_{fer}(t) = \delta f_{npp} - \delta f_{decay} = \delta f_{npp}(t) - \int_{-\infty}^t \delta f_{npp}(t')r_b(t-t')dt' \tag{9}$$

The uptake of atmospheric CO_2 from increase in net primary production is modelled as

$$\delta f_{npp}(t) = 60\,[GtC/yr]\,\beta \ln\frac{CO_{2,a}(t)}{278} \tag{1}$$

where $\beta = 0.287$ is the fertilization factor and the CO_2 concentration level is measured in ppm. The unit of $\delta f_{npp}(t)$ (and f_{fer}) is $GtC = 10^{15}gC = PgC$.

Net primary production response to CO_2 in the atmosphere

For $t > 0$ we use the following expression for the biotic decay response function

$$r_b(t) = 0.70211e^{-0.35t} + 13.414\cdot 10^{-3}e^{-\frac{t}{20}} - 0.71846e^{-\frac{55t}{120}} + 2.9323\cdot 10^{-3}e^{-\frac{t}{100}}$$

The form of the function is as follows:

Biosphere decay response function r_b

4 Procedure of calculation of atmospheric CO_2 concentration

In solving the above system of equations on a computer, a discretization is necessary. One possible discrete version of the equations 5-7 is (trapezoidal rule)

$$\begin{aligned} x_i - x_0 &= \Delta t[\frac{1}{2}(e_1 - Ak(x_1 - y_1)) \\ &\quad + \sum_{j=2}^{i-1}(e_j - Ak(x_j - y_j)) \end{aligned} \tag{11}$$

$$+\frac{1}{2}(e_i - Ak(x_i - y_i))],$$

$$y_i = F\left(z_i\right), \tag{12}$$

$$\begin{aligned}
z_i &= \frac{ck}{h}\Delta t \times [\frac{1}{2}\left(x_1 - y_1\right) r_i + \sum_{j=2}^{i-1}\left[x_j - y_j\right] r_{i-j} + \frac{1}{2}\left(x_i - y_i\right) r_1] \qquad (13)\\
&= \frac{ck}{h}\Delta t \times [\sum_{j=1}^{i-1}\left[x_j - y_j\right] r_{i-j} - \frac{1}{2}\left(x_1 - y_1\right) r_i + \frac{1}{2}\left(x_i - y_i\right) r_1].
\end{aligned}$$

Equation 11 can be solved with respect to x_i to yield

$$\begin{aligned}
x_i &= c_1\left[\sum_{j=2}^{i-1}\left(\frac{e_j}{Ak} - (x_j - y_j)\right) + \frac{1}{2}\left(\frac{e_1}{Ak} - (x_1 - y_1) + \frac{e_i}{Ak} + y_i\right)\right] \qquad (14)\\
&= c_1\left[\sum_{j=1}^{i-1}\left(\frac{e_j}{Ak} - (x_j - y_j)\right) - \frac{1}{2}\left(\frac{e_1}{Ak} - (x_1 - y_1)\right) + \frac{1}{2}\left(\frac{e_i}{Ak} + y_i\right)\right],
\end{aligned}$$

where the coefficient c_1 is given by

$$c_1 = \frac{\Delta t Ak}{1 + \Delta t Ak/2}.$$

Using

$$s_i = x_i - y_i$$

we get

$$\begin{aligned}
s_i &= c_1\left[\sum_{j=1}^{i-1}\left(\frac{e_j}{Ak} - s_j\right) - \frac{1}{2}\left(\frac{e_1}{Ak} - s_1\right) + \frac{1}{2}\left(\frac{e_i}{Ak} + y_i\right)\right] - y_i \qquad (15)\\
&= c_1\left[\sum_{j=1}^{i-1}\left(\frac{e_j}{Ak} - s_j\right) - \frac{1}{2}\left(\frac{e_1}{Ak} - s_1\right) + \frac{1}{2}\frac{e_i}{Ak} - \frac{y_i}{\Delta t Ak}\right]
\end{aligned}$$

$$z_i = \frac{ck}{h}\Delta t \times [\sum_{j=1}^{i-1} s_j r_{i-j} - \frac{1}{2}s_1 r_i + \frac{1}{2}s_i r_1] \tag{16}$$

The recursive procedure is then:

1. from given y_i, calculate s_i from eq. (15),
2. calculate z_i from eq. (16)
3. finally calculate a new y_i from eq. (12).

4.1 Calculating the biosphere response

Similarly, using a trapezoidal scheme for integration of the biosphere equation, and approximating equation 10 by

$$\delta f_{npp,i} = 60\beta \ln \frac{CO_{2,a,i-1}}{278},$$

and further noting that $r_b(0) = 0$ and $\delta f_{npp}(0) = 0$, we get a discretisized version of eq. 10 as follows

$$f_{fer,i} = \delta f_{npp,i} - \Delta t \sum_{j=1}^{i-1} \delta f_{npp,j} r_{b,i-j}. \tag{17}$$

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5

A Fortran program

Below follows a Fortran-subroutine incorporating the above relations.

```
SUBROUTINE CO2EM2CONC(NY,NYE)
C
C — CO2 model based on Joos et al., Tellus, 48B, 397-417, 1996.
C — Parameter NY is number of years since NSTART(=1750)
C
 INCLUDE 'scm.cmn'
C
 REAL*8 SUMZ, SUMF, SS2, SS1
 REAL*8 K,CC1
C
C — Fertilization factor
 BETAF = 0.287
C — Area of the ocean (m^2)
 A = 3.62E+14
C — Gas exchange coefficient (air <–> ocean) (yr^-1*m^-2)
 K = 1./(A*9.06)
C — TIMESTEP (YR)
 DT = 1./IDTM
C — Conversion factor ppm/kg –> umol*/m3
 C = 1.722E+17
C — Depth of mixed layer in the ocean (75 m)
 H = 75.
C
 CC1 = DT*A*K/(1.+DT*A*K/2.)
C
 DO IDT = 1,IDTM
 IT = (NY-1)*IDTM + IDT
C
C — Net emissions, including biogenic fertilization effects ———
```

```
C
 EMFF = FLOAT(IDT)/IDTM
 SUMF = 0.0
 IF (IT .GT.1) THEN
 DFNPP(IT) = 60.0*BETAF*LOG(XCO2(IT-1)/278.)
 DO I=2,IT-1
 ITX = IT-I
 SUMF = SUMF + DFNPP(I)*RB(ITX)
 END DO
 ELSE
 SUMF = 0.0
 END IF
 FFER = DFNPP(IT) - DT*SUMF
C
 IF (NY .LT. 241 ) THEN
 EMCO2(IT) = (DFCO2F(NY)+EMCO2F(NY))*(1.-EMFF) +
 . (DFCO2F(NY+1)+EMCO2F(NY+1))*EMFF - FFER
 ELSE
 IF (NYE .LT. NYMAX ) THEN
 EMCO2(IT) = EMISSION(1,NYE)*(1.-EMFF) +
 . EMISSION(1,NYE+1)*EMFF - FFER
 ELSE
 EMCO2(IT) = EMISSION(1,NYE) - FFER
 END IF
 END IF
C — 10ˆ15 gC/yr) –> ppm/yr
 EMCO2(IT) = EMCO2(IT)/2.123
C
C — End emissions ——————————————————————
C
C — Calculate SCO2(IT) (=dpCO2,a - dpCO2,s)
 SS1 = 0.5*EMCO2(1)/(A*K)
 IF (IT .EQ. 1) THEN
```

```
 SS2 = 0.5*EMCO2(IT)/(A*K)
 SUMS = 0.0
 ELSE
 SS2 = 0.5*EMCO2(IT)/(A*K) - YCO2(IT-1)/(DT*A*K)
 SUMS = SUMS + EMCO2(IT-1)/(A*K)-SCO2(IT-1)
 END IF
 SCO2(IT) = CC1*(SUMS + SS1 + SS2)
C
C — Calculate ZCO2(IT) (=dSCO2, change in total inorganic carbon in the mixed layer)
C
 SUMZ = 0.0
 DO I=1,IT-1
 ITX = IT-I
 SUMZ = SUMZ + SCO2(I)*RS(ITX)
 END DO
C
C — Since RS(IT=0) = 1, and SCO2(0) = 0
 ZCO2(IT) = C*K*DT/H *(SUMZ + 0.5*SCO2(IT))
c
C — Calculate new YCO2(IT) (= dpCO2,s)
c
 YCO2(IT) = 1.3021*ZCO2(IT) + 3.7929E-3*(ZCO2(IT)**2) +
 . 9.1193E-6*(ZCO2(IT)**3) + 1.488E-8*(ZCO2(IT)**4) +
 . 1.2425E-10*(ZCO2(IT)**5)
C
C — Atmospheric concentatration of CO2 (ppm)
 XCO2(IT) = SCO2(IT) + YCO2(IT) + 278.
C
 END DO
C
C — Update CO2 concentrations in the CONC array.
 CONC(1,NY) = XCO2(IT)
```

```
c
 RETURN
 END
C ——————————————————————————————-
 SUBROUTINE SETUP_CO2
C
 INCLUDE 'scm.cmn'
C
 YCO2(1) = 0.0
 SCO2(1) = 0.0
 XCO2(1) = 278.
 DFNPP(1) = 0.0
 SUMS = 0.0
c
C — Tabulate values of response/decay functions which are only a function of time.
 DO NYEAR=NYSTART,NYEND
 IYR = NYEAR-NYSTART+1
 DO IDT = 1,IDTM
 IT = (IYR-1)*IDTM + IDT
 TIME = (IYR-1)+FLOAT(IDT-1)/IDTM
C — Pulse respons function for the mixed layer (t<2 yr)
 IF (TIME .LE. 2.0) THEN
 RS(IT) = 0.12935 + 0.21898*exp(-TIME/0.034569) +
 . 0.17003*exp(-TIME/0.26936) +
 . 0.24071*exp(-TIME/0.96083) +
 . 0.24093*exp(-TIME/4.9792)
C — Pulse respons function for the mixed layer (t>2 yr)
 ELSE
 RS(IT) = 0.022936 + 0.24278*exp(-TIME/1.2679) +
 . 0.13963*exp(-TIME/5.2528) +
 . 0.089318*exp(-TIME/18.601) +
 . 0.03782*exp(-TIME/68.736) +
```

```
 . 0.035549*exp(-TIME/232.3)
 END IF
C — Biotic decay function
 RB(IT) = 0.70211*EXP(-0.35*TIME) +
 . 13.4141E-3*exp(-TIME/20.) -
 . 0.71846*exp(-55*TIME/120.) +
 . 2.9323E-3*exp(-TIME/100.)
 END DO
 END DO
 RETURN
 END
```

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