CICERO Working Paper 2001:1

# Impatience and climate policy

**H. Asbjørn Aaheim**

December 2000

**CICERO**
Center for International Climate
and Environmental Research
P.O. Box 1129 Blindern
N-0318 Oslo, Norway
Phone: +47 22 85 87 50
Fax: +47 22 85 87 51
E-mail: admin@cicero.uio.no
Web: www.cicero.uio.no

**CICERO Senter for klimaforskning**
P.B. 1129 Blindern, 0318 Oslo
Telefon: 22 85 87 50
Faks: 22 85 87 51
E-post: admin@cicero.uio.no
Nett: www.cicero.uio.no

# 1 Introduction

On several occasions, it has been pointed out that the optimal level of action to mitigate climate change is completely governed by the choice of discount rate [6]. This is why the question of which discount rate to apply in economic analyses of climate policy has become a very controversial issue. Climate policy is sensitive to the choice of discount rate because of the long time span from when greenhouse gases are emitted to the atmosphere to the effects of these emissions have disappeared. For $CO_2$, which constitutes the bulk of greenhouse gas emissions, it takes more than 200 years before the concentrations added from a ton of emissions has been reduced to 1/3 of the initial increase. More than 800 years may pass before the concentrations are reduced to 10 percent. The major part of the benefits of abatement therefore diminish as a result of the discounting.

This is why economic analyses seldom support strong measures to mitigate climate change. However, many people find it difficult to accept that it should be irrational to cut present emissions if it is considered likely that these emissions represent a serious threat to the welfare of future generations. In recent years, new arguments for applying lower discount rates for climate policy analysis than for other decisions have been put forward. The renewed interest in the discounting issue is due to the extension of the time frame in analyses of climate change, compared with conventional economic analyses. Assumptions that are acceptable within the normal time frame of 15 - 30 years may be inappropriate when the perspective extends to hundreds or thousands of years.

Perhaps the most debated issue is whether or not intertemporal comparisons of social welfare should reflect impatience. This seems to be more difficult to accept because the change of time frame turns the presence of impatience in the welfare function into a question of intergenerational equity, while it is usually explained with reference to the preferences of single individuals. Clearly, impatience denies intertemporal equity from the outset. So why do economists insist on impatience if they consider discounting to be a problem in analyzing abatement of long-term environmental problems? First, some economists also discard impatience. Ramsey [20], who is the father of modern economic growth theory, claims that it is unethical, and Cline [6] considers discount rates without impatience. On the other hand, one cannot impose intergenerational equity without problems. In developing intertemporal recursive utility functions, Koopmans [15], [16] showed that impatience is embedded in a standard welfare function by default, if extended to an infinite number of periods. Later, Diamond [8] showed that one cannot claim both intergenerational equity and efficiency of a utility function over an infinite time horizon. For natural reasons, economists find it hard to give up efficiency. Manne [17] also shows that a zero rate of impatience leads to an unrealistically high savings ratio.

Other arguments favoring an adjustment of the discount rate in analyses of climate policy relate to the properties of the economy that is affected by global warming. Weitzmann [26] shows why unequal development of capital and environmental stocks leads to systematically different discount rates for ordinary investments and investments in environmental preservation. According to Weitzmann [27], uncertainty about environmental impacts of present actions in the very long term should be discounted at the lowest possible discount rate, because the present value is completely dominated by the outcome of the most unfortunate case. More intuitive arguments for applying low discount rates for long-term environmental benefits are provided by e.g. Hasselman et. al. [13] and Hasselmann [12]. They claim that the irreversibility of a catastrophe, and uncertainty as to when the damage from climate change occurs justifies the use of low discount rate for investments in the mitigation of climate change. Nordhaus [18] and Heal [14] respond that it is difficult to achieve consistency by the use of different discount rates. The choice of discount rate is therefore left to the analyst, which is also problematic.

This debate commonly centers on whether or not it is justifiable to make considerable investments in the present to mitigate future climate change. I question this point of departure. I argue that the present "now or never" perspective overlooks the issue of *timing*. The choice of discount rate not only indicates whether a given investment might be cost-effective at the present time, but also indicates precisely when it will become so. In other words, a discount rate acts as an incentive to allocate consumption and abatement over time. A change the perspective of this debate is important because the current approach, with its emphasis of present value, tends to ignore the consequences of present inaction.

The question of how much to do over a certain period of time, on the other hand, is primarily a question of sustainability. The problem is clearly illustrated by the efforts made by those who favor low discount rates in order to make climate policy become beneficial 'enough'. The common view is that if a low discount rate can be justified, so can considerable abatement. But this does not match the interpretation of the discount rate as reflecting the return on investments. A low discount rate means that the return is low, which indicates that the investments should be reduced.

Can these two signals be joined? The answer is no, unless the amount of present investment is compared with the amount of future investment. To make such a comparison, most of the factors known from the previous debates on the discount rate, such as the rate of impatience, the preferences for intergenerational equity, and the return on capital are important, but their roles are to some extent different from what is usually believed. The marginal cost of abatement, for example, is important, but has hardly been touched upon in the debate.

This paper aims to assess the discount rate for analysis of climate policy from the perspective of the level and path of abatement. For such an analysis, we need to assess both the discount rate and the sustainability criterion. In the next section I present the traditional approach based on the Ramsey-model of economic growth, and extend the model to include mitigation and impacts of climate change. The discount rates are implicit in the endogenous paths for the shadow prices on capital and concentrations of climate gases. The sustainability criterion can naturally be defined by the steady state solution. In the following sections, particular attention is paid to the welfare function, and to the role of impatience. It is shown that the traditional additive welfare function, which includes a rate of impatience, is poorly founded but difficult to replace with alternatives that meet more intuitive and ethically acceptable properties of social preferences. The message is that an intertemporal additive welfare function with a given rate of impatience fails to provide a perfect description of social preferences, but it is difficult to come up with good alternatives. Moreover, the conventional approach to discounting is not as discriminating towards long-term environmental impacts as is often claimed.

# 2 The basic approach to discounting

Over the past decades a consensus has emerged about the main determinants for the discount rate. This is mainly due to Arrow [1], whose framework is extended to include environmental consequences in Dasgupta et al. [7]. However, the basics stem from Ramsey's [20] modelling of the consumption-saving decision. Ramsey's point of departure was to find the optimal allocation of present and future consumption, given declining marginal utility of instantaneous consumption and a return on savings. In its simplest form, Ramsey's model aims at maximising utility over an infinite time horizon, given that economic growth is constrained by the increase in the capital stock. Climate change can be included in the Ramsey model by extending the production function to depend on the level of global warming, which leads to a damage. Global warming is determined by adding emissions to the concentrations of greenhouse gases over time. We first discuss the discount rate in the simplest Ramsey model, later extending it to the case of climate change.

## 2.1 The Ramsey model

Denote by $x_t$ consumption at $t$. The production of goods and services are generated by the production function $f(k_t)$, where $k_t$ is real capital. The instantaneous utility function, $u(x_t)$, has the traditional (weak) properties ($u'_x \geq 0$, $u''_{xx} \leq 0$), and comparisons of consumption levels over time are adjusted by a constant rate of pure time preference, or impatience, $\delta \geq 0$. As a curiosity, it should be mentioned that Ramsey argued that $\delta = 0$ for ethical reasons. The model is

$$\max_x \int_0^\infty u(x_t)e^{-\delta t}dt,$$

subject to

$$\dot{k} = f(k_t) - x_t. \tag{1}$$

The key factor to the solution of this problem is the development of the price of capital along the optimal path. This price expresses the value of capital at each point in time. Hence, the rate of change in the value of capital provides the appropriate rate at which the stock of capital can be compared at different times, that is, the discount rate. The intertemporal equilibrium condition for this problem is (see Blanchard and Fisher [4])

$$\rho = f'_k = \mu_x \frac{\dot{x}}{x} + \delta. \tag{2}$$

This is the so-called Ramsey rule for optimal saving. The factor $\mu_x = -xu''_{xx}/u'_x$ characterizes the curvature of the utility function. It is sometimes called the elasticity of intertemporal substitution because it measures the utility of moving consumption from one point in time to another. A high $\mu_x$ indicates a strongly curved utility function, and vice versa. If $\mu_x$ is high, the utility is strongly affected by a change of consumption over time. In the context of intertemporal models, a high $\mu_x$ is usually interpreted as a result of high preferences for intergenerational equity because it means that the marginal utility of consumption growth is rapidly declining.

The Ramsey rule is an equilibrium condition telling how to balance present and future consumption. When (2) applies, the choice between adding a unit of output to present consumption versus saving and investing in the capital stock to gain more in the future are equally weighted. The marginal utility of present consumption equals the gain of substituting consumption from a future point of time plus the gain that follows from impatience. An assessment of the marginal utility of adding to present consumption thereby provides one way to assess the discount rate. The alternative is to estimate the return on capital.

Some properties of the discount rate assessed by (2) are worth noting. First, the discount rate is a result of an equilibrium condition. Hence, one cannot choose a discount rate on the basis of, e.g., some indication of the return on capital without considering the consequences for the marginal utility of present consumption. If, for example, the utility function is logarithmic, $\mu_x$ is constant and independent of the consumption level. If a high discount rate is chosen because it believed that the return on capital is high, it follows that consumption growth also has to be high. With a given initial capital stock, it is therefore necessary to start at a low level of consumption, to manage the required growth.

Correspondingly, a low rate of impatience leads to a high growth rate in consumption. Again, the initial level of consumption will have to be low. One may test the justification of a zero rate of impatience by asking whether the resulting optimal level of initial consumption is compatible with observations. Manne [17] argues that it is not. He therefore concludes that to be realistic, one needs to include impatience in the preference structure.

A second property following from (2) is that the discount rate is not constant over time. It changes according to the increase in the capital stock in a growth process. With a slight extension of the model, one could also include technological change, which also implies a change in the marginal return on capital over time. However, technological change may counteract a declining marginal return on capital as the capital stock increases. Over a period of 10 to 20 years, one may therefore defend a constant discount rate as an approximation. When analyzing long-term issues, such as climate change, this assumption is generally too strong.

The fact that the discount rate is variable over time makes it inconsistent to claim that discounting may lead to future disasters. A future disaster will lead to a decline in consumption. If the rate of impatience is close to zero, declining consumption will reflect a negative return on capital, which cannot be optimal because it will always be better to refrain from investments and consume immediately unless the marginal return on capital is positive.

The rate of impatience is, however, crucial. If it is high, present consumption may be preferred to such an extent that it becomes optimal to reduce consumption in the long term. To take a closer look at this case, consider figure 1, which displays a graphical illustration of the solution of the Ramsey model. The dashed line demarcates the size of the capital stock at which consumption shifts from an increasing path to a decreasing path. This is where the rate of impatience is lower or higher than the return on capital. The capital stock increases or decreases depending on whether consumption is lower or higher than the output, $f(k)$. The arrows indicate directions for the growth in consumption

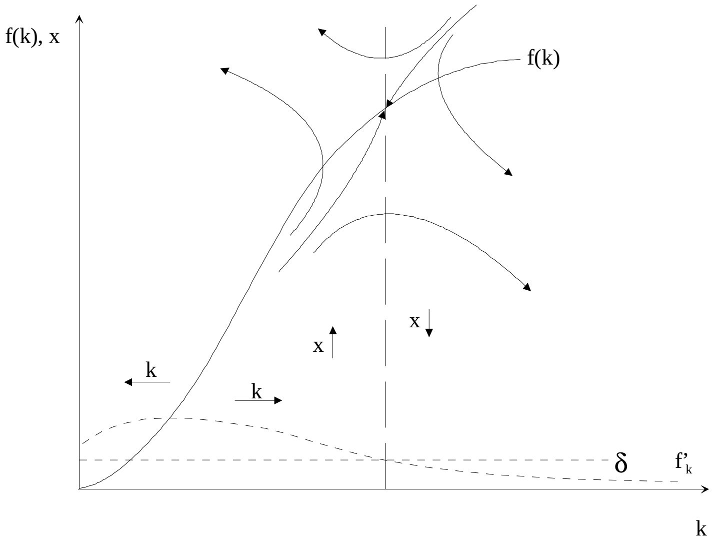


Figure 1: *Optimal economic growth in the Ramsey model*

and stock of capital.

With an infinite time horizon, the case where the capital stock approaches zero is clearly suboptimal. Along an optimal consumption path with declining capital stock, $x$ grows at an accelerating rate. Hence the system will sooner or later collapse. This cannot be optimal if the time horizon extends to infinity. Since the rate of return in that case exceeds the rate of impatience, the marginal utility of adding to the capital stock in order to postpone the collapse is positive. The case of a declining consumption rate is also suboptimal when the time horizon is infinite because $x$ approaches zero while $k$ grows to infinity. This cannot be optimal as long as the utility depends on consumption only.

We are therefore left with the long-term stationary solution which gives a constant consumption rate and a constant capital stock. This implies that the discount rate equals the rate of impatience. That is, in the very long term, the discount rate is implicitly chosen by the chosen rate of impatience. As noted above, the economy may never approach the stationary solution because of technological change, but this is not, strictly speaking, considered in this model.

The question is whether $\delta$ could approach zero. In the case of an unbounded production function ($f_k' > 0$), the stationary solution would then shift infinitely far out to the right in the diagram. We have concluded already that the optimal consumption path has to be non-decreasing. Assume that for a given time horizon, $T$, the optimal initial consumption is $x_0^T$. Extending the time horizon without adding to the initial capital stock, initial optimal consumption would have to be lowered. Extending the time horizon further to infinity means that initial consumption would have to be lowered to zero. But initial optimal consumption equal to zero is not of any interest. There has to be something wrong with the model. The most frequent solution is also the simplest, namely to accept a positive rate of impatience.

## 2.2 Including climate change

The previous section summarizes the basic elements that go into determining the discount rate: the social return on investments, the preferences for intergenerational equity, and the preferred rate of impatience. These issues are standard in discussions about the choice of discount rate to be applied on the benefits of measures to mitigate climate change. This is intuitively reasonable, because it is evident that if the issues are important for an assessment of the discount rate for investments in real capital, they are also important for the benefits of climate measures. In an economic perspective, it is important to attain intertemporal efficiency, which is the aim of equation (2). Measures to mitigate climate change should, therefore, be subject to the same constraints as social costs imposed to achieve other means.

On the other hand, the assessment above uses a very simple model, which does not include climate impacts or climate policy. The discount rate was linked directly to real capital, which is the only stock variable. Benefits or costs of investments were supposed to be added to production in accordance with the production function. The aim of climate measures is, however, to avoid damage from the increase in the concentrations of greenhouse gases. In other words, we have not yet told the whole story about the discounting of climate measures.

To include climate explicitly, we assume that the level of concentrations of greenhouse gases, $s_t$, affects productivity such that the production function is $f(k_t, s_t)$, and $f'_s < 0$. The problem can now be reformulated to

$$\max_{x,y} \int_0^\infty u(x_t)e^{-\delta t}dt,$$

subject to

$$\dot{k}_{1t} = f(k_{1t}, s_t) - x_t - y_t, \tag{3}$$

$$\dot{s} = \alpha(s_t) + \gamma f(k_{1t}, s_t) - g(y_t). \tag{4}$$

Equation (4) describes the development of concentrations of greenhouse gases in the atmosphere. The term $\alpha(s_t)$ represents the chemistry of the atmosphere, and $\alpha'_s < 0$ is usually called the natural rate of decay for greenhouse gases. This is a complex function, indeed, but we will not explore its properties further, except to assume that the derivative is negative. The coefficient $\gamma$ gives emissions per unit of production and $g(y_t)$ represents the productivity of abatement measures imposed at $t$. We assume that $g'_y > 0$ and $g''_{yy} < 0$.

The static equilibrium conditions for the problem are

$$q_1 = u'_x e^{-\delta t}, \tag{5}$$

$$q_s = -\frac{q_1}{g'_y}. \tag{6}$$

The variables $q_1$ and $q_s$ are the shadow prices of real capital and greenhouse gas concentrations, respectively. According to (5) the value of a unit of capital is equal to the discounted marginal utility of optimal consumption, which is the basic condition for the assessment of the discount rate, as pointed out by

Arrow [1]. Equation (6) implies that the value of greenhouse gas concentrations is equal to the marginal cost of abatement, which is expressed on the right hand side.

The discount rate is defined by the rate at which these prices change over time, that is $\rho_1 = \dot{q}_1/q_1$ and $\rho_s = \dot{q}_s/q_s$. They are expressed by means of the intertemporal equilibrium conditions. For real capital, we have

$$\rho_1 = (1 - \frac{\gamma}{g'_y})f'_k = \mu_x \frac{\dot{x}}{x} + \delta, \tag{7}$$

and for the concentrations of greenhouse gases,

$$\rho_s = \alpha'_s + (\gamma - g'_y)f'_s = -\mu_y \frac{\dot{y}}{y} + \rho_1. \tag{8}$$

The term $\mu_y = -yg''_{yy}/g'_y$ characterizes the curvature of the productivity of abatement costs.

We recognize the Ramsey rule in (7), except that the return on capital has been reduced by the rate $\gamma/g'_y$. The marginal cost of emission cut in optimum can be expressed as $1/g'_y$, and $\gamma/g'_y$ can therefore be interpreted as the abatement cost rate. The term on the left hand side is the social return on capital. It may be worth noting that the difference between the 'private' return, $f'_k$, and the social return depends on the productivity of abatement efforts and not directly on the damage. The importance of this result, which goes back to Arrow [1] and Arrow and Kurtz [?], is that one cannot, in principle, discount the benefits of climate measures only on the basis of the private return on capital and the social damage of climate change. One needs to know what the optimal abatement efforts are in order to find the discount rate.

The level of abatement is not determined by the discount rate for productive capital from (7), but from the intertemporal equilibrium condition for the concentrations of greenhouse gases, (8). The left hand side of (8) consists of the rate of decay, which is negative. Moreover there are two terms related to the marginal damage cost, $f'_s$. The first, $\gamma f'_s$, is emission reductions caused by higher damage related to climate change. This term arises from the assumption that emissions are dependent on the output, and not on the use of input factors. If emissions depend on the use of input factors, e.g. $k$ only, this term would be zero. The second term, $g'_y f'_s$ is the effect on damage from abatement, which is the shadow price of damage in terms of concentration units. The left hand side represents the net 'return' on the climate system. Its two components are the natural change in concentrations and the change caused by human activities. Whether the overall return is positive or negative in a given state of the world depends on the level of abatement efforts.

With two state variables and two controls, it is difficult to analyze the long-term development and stability conditions of the system. Pezzey and Withagen [19] showed that an economy based on the extraction of a non-renewable natural resources, and with a Cobb-Douglas technology with diminishing returns, the optimal consumption path is either single-peaked or stationary. Although one may justify an interpretation of the climate system as a non-renewable natural resource, the control of this resource, emissions of greenhouse gases, is much more complex than in Pezzey and Withagen's study, where the rate of change in the stock of natural resources is equal to the extraction. To study some

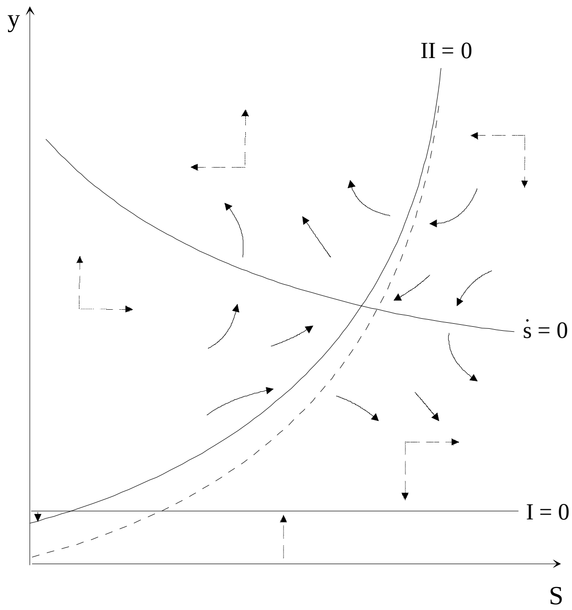


Figure 2: *Directions of optimal abatement paths for given levels of consumption and capital stock.*

properties of the solution of the present model, we therefore start with a partial investigation of the dynamics of each state variable.

The consumption path in the model used here is governed by the same conditions as in the Ramsey model. However, the output in the present model is applied for two purposes. The capital stock then declines if the sum of consumption and abatement exceeds output, $f(k_t, s_t)$. Considered in isolation, the stationary level of consumption therefore becomes lower. An increase in $s$ will reduce the marginal 'private' return on capital. The $f_k'-$curve in figure 1 thereby shifts downwards. The point for the stationary state thereby moves towards the left, with a lower capital stock and less left for consumption and abatement. In addition, the social return also depends on the level of $y$. Since $g_y'$ decreases as $y$ increases demarcation line for increasing and decreasing consumption slopes 'backwards' if $y$ increases, and an 'outwards' if $y$ decreases.

Similarly, we can investigate the relationship between abatement paths and the level of concentrations for unaltered consumption and capital. Inserting for $\rho_1$ from (7) in (8), and rearranging terms yields

$$-\alpha - (\gamma - g_y')(f_s' + \frac{1}{g_y'} f_k') = \mu_y \frac{\dot{y}}{y}. \tag{9}$$

The direction of the abatement path is governed by three terms. The rate of natural decay adjusts the abatement cost path upwards. The higher the rate, the more rapidly the abatement costs will rise. The demarcation of rising and declining abatement paths in the $(s, y)$ diagram is a constant, positive shift of the relationship $(\gamma - g_y')(f_s' + \frac{1}{g_y'} f_k') = 0$. The term $(\gamma - g_y')$ is independent of the level of concentrations, and represents a horizontal line in the $(s, y)$ diagram. As noted above, one may question the realism of including $\gamma$ at all in this expression. With a reasonable choice of parameter the limit for the sign of the parenthesis is found at very low levels of abatement. Hence, we assume that $(\gamma - g'_y) < 0$ below.

The last term is, on the other hand, crucial for the long-term direction of abatement costs. Recall that $1/g'_y$ is the marginal cost of abatement. Thus, $f'_k/g'_y$ measures the loss in income of replacing one unit of capital by abatement. We can call this term the opportunity cost of abatement. When the opportunity cost of abatement is equal to the marginal damage cost, the last term in (9) is zero. If the rate between the opportunity cost of abatement and the marginal damage cost is lower than $\alpha/(\gamma - g'_y)$ and $\gamma > g'_y$ in optimum, the abatement cost curve increases. Correspondingly, if, under the same condition, the rate between the opportunity cost and the damage cost is higher, we are on a declining abatement cost curve. Both $f'_s$ and $1/g'_y$ have positive derivatives. The relationship between $y$ and $s$ is therefore an increasing curve (II $= 0$) in the diagram. The $\dot{s} = 0$ curve follows directly from equation (4). It leads to a declining curve in the diagram, reflecting the fact that the higher the concentrations the more emissions can be accepted under stabilization of concentrations. The arrows in figure 2 indicate the direction of abatement in the $(s, y)$-diagram. The abatement costs increase in a large area, but, for constant $k$, there exists a steady state where $\dot{s} = \dot{y} = 0$.

A full illustration of the simultaneous evolution of the entire system cannot be shown in a phase diagram, because the variables represent four dimensions. However, the system can be represented in a three-dimensional phase diagram, where consumption and abatement are measured along one dimension. The lines drawn in figure 1 and figure 2 are then represented by planes in the $(k, s, (xy))$ diagram drawn in figure 3. The light grey plane is the production function. The dark plane is the demarcation between increasing and decreasing consumption. The dashed plane below the production function represents the stabilization of concentrations. This plane increases in $k$ and decreases in $s$. If $y$ is above this plane, $s$ increases. From (4) we see that this plane increases in the capital stock according to the emission coefficient $\gamma$. If this plane is above the production function, we cannot find a stationary solution. The third plane represents the demarcation of increasing and decreasing abatement. This plane increases in $s$, and corresponds to the increasing curve II=0 in figure 2. From equation (9) this plane decreases in $k$ because of the declining marginal productivity of capital.

The dark grey plane is the $\dot{x} = 0$ curve in figure 1. It cuts the other three planes at the curves where the economy is in a stationary state. As mentioned above, this curve bends somewhat 'backwards' from the intersection with the $\dot{y} = 0$ plane. Moreover, the stationary capital stock decreases as $s$ increases. The figure is an imagination of the $\dot{x} = 0$ plane cutting the three other planes. The two lines drawn across this plane, intersecting in point A, correspond to the lines crossing in figure 2. Point A therefore represents the stationary state of the entire system, where the vertical distance from the 'floor' to point A is abatement, and from point A to the production plane is consumption. For the stationary state to exist, the two planes drawn with dashed lines have to be below the production function in this point. For example, if A is 'behind' line B, a stationary solution does not exist.

The role of the discount rate in this model is quite different from the usual understanding of how discounting affects climate policy. In this model, the

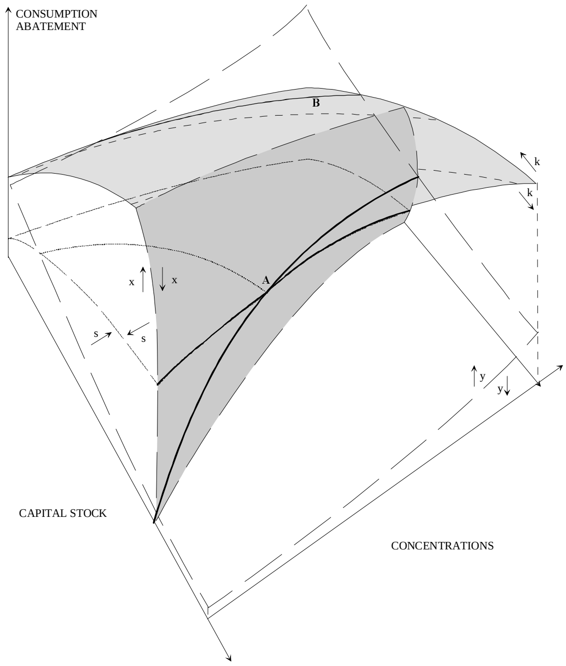


Figure 3: *3-D phase diagram for capital stock, concentrations and consumption/abatement*

discounting of both investments in productive capital and of benefits of climate policy can be characterized by two tasks. One is to identify the stationary state. The other is to allocate consumption and abatement over time to approach the stationary solution and attain the highest possible total welfare.

The stationary state is characterized by a discount rate for real capital equal to the rate of impatience, $\delta$. According to (8), $\rho_s$ and $\rho_1$ also coincides, since $\dot{y}/y = 0$. That is, if the benefits from abatement are discounted at the same rate as benefits from investments in real capital, it is implicitly assumed that we are in a stationary state, and the discount rate is equal to the rate of impatience. If not, abatement should be allocated over time to account for the change in the marginal productivity of abatement, which depends on the marginal abatement cost. If the abatement cost is sharply curved ($\mu_y$ is 'large'), the abatement profile might be very different from the abatement profile following a smoothly curved abatement cost function. The discount rate for climate measures therefore has a very different role to play than simply indicating whether or not present actions to mitigate climate change are beneficial. Rather, it can function as a means to allocate abatement over time so that a stationary solution can be approached in the most efficient way.

The stationary solution can be found by setting the time derivative of all the control and state variables equal to zero. Then the optimal stationary solution for the abatement policy follows:

$$(1 - \frac{\alpha}{\delta})\frac{f'_k}{g'_y} = f'_s \tag{10}$$

Again, we see the crucial role played by the relationship between the opportunity cost of abatement and damage costs. In a static world, one would expect that the marginal opportunity cost of abatement should be equal to the marginal damage cost. In a long-term dynamic perspective, this has to be adjusted for the two parameters that depend solely on the passing of time, the natural rate of decay and the rate of impatience. Since $\alpha < 0$, the optimal solution prescribes a lower marginal opportunity cost of abatement than marginal damage cost. The intuition behind this result is that abatement has to be initiated before damage is affected, and $\alpha/\delta$ is the rate at which the welfare is lost because of this inertia. Hence $\alpha/\delta$ is, in fact, the steady state discount rate for abatement measures. The lower the rate of impatience is, or the more quickly the nature stabilizes after, the larger the difference is between marginal opportunity cost and the marginal damage. For a two-percent rate of impatience, and a one-percent natural rate of decay, the marginal damage cost has to be 1.5 times the marginal opportunity cost of abatement. This is drastically lower than the ratio between total costs and total benefits in the standard discounting approach.

From equation (10), we see that the rate of impatience has a direct impact on the stationary solution. Beyond that, the discount rates for investments in real capital and benefits from mitigation of climate change are primarily subject to the curvatures of the welfare and abatement cost functions. This is important for how long it takes before the stationary state is approached, but it does not affect the long-term optimal combination of consumption, abatement and investment.

The discount rate is usually explained entirely with reference to the return on capital and intertemporal preferences, with the role of allocating consumption over time as means of maximizing total welfare. The model shows that this is only a part of the story. Alternatively, discounting can be characterized by two tasks: The first is to is to determine the level of abatement and consumption with reference either to the stationary state, or another sustainability criterion. Not surprisingly, this level is found by equating marginal costs and marginal benefits. What is not so well-known is that in a stationary state, or a sustainable world, the passage of time is accounted for by the ratio between the natural rate of decay and the rate of impatience. The second task is to allocate consumption and abatement over time from the initial state to the stationary, or sustainable, state. This task is identified by the curvatures of the welfare function and the abatement cost curve. The curvature of the welfare function does not affect the discount rate for benefits of abatement directly, and this is why the discount rates for investments in capital and abatement are different.

# 3 Can impatience be justified?

Although impatience implies that the distant future has a negligible impact on present decisions, it was shown above that an ambitious programme for the abatement of long-lived greenhouse gases may nevertheless be justified. On the other hand, the long-term solution of the system is clearly sensitive to the chosen rate of impatience, and it is difficult to find any empirical point of reference for this choice. Moreover, an ethical justification for treating generations differently is not provided just because we can show that it is optimal to invest in abatement for the advantage of our descendants. Even though the role of impatience is different from what most critics claim, one may, therefore, still question why the social welfare measure should reflect impatience.

Arguments in the defense of impatience can be divided into two categories. One category uses descriptive arguments. These may be related to the growth patterns of economies, and refer to the Ramsey model, which may abolish present consumption over an infinite time horizon if the rate of impatience is zero. Another descriptive defense of impatience refers to the 'obvious' fact that people are impatient in their consumption. The second category is of a technical nature, and is based on Diamond (1965) who showed that one cannot claim efficiency and intergenerational equity of the same welfare function. Since the properties of the Ramsey model was discussed in detail above, the discussion in this section is limited to the reference of individual impatience and the issue of efficiency and intergenerational equity.

## 3.1 Individual impatience and social welfare

Böhm-Bawerk [3] provided the first formal treatise on the implications of impatience in his famous exposition of why capital creates value. He argued that there are three grounds for a positive interest. First, people expect income to increase in the future; second, people are impatient; and third, a delay of consumption means that money can be invested and yield a return in the future. We may now roughly identify these grounds by the three components in the Ramsey rule in equation (2), but Böhm-Bawerk's three grounds has been debated ever since they were presented with the aim of finding the core driving force behind the interest rate. von Mises [25], among others, claimed that impatience plays this role alone. As Böhm-Bawerk himself admitted, one may find people whose preferences are not impatient, but it applies as a general assumption, he argued. Today, there seems to be a general consensus about this view, so let us accept it.

Important for the discussion here, however, is the attempt to draw an immediate link between individual preferences and social phenomena. The common view among most economists is that as long as individuals are impatient, the social welfare function should also reflect impatient preferences (Fankhauser [10]), because the social welfare function ought be based on individual preferences. This conclusion is erroneous, however. If built upon individual preferences, social preferences are aggregated over individual preferences in space *and* time. Social welfare therefore represents different people, or cohorts, at different points in time. The impatience of an individual, or a cohort, is not necessarily reflected in the aggregate because the cohorts change place within the aggregate from one point in time to the other. In such a setting, van der Burg [24] shows that to end up with a positive social rate of impatience, individuals will have to have an infinitely high rate of impatience over small time intervals.

To see how individual preferences affect the social welfare function, assume that the social welfare at $t$ is a sum of the utility of all individuals living at $t$. Social impatience can be defined as the rate of change in total welfare provided that there are no changes in consumption level and size of each cohort. For simplicity, I assume that individual utility is linear in consumption. Non-linear utility does not change the conclusions below.

Denote by $u_i$ the utility of an individual, here defined as a cohort of people born in year $i$. The total utility of this cohort over its lifetime is expressed in the usual terms:

$$u_i = \sum_{t=i}^{T+i} x_{it}\delta^i(t-i).$$

The term $T$ denotes the lifetime of each cohort, which means that each cohort is expected to live equally long. The utility of consumption for cohort $i$ at time $t$ is denoted $x_{it}$, and $\delta^i(t-i)$ is cohort $i$'s rate of impatience, which reflects the cohort's impatience in consumption. I will assume that this rate is constant for each cohort. Total welfare is the aggregate over the utility of each cohort for the total period $(1, T+n)$, that is

$$w_t = \sum_{i=i}^{n} u_i = \sum_{i=1}^{n}\sum_{t=i}^{T+i} x_{it}\delta^i(t-i) = \sum_{t=1}^{T+n}\sum_{i=1}^{n} x_{it}\delta^i(t-i).$$

The last term, $\sum_{i=1}^{n} x_{it}\delta^i(t-i)$, is the utility of all cohorts at a given point in time, $t$. Hence, if $t > i+T$, the cohort is dead at $t$, which means that $x_{it} = 0$. If $t > i$ the cohort is not yet born, and also then, $x_{it} = 0$. Hence, we can write the aggregated welfare as

$$w_t = \sum_{t=1}^{T+n} \sum_{t-T<i<t}^{n} x_{it}\delta^i(t-i).$$

In other words, for all cohorts born a lifetime or less before year $t$, and for all cohorts born before $t$, the contribution to welfare is positive. For all other cohorts, the contribution is zero. The total welfare in a given year, $s$, is

$$w_s = \sum_{s-T<i<s}^{n} x_{is}\delta^i(s-i), \tag{11}$$

and, correspondingly,

$$w_{s+1} = \sum_{s+1-T<i<s+1}^{n} x_{i,s+1}\delta^i(s+1-i). \tag{12}$$

Recall that the impatience term, $\delta^i$, relates to each cohort $i$. This means that, for example, the second term in the sum in (11), $\delta^{s-(T+1)}(1)$, differs from the second term in the sum in (12), $\delta^{s+1-(T+1)}(1)$, only if those who were born at $s-(T+1)$ have a different rate of impatience than those who were born at $s+1-(T+1)$.

Following Koopmans (1960), impatience implies that an interchange of two consumption bundles within a consumption program $(x_0, ..., x_t, x_{t+1}, ...)$ increases total welfare if the most preferred bundle is advanced at the expense of a less preferred bundle. That is

$$w(x_0, ..., x_t, x_{t+1}, ...) > w(x_0, ..., x_{t+1}, x_t, ...) \text{ if and only if } x_t \succ x_{t+1}.$$

The only concern here is whether the passing of time has an impact on the total welfare of the society. We can therefore consider only constant consumption programs. That is, we have

$$w_s = x \sum_{s-T<i<s}^{n} \delta^i (s-i)$$
$$w_{s+1} = x \sum_{s+1-T<i<s+1}^{n} \delta^i (s+1-i)$$

Because the discount rates relates to the cohorts, as stated above, impatience, as defined by Koopmans, is present only if the individual cohort has different rates of impatience. Generally it is therefore not true that impatience must be reflected in the social welfare function if individuals exhibit impatience. In fact, to account for individual impatience, the social welfare function could equally well exhibit expressed patience as expressed impatience. If people born in an early period have a higher rate of impatience than people born later, the social welfare aggregate should, in fact, reflect patience. During a growth process, one might even argue that patience is more realistic than the opposite, since many would argue that low-income groups are more impatient than high-income groups (see section 3.2.2).

## 3.2 Intertemporal preference structures

Nearly all studies of economic development, and of climate policy in particular, apply an additive welfare function with an exogenously chosen rate of impatience, similar to the welfare function in section 2. The intertemporal properties of this function are hampered with certain drawbacks. For example, the relationship between consumption and welfare at two time points varies only as a result of the passing of time. In principle, an increase of consumption is therefore treated in the same manner as a reduction. This property is unsatisfactory, in particular in studies of climate change, where the possibility for severe damage and declining consumption is one possibility for which preferences need to be stated. The direction in which consumption changes is probably important for the evaluation of consumption programs.

### 3.2.1 Impatience and efficiency

The standard atemporal utility function defines preferences over consumption bundles at given points in time. Intertemporal preferences can instead be defined over consumption programs. In Koopmans [15] and Koopmans et al. [16] it is shown that with a seemingly innocent extension of the definition of the continuity property of consumption bundles in the atemporal utility function, one may define utility functions over infinite consumption programs. Koopmans sets up two requirements to an ordinal intertemporal utility function. The first relates to the stability of preferences and the second to preferences not changing as a result of the passing of time. In addition, utility functions need be differentiable, so a definition of what is meant by continuity of consumption programs is required. Below, the building blocks of the intertemporal utility function is presented. For proofs, see Koopmans [15].

The first requirement is that the underlying preferences are stable. Define by $x_t$, consumption at $t$, and ${}_{t+1}x = x_{t+1}, x_{t+2}, \ldots$ consumption from $t+1$

and beyond. Then, the consumption program ${}_1x$ can be written as $(x_1, {}_2x)$. Underlying stability of preferences means that

$$\begin{aligned} U(x_1, {}_2x) &\geq U(x'_1, {}_2x) \Rightarrow U(x_1, {}_2x') \geq U(x'_1, {}_2x'), \\ U(x_1, {}_2x) &\geq U(x_1, {}_2x') \Rightarrow U(x'_1, {}_2x) \geq U(x'_1, {}_2x'). \end{aligned}$$

The second line, for example, says that the preference ordering within a class of programs ${}_1x$ with a first–period consumption vector $x_1$ does not depend on what that vector $x_1$ is. Consequently, there exists an aggregator $V$ such that $U(x_1, {}_2x) = V(u(x_1), U_2({}_2x))$.

We also require that preferences are independent of the passing of time. Hence, for some $x_1$ and all ${}_2x, {}_2x'$

$$U(x_1, {}_2x) \geq U(x_1, {}_2x') \text{ if and only if } U({}_2x) \geq U({}_2x').$$

Since the sensitivity of changes in ${}_2x$ to total utility is independent on $x_1$, it is possible to write the utility of ${}_2x$ as a linear transformation of $U$, that is, $U_2({}_2x) = G(U({}_2x))$. Hence, we can write the utility of a consumption program as the recursion

$$U({}_1x) = V(u(x_1), U({}_2x)). \tag{13}$$

According to the Weierstrass theorem, an optimal program exists if the set over which the utility function is defined is compact. In other words, there has to be a continuum of feasible consumption paths, and we therefore need to know what is meant by the continuity of a consumption programme. Following Koopmans [15], let the distance between two consumption programmes, $x$ and $y$, be defined as

$$\Delta = \sup_{x_t} |x_t - y_t|, \forall t$$

Continuity means that there exist programs such that $\Delta$ can be made as small as we want. However, the need for continuity restricts the possibilities for including other properties on the intertemporal welfare function. In particular, Diamond [8] show that we cannot require that this utility function prescribe intergenerational equity. To see this, note that equity implies that an interchange of consumption bundles do not affect total utility, i.e.,

$$U(x_1, ..., x_{t-1}, x_t, ...) = U(x_1, ..., x_t, x_{t-1}, ...) \tag{14}$$

Assume, first, that $x_{t-1} < x_t$. The distance between these two programmes is clearly $\Delta = x_t - x_{t-1}$. Continuity implies that there exists a program $(x_1, ..., x'_{t-1}, x'_t, ...)$ such that $0 < \Delta' = \sup(x'_{t-1} - x_{t-1}, x'_t - x_1) < \Delta$.

Define a set of programmes for which $x'_{t-1} \geq x_{t-1}$ and $x'_t \geq x_1$, where inequality applies for at least one of them. Clearly, this set is non-empty if the set of feasible consumption paths is continuous. All consumption bundles in this set are either equal to or larger than the original programme. In order to be efficient, the utility function therefore ought to give a higher utility for this set of programmes. But this is a contradiction to (14), which gives the same utility for two programmes with distance $\Delta$. A similar argument applies if $x_{t-1} > x_t$. Hence, we cannot require efficiency and equity of the same utility function.

To solve this problem, one may either discard equity, or limit the scope of the study to a finite time horizon. Koopmans' contribution was to show that impatience is embedded in the intertemporal utility function with an infinite time horizon. Roughly speaking, impatience introduces a ranking of consumption programs according to the timing of consumption, which closes the efficiency gap between two programs with equal total utility. A similar effect is obtained by the choice of terminal conditions with a finite time horizon. Instead of explicitly ranking the timing of consumption, a terminal condition can, however, be related to a broader system, such as a growing economy.

This way of thinking can be considered the background for some studies in recent years, where the continuity of the utility function *per se* has been questioned. As noted by Epstein [9], continuity of the utility function is a sufficient, but not necessary condition for the optimal solution to exist. Following this idea, Asheim et al. [2] show that one may combine efficiency and equity if, by efficiency, one requires that the economy is intertemporally efficient.

### 3.2.2 Optimal policy with endogenous impatience

Because of the need to define what is meant by the continuity of consumption programs, we do not automatically avoid ethical problems using intertemporal preference structures. On the other hand, stating preferences over consumption programs rather than atemporal consumption bundles allows for a more realistic representation of preferences that are worth a closer look, especially in cases of very long-term issues. In particular, a statement of preferences over consumption programs allows for studies of how habits may affect long-term economic development (see e.g. Ryder and Heal [21] or Shi and Epstein [22]). However, there are to date relatively few studies of optimal policy with recursive welfare, and the availability of functions are scarce. In this section, some properties of the most frequent choice of recursive welfare function, developed by Uzawa [23], are discussed. Moreover, the model developed in section 2 is solved with application of Uzawa's function.

Uzawa's function is written as

$$W({}_t x) = \int_t^{\infty} w(x_\tau) e^{-\int_t^\tau \delta(x_s) ds} d\tau, \tag{15}$$

The recursive property of this function is shown in the appendix. The standard welfare function can be regarded as a special case of (15), that is, when the discount factor is independent of the consumption level. Then, $-\int_t^\tau \delta(x_s)ds = -\bar{\delta}t$, where $\bar{\delta}$ is a constant. The distinction between the traditional intertemporal welfare function and the recursive welfare function therefore depends on the properties of $\delta(x_t)$. There are two issues of importance.

First, we need to make an assumption as to whether impatience increases or decreases with consumption, that is, whether $\delta_x'$ is positive or negative. Blanchard and Fisher [4] assume by intuition that the richer people get, the less impatient they become. This is to some extent supported if people save more as they get wealthier, which is likely. To apply (15) in a model for optimal growth then means that the rich grow richer over time, which will lead to an unstable solution. Hence, we have to assume $\delta_x' > 0$. This is the reason why Blanchard and Fisher discommend a general use of (15).

But discarding an intertemporal welfare function for the reason that it leads to a counterintuitive solution is also problematic. If it is impossible to accept that the world approaches instability in the long run, one must believe either that the rate of impatience is, in fact, constant, or that there are no welfare functions available that describes intertemporal welfare in a satisfactory manner. Neither of these alternatives are very attractive. Another possibility might that there is something wrong with our intuition. One reason why the intertemporal welfare function might exhibit increasing impatience in consumption is habit formation. If $\delta'_x > 0$, (15) reflects an asymmetry between the change of welfare following an increase and a decrease in consumption. This is because a drop in consumption reduces the rate of impatience, and thereby 'dampens' the effect of the fact that consumption occurs at a later date. Note that this representation of habit does not mean that a postponement is regarded disadvantageous if consumption declines, only that it takes away some of the attractiveness in postponing consumption. For this reason, I find it worthwhile to study optimal policy with recursive welfare more closely.

The second assumption about the shape of $\delta(x_t)$ is that $\delta''_{xx}/\delta'_x > w''_{xx}/w'_x$. If not, the marginal instantaneous welfare from increasing consumption will fall more rapidly than the marginal reduction in impatience. Over time, when consumption grows 'large', the reduction in total welfare due to the passing of time will become stronger than the increase due to consumption growth. As a consequence, total welfare may decrease on an increasing consumption programme. To avoid this, it is assumed that the impatience function is more strongly curved than the instantaneous utility function.

Optimal climate policy is found by maximising (15), subject to (3) and (4). The atemporal and the intertemporal conditions for optimum are developed in the appendix. Here, the discussion is limited to comparing these conditions with the solution of the traditional approach in section 2. Define the optimal solution by:

$$V_t = \max W({}_t x)$$

The atemporal optimum conditions are

$$V'_k = w'_x - \delta'_x V(k, s, t), \tag{16}$$

$$V'_S = -\frac{V'_k}{g'_y}. \tag{17}$$

The terms $V'_k$ and $V'_s$ denote the values of capital and the damage indicator $s$ in optimum. They represent shadow prices, but differ from $q_k$ and $q_s$, respectively, because the point of reference for the evaluation is $t$. Hence, $V'_i$ corresponds to $q_i e^{\delta t}$.

Denote the elasticity of consumption in optimum by $el_x W^* = \frac{\partial(w+\delta V)}{\partial x}\frac{x}{w+\delta V}$ and the elasticity of impatience by $el_x\delta = \frac{\partial\delta}{\partial x}\frac{x}{\delta}$. Moreover, let $\sigma(x)$ be a measure for the curvature of the welfare of instantaneous consumption, similar to the interpretation of $\mu_x$, but applied to the recursive welfare function $W({}_t x)$ instead of $u(x_t)$. As in traditional welfare function, $\sigma(x)$ may express the preferences for intertemporal equity. Then, the intertemporal optimum conditions can be written as:

$$\rho_1 = \sigma(x)\frac{\dot{x}}{x} + \delta(x)(1 + \frac{el_x\delta}{el_x W^*}) \tag{18}$$

$$\delta - \alpha + f_S'(g_y' - \gamma) = -\upsilon_1 - \mu_y \frac{\dot{y}}{y} \tag{19}$$

We note, first, that the conditions related to the concentrations of greenhouse gases and abatement are exactly the same as in the traditional model. Hence, climate policy is affected by the introduction of recursive welfare only indirectly by the new conditions for the consumption-savings decision. The static condition (16) contributes to lower the shadow price, since $\delta_x' > 0$. This can be considered as a reduction in the supply of savings, and leads, in isolation, to a lower growth rate and higher initial consumption. The reason is simply that the marginal welfare of increased consumption is reduced as a result of a higher rate of impatience.

The intertemporal condition is changed to reflect the relative elasticities of impatience and welfare. Since $\delta_x' > 0$, there are two reasons why endogenous impatience contributes to increasing the discount rate. This is partly because impatience increases by increasing consumption. In addition, when impatience increases, the importance of impatience also increases, which is why the last term in the parenthesis is included. This latter effect declines over time, however, because we have assumed that $\delta_{xx}''/\delta_x' > w_{xx}''/w_x'$.

As discussed by Chang [5], this change in the rate of impatience can be illustrated by a backward sloping intersection point for $f_k'$ and $\delta(x)$ in Figure 1. If we use the present day rate of impatience as a reference for comparing long-term solutions of traditional and recursive welfare functions, this would lead to a steady state solution at a lower level of production and consumption. As noted, the effect on climate policy would follow indirectly from this new situation. For the steady state solution, we can see the effect by considering figure 3. Recursive welfare means that the dark plane slopes backwards, pushing point A 'inwards' along the dotted curve that represents the intersection of the two planes. Without a formal proof, we see that this leads to lower level of capital, and lower concentrations. The steady-state level of abatement will be drawn upwards as the capital stock is reduced, but downwards because the concentrations are lower. Which effect is the strongest cannot be concluded without further specifications of the model.

# 4 Conclusions

Economists have had trouble explaining to other people why benefits of climate measures ought to be discounted because it seems clear to anyone that discounting will make the bulk of benefits from climate policies disappear. Few seem to be convinced by the usual assertion that one should compare the return of any investment with the return of the best available alternative. This is why it is useful to go more deeply into the determinants of the discount rate, and ask whether there is something with climate change that may spur a reorientation with respect to how discounting benefits of climate measures is explained.

The main answer to this last question is yes, and this has been known for quite some time (see e.g. Haavelmo [11]). Climate measures (as well as many other environmental policy measures) aim at controlling the development of a stock, the concentrations of greenhouse gases. Most other investments aim at controlling a different stock, the stock of productive capital. The concentrations of greenhouse gases develop at a different speed than the capital stock. As a result, also the value of the concentrations evolves differently from the value of real capital. This implies, taking into consideration how discount rates are defined, that the discount rate for benefits of climate measures differs, in general, from the discount rate for other investments.

The major objective of discounting is to provide the right incentives for steering the concentrations of greenhouse gases and the stock of real capital in such a way that the total welfare is maximized. In such a perspective, the different rates of discount for investments in real capital and abatement of greenhouse gases can be explained by how consumption and abatement affect total welfare, directly or indirectly. The direct welfare gain from consumption growth can be characterized by the curvature of the welfare function. This reflects peoples' evaluation of present consumption versus future wealth, and is a well-established result in economics. Similarly, the indirect welfare gain from rising abatement is characterized by the 'productivity' of abatement costs, that is, how much emissions are cut by abatement at different stages of the abatement level. The different discount rates can therefore be explained by the fact that the curvatures of the welfare function and the abatement cost function are different. Put in a simpler way, abatement and other investments are discounted differently because it is advantageous to follow different time profiles in order to reach the overall goal of maximum total welfare over time.

Different discount rates matter, therefore, only to the extent that consumption and abatement are actually increasing (or changing). The level of the discount rates, and thereby the level of consumption and abatement over time, depends on how people evaluate the advantage of having a given amount consumer goods today compared with having then provided some time in the future. The assessment of a level for abatement and consumption can be addressed by the stationary state solution. Then, it turns out that the intuitive rule for abatement applies, namely that the marginal cost of abatement should be related directly to the marginal benefits in terms of higher productivity. However, the passing of time affects the value of these benefits for two reasons. First, nature itself reduces the concentrations by a certain rate, $\alpha$, and the welfare of the productivity yield is reduced by the rate of impatience if we compare with the time at which abatement has to take place. This explains the adjusted optimum criterion for abatement in steady state, shown in equation (10). Therefore, a high discount rate does not necessarily mean that future benefits are neglected. In fact, in most cases it will not reflect such view, but rather a consideration of *when* major abatement should be initiated.

On the other hand, discounting is controversial from an ethical point of view because it reflects an acceptance of the assumption that generations should not be treated equally. As we have seen, there is no ethical justification for this, not even if we insist that welfare functions should be based on preferences of impatient people. The assumption of impatience is more a question of technical convenience. One may, however, question the standard assumption of a constant rate of impatience, especially since certain time-specific properties of preferences, in particular habit formation, are disregarded. The inclusion of an endogenous rate of impatience, reflecting habit formation, changes optimal policy slightly. The stationary level to which the stock of real capital and concentrations of greenhouse gases rises goes down, but we cannot say for sure whether the level of abatement increases or decreases.

It should be added, finally, that to include habit formation may reflect preferences in a more realistic way, but it does not solve the ethical problem of intergenerational equity. Rather, the opposite is the case. Habit formation implies an acceptance of the claim that people who are rich have the right to remain so, a so-called grandfathering principle. This solution of the problem of burden sharing can also be traced in the Kyoto Protocol, where countries are committed to reducing emissions in proportion to present levels. Still, there are reasons to request further development in this area.

# A Optimal policy with recursive welfare

A function that is consistent with Koopman's [15] definition of intertemporal utility was developed by Uzawa [23], and can be written in the form,

$$W({}_t x) = \int_t^\infty w(x_\tau) e^{-\int_t^\tau \delta(x_s) ds} d\tau, \tag{20}$$

where ${}_t x \equiv \lim_{\Delta t \to 0}(x_t, x_{t+\Delta t}, x_{t+2\Delta t}, ...)$ is the consumption programme starting from $t$. To simplify notation below, define the discount factor

$$D(x; t, \tau) = e^{-\int_t^\tau \delta(x_s) ds} \tag{21}$$

The discount factor can be separated into time intervals,

$$D(x; t, \tau) = e^{-\int_t^{t+\Delta t} \delta(x_s) ds - \int_{t+\Delta t}^\tau \delta(x_s) ds} = e^{-\int_t^{t+\Delta t} \delta(x_s) ds} e^{-\int_{t+\Delta t}^\tau \delta(x_s) ds}.$$

Hence, $D(x; t, \tau) = D(x; t, t + \Delta t) D(x; t + \Delta t, \tau)$. Note that $D(x; t, t + \Delta t) = D(x_t)$ when $\Delta t \to 0$. We may therefore write the welfare function as the recursion

$$\lim_{\Delta t \to 0} W({}_t x) = w(x_t) + D(x_t) W({}_{t+\Delta t} x). \tag{22}$$

Now, consider the optimization problem in section 3:

$$\max_{x,y} W({}_t x),$$

s.t.

$$\begin{aligned} dk_t &= [f(k_t, S_t) - x_t - y_t] dt, \\ dS_t &= [\alpha S_t + \gamma f(k_t, S_t) - g(y_t)] dt. \end{aligned}$$

First, we form the value function

$$V(k, S, t) = \max_{x,y} \{ \int_t^T w(x_s) D(x_s; t, T) ds \},$$

We assume that $V(k; S, t)$ is continuous and twice differentiable over any admissible path. Because of the separability properties assumed above, we can develop the value function $V(k, S, t)$ to

$$\begin{aligned} &\max_{x,y} \{ \int_t^{t+\Delta t} w(x_s) D(x_s; t, t + \Delta t) ds + \int_{t+\Delta t}^T w(x_s) D(x_s; 0, t + \Delta t) D(x_s; t + \Delta t, T) ds \} \\ &\max_{x,y} \{ \int_t^{t+\Delta t} w(x_s) D(x_s; t, t + \Delta t) ds + D(x_t; t, t + \Delta t) \int_{t+\Delta t}^T w(x_s) D(x_s; t + \Delta t, T) ds \} \\ &\max_{x,y} \{ \int_0^{\Delta t} w(x_s) D(x_s; t, t + \Delta t) ds + D(x_t; t, t + \Delta t) V(k + \Delta k, S + \Delta S, t + \Delta t) \}, \end{aligned} \tag{23}$$

which is the fundamental recurrence relation. Subtracting $V(k, S.t)$ on both sides, and inserting a 'neutral' element $V(k + \Delta k, S + \Delta S, t + \Delta t)$ gives

$$\begin{aligned} 0 \;=\; &\max_{x,y} \{ \int_0^{\Delta t} w(x_s) D(x_0; 0, \Delta t) ds + D(x_0; 0, \Delta t) V(k + \Delta k, S + \Delta S, t + \Delta t) \\ &- V(k + \Delta k, S + \Delta S, t + \Delta t) + V(k + \Delta k, S + \Delta S, t + \Delta t) - V(k, S, t) \}. \end{aligned}$$

We now let $\Delta t \to 0$. The integral on the right hand side then approaches $w(x)dt$. This is readily understood if we take the derivative of this term w.r.t. $\Delta t$ :

$$\frac{\partial}{\partial \Delta t}[\int_t^{t+\Delta t} w(x_s)D(x_t;t,t+\Delta t)ds = w(x)D(x_t;t,t+\Delta t) + \int_t^{t+\Delta t} \frac{\partial D}{\partial \Delta t}ds.$$

The first term expresses the discounted welfare of consumption added (subtracted) because of a longer (shorter) period. The second term is the lowered (increased) discount term of remaining consumption. This term is obviously 0. Hence,

$$\lim_{\Delta t \to 0} \int_t^{t+\Delta t} w(x_s)D(x_t;t,t+\Delta t)ds = w(x)dt. \tag{24}$$

The next two terms can be written $(D(x_t;t,t+\Delta t)-1)V(k+\Delta k, S+\Delta S, t+\Delta t)$. Since $D(x_t;t,t) = 1$, the first set of parenthesis is the change of $D$ at point $t$, that is, the discount rate in a given point of time, $\delta(x_t)$. Hence,

$$\begin{aligned} & \lim_{\Delta t \to 0}(D(x_t;t,t+\Delta t) - 1)V(k+\Delta k, S+\Delta S, t+\Delta t) \\ = \quad & D'V(k,S,t) = \delta(x)V(k,S,t). \end{aligned} \tag{25}$$

Finally, the two last terms give the change in the value function over time. Since we assume that $V(k,S,t)$ is differentiable, we have, by definition

$$\lim_{t \to 0} V(k+\Delta k, S+\Delta S, t+\Delta t) - V(k,S,t) = V_k'dk + V_S'dS + V_t'dt. \tag{26}$$

Inserting from (24), (25) and (26) into (23), and dividing through by $dt$ we get the Bellman equation:

$$0 = \max_{x,y}\{w(x) - \delta(x)V(k,S,t) + V_t' + V_k'\dot{k} + V_S'\dot{S}\}. \tag{27}$$

Replacing for $\dot{k}$ and $\dot{S}$, we obtain

$$\begin{aligned} 0 \quad = \quad & \max_{x,y}\{w(x) - \delta(x)V(k,S,t) + V_t' \\ & +V_k'[f(k,S) - x - y] + V_S'[\alpha S + \gamma f(k,S) - g(y)]\}. \end{aligned} \tag{28}$$

The first order conditions for optimum are found directly from this maximisation:

$$V_k' \quad = \quad w_x' - \delta_x' V(k,S,t), \tag{29}$$

$$V_S' \quad = \quad -\frac{V_k'}{g_y'}. \tag{30}$$

Equation (30) is recognized from the static condition for the shadow price of the damage indicator in the problem studied in section 2.2, except that the derivatives of the value function have replaced the shadow price $q_S$. This makes sense, because $V_S'$ is the marginal value of $S$, which, by definition, is the shadow price. Similarly $V_k'$ corresponds to $q_1$ in section 2. Note however, that $V_i'$ ($i = k, S$) expresses the shadow price in terms of present value. In other words, we can compare the impact of the alternative welfare functions on the static condition directly by comparing the shadow prices of capital, where $V'_k(k, S, t)$ compares to $q_1 e^{-\delta t}$.

The intertemporal optimum conditions can be traced from the change in the value of a marginal shift of the whole system, which is characterized by changes in the shadow prices resulting from the optimal solution. This change can be expressed by an expansion of the shadow prices:

$$
\begin{aligned}
dV'_k &= V''_{kt}dt + V''_{kk}dk + V''_{kS}dS, && (31)\\
dV'_S &= V''_{St}dt + V''_{Sk}dk + V''_{SS}dS. && (32)
\end{aligned}
$$

For the optimality condition to be sustained for changes in the state variables, the derivatives on both sides of the Bellman equation with respect to $k$ and $S$ will, moreover, have to be unaltered, that is,

$$
\begin{aligned}
0 &= w'_x x'_k - \delta'_x x'_k V - \delta(x)V'_k + V''_{tk} + V''_{kk}\dot{k}\\
&\quad + V'_k[f'_k - x'_k - y'_k) + V''_{Sk}\dot{S} + V'_S[\gamma f'_k - g'_y y'_k]\\
0 &= w'_x x'_S - \delta'_x x'_S V - \delta(x)V'_S + V''_{tS} + V''_{kS}\dot{k}\\
&\quad + V'_k[f'_S - x'_S - y'_S) + V''_{SS}\dot{S} + V'_S[\alpha + \gamma f'_S - g'_S y'_S].
\end{aligned}
$$

from which expressions for $V'_{kt}$ and $V'_{St}$ can be found. By inserting these into (31) and (32), eliminating terms and dividing through by $dt$ we obtain

$$
\begin{aligned}
\frac{\frac{d}{dt}\{V'_k\}}{V'_k} &= \upsilon_1 = -\delta + f'_k(1 - \frac{\gamma}{g'_y}), && (33)\\
\frac{\frac{d}{dt}\{V'_S\}}{V'_S} &= \upsilon_S = \delta - \alpha + f'_S(g'_y - \gamma), && (34)
\end{aligned}
$$

which are similar to the social returns in the basic model with a constant rate of impatience, except that $\upsilon_i$ reflects evaluations at present, while $\rho_i$ reflects evaluations at the current point in time.

Intertemporal equilibrium is found by the expressions for the evolution of the first order conditions, (29) and (30):

$$
\begin{aligned}
\frac{d}{dt}\{V'_k\} &= w''_{xx}\dot{x} - \delta''_{xx}\dot{x} - \delta'_x V'_t, && (35)\\
\frac{d}{dt}\{V'_S\} &= -\frac{\frac{d}{dt}\{V'_k\}g'_y - V'_k g''_{yy}\dot{y}}{(g'_y)^2}. && (36)
\end{aligned}
$$

Recall that (36) is similar to the rate of change for the shadow price of $S$ in the basic model. Hence, the discount rate can be written as

$$
\frac{\frac{d}{dt}\{V'_S\}}{V'_S} = \upsilon_S = -\upsilon_1 - \mu_y \frac{\dot{y}}{y}, \qquad (37)
$$

which, together with (34) gives the intertemporal equilibrium condition for the damage indicator.

To solve for the evolution of $V'_k$ in (35), we have to find an expression for $V'_t$. From the definition of the value function we can find the derivative directly, considering $V$ as an integral depending on the parameters. Then

$$\begin{aligned} V'_t &= -w(x) + \int_t^\infty w(x_\tau) \frac{\partial}{\partial t} \{e^{\int_t^\tau \delta(x_s) ds}\} d\tau \\ &= -w(x) + \int_t^\infty w(x_\tau) e^{\int_t^\tau \delta(x_s) ds} (-\delta(x_t)) d\tau \\ &= -[w(x_t) + \delta(x) V(k, S, t)]. \end{aligned}$$

Insert in (35) we get

$$\frac{\frac{d}{dt}\{V'_k\}}{V'_k} = \upsilon_1 = x \frac{w''_{xx} - \delta''_{xx} V}{w'_x - \delta'_x V} \frac{\dot{x}}{x} + \delta'_x \frac{w(x) - \delta(x) V}{w'_x - \delta'_x V} \tag{38}$$

The first term on the right hand side may be considered a characterization of the taste (see Chang [5]), expressing the saturation of instantaneous welfare. Denote this term by $\sigma(x)$. Note that in the case of constant $\delta$ this expresses the curvature of the welfare function, and equals the intertemporal elasticity of substitution, denoted $\mu_x$ earlier. Note also that in this case the intertemporal optimum condition reduces to the familiar Ramsey rule, $\upsilon_1 = \mu_x \dot{x}/x$. When impatience is dependent on consumption, we can express the intertemporal equilibrium condition by means of the elasticities of total welfare $W^* = (w + \delta V)$ and the impatience function $\delta(x)$. Denote by $el_x W^* = \frac{\partial(w+\delta V)}{\partial x} \frac{x}{w+\delta V}$ and $el_x \delta = \frac{\partial \delta}{\partial x} \frac{x}{\delta}$. Then,

$$\rho_1 = \sigma(x) \frac{\dot{x}}{x} + \delta(x)(1 + \frac{el_x \delta}{el_x W^*}) \tag{39}$$

# References


[1] Arrow, K. J. "Discounting and public investment criteria." *Water Research* edited by A:V. Kneese and S.C. Smith, Baltimore, Maryland: John Hopkins University Press for Resources for the Future, 1966.

[2] Asheim, G., et al. *Justifying Sustainability*. mimeo, Univ. of Oslo, Univ of Regenburg and Norwegian School of Economics and Business Administration, 1999.

[3] Böhm-Bawerk, E. von. *THe positive theory of capital*. London: Macmillan, 1891.

[4] Blanchard, O. J. and S. Fischer. *Lectures in macroeconomics*. Cambridge, Mass.: The MIT Press, 1989.

[5] Chang, F.R. "Optimal Growth and Recursive Utility: Phase Diagram Analysis," *Journal of Opimization Theory and Practice*, *80*(3) (1994).

[6] Cline, W.R. *The Economics of Global Warming*. Institute for International Economics. Washington, 1992.

[7] Dasgupta, P., et al. "Intergenerational equity, social discount rates and global warming." *Discounting and intergenerational equity* edited by P. Portney and J. Weyant, Resources for the Future, Wash. DC., 1999.

[8] Diamond, P. "The evaluation of infinite utility streams," *Econometrica*, *33*:170–177 (1965).

[9] Epstein, L.G. "Impatience." *The New Palgrave. A Dictionary of Economics. Vol. 2* edited by Milgate M. Newman P. Eatwell, J., 720–723, The Macmillan Press Ltd, London, 1987.

[10] Fankhauser, S. *Valuing Climate Change. The Economics of the Greenhouse*. Earthscan Publishers Ltd., London, 1995.

[11] Haavelmo, T. "Forurensingsproblemet fra en samfunnsøkonomisk synspunkt (The pollution problem from a social perspective)," *Sosialøkonomen*, *24* (1970).

[12] Hasselmann, K. "Intertemporal accounting of climate change - hamonizing economic efficiency and climate stewardship," *Climatic Change*, *41*:333–350 (1999).

[13] Hasselmann, K., et al. "Sensitivity Study of Optimal $CO_2$ Emission Paths Using a Simplified Structural Integrated Assessment Model (SIAM)," *Climatic Change*, 345–386 (1997).

[14] Heal, G. "Discounting and Climate Change. An Editorial Comment," *Climatic Change*, *37*:335–343 (1997).

[15] Koopmans, T. C. "Stationary ordinal utility and impatience," *Econometrica*, *28*(2):287–309 (1960).

[16] Koopmans, T. C., et al. "Stationary utility and time perspective," *Econometrica*, *32*(1-2) (1964).

[17] Manne, A. S. "The Rate of time preference. Implications for the greenhouse debate," *Energy Policy*, *23*(4/5):391–394 (1994).

[18] Nordhaus, W. D. "Discounting in Economics and Climate Change. An Editorial Comment," *Climatic Change*, *37*:315–328 (1997).

[19] Pezzey and Withagen. "Single-peakedness and initial sustainability in capital-resource economies," *Scandinavian Journal of Economics*, *100*(2):513–527 (1998).

[20] Ramsey, F. "A Mathematical Theory of Savings," *Economic Journal*, *38*:543–549 (1928).

[21] Ryder, H. E. and G. M. Heal. "Optimal growth with Intertemporally dependent preferences," *Review of Economic Sutides*, *40*(1):1–31 (1973).

[22] Shi, S. and L. G. Epstein. "Habits and time preference," *International Economic Review*, *34*(1):61–84 (1993).

[23] Uzawa, H. "Time preference, the consumption function and optimal asset holdings." *Value, capital and growth: Papers in honour of Sir John Hicks* edited by J.N. Wolfe, Edinburgh: University of Edinburgh, 1968.

[24] van der Burg, T. *On the short-term and long-term social rate of discount*. Technical Report, Presented at the Ninth Annual meeting of the IAEE in Oslo, 1999.

[25] von Mises, L. *Human action: a treatise on economics* (3rd, 1966 Edition).

[26] Weitzmann, M.L. "On the 'Environmental' Discount Rate," *Journal of Environmental Economics and Management*, *26*(2):200–209 (1994).

[27] Weitzmann, M.L. "Why the Far-Distant Future Should be Discounted at its Lowest Possible Rate," *Journal of Environmental Economics and Management*, *36*:201–208 (1998).