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Investment in flood protection measures under climate change uncertainty

An investment decision model

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Abstract: Recent river flooding in Europe has triggered debates among scientists and policymakers on future projections of flood frequency and the need for adaptive investments, such as flood protection measures. Because there exists uncertainty about the impact of climate change on flood risk, such investments require a careful analysis of expected benefits and costs. The objective of this paper is to show how climate change uncertainty affects the decision to invest in flood protection measures. We develop a model that simulates optimal decision making in flood protection, it incorporates flexible timing of investment decisions and scientific uncertainty on the extent of climate change impacts. This model allows decision-makers to cope with the uncertain impacts of climate change on the frequency and damage of river flood events and minimises the risk of under- or over-investment. One of the innovative elements is that we explicitly distinguish between structural and non-structural flood protection measures. Our results show that the optimal investment decision today depends strongly on the cost structure of the adaptation measures and the discount rate, especially the ratio of fixed and weighted annual costs of the measures. A higher level of annual flood damage and later resolution of uncertainty in time increases the optimal investment. Furthermore, the optimal investment decision today is influenced by the possibility of the decision-maker to adjust his decision at a future moment in time.

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1 Introduction

The August 2002 flood in the Elbe basin was a showcase of a flood event, with estimated damage costs of approximately US$ 12 billion (Becker and Grünewald, 2003). The Elbe flood, jointly with other severe floods in Europe, provided a stimulus to two ongoing scientific debates. The first debate takes place among hydrologists and concerns historical observations and future projections of flood frequency, and its relation to the possible impacts of climate change on river flow. The second debate takes place among river basin decision-makers and concerns the need for additional (adaptive) investments in flood protection measures. In this paper we link the two debates in a model that assesses optimal investments in flood protection measures under uncertain climate change impacts on flood risk.

There is mixed evidence on the impact of climate change on flood risk and extreme flood events in river basins. On the one hand, Petrow and Merz (2009) analysed historical observations for different river basins in Germany for the period 1951-2002, and concluded that a large share of these basins show significant upward flood trends, and Milly et al. (2002) showed "significant trends towards more extreme flood events" in 29 basins. On the other hand, Mudelsee et al. (2003) analysed flood frequency in the Oder and Elbe rivers and concluded that "although extreme floods with return periods of 100 year and more occurred in central Europe in July 1997 (Oder) and August 2002 (Elbe), there is no evidence from the observations for recent upward trends in their occurrence rate". Kundzewicz et al. (2005) found varying results, with "increases, decreases as well as no significant long-term changes in annual extreme flows" for a sample of 195 rivers (Trenberth et al., 2007). The same ambiguity is present in projections of climate change effects on flood frequency. The frequency of flood events is influenced by, among others, precipitation intensity and the discharge regime, both of which might be affected by climate change. It is unclear, however, to what extent climate change will affect extreme peak discharges, which under normal circumstances result in flood events. Climate models generally project changes in seasonal average discharge regimes of rivers, with higher discharges in winter and lower discharges in summer (Te Linde et al., 2008). In addition, these models project an overall decrease in precipitation in Europe, although flooding may well become more frequent in summertime (Christensen and Christensen, 2003). These types of projections, however, have to be used with care as they are not supported by historic flooding trends (Helms et al., 2002; Mudelsee et al., 2003), are typically made at scales that are larger than those relevant for decision-making (Towler et al., 2010), and it remains difficult to link individual extreme weather events to a change in the climate (Kundzewicz, 2005; Trenberth et al., 2007).

Thus, there exists uncertainty about the impact of climate change on flood risk in river basins. Therefore, the relevant question for decision-makers responsible for flood protection is how to deal with this uncertainty. In response to the 2002 flood, decision-makers in the Elbe basin started to adapt their flood protection infrastructure. Relevant flood protection measures were identified, including increased storage capacity in upstream reservoirs and upgrading of the existing river dikes (De Kok and Grossmann, 2010). The implementation of these measures remains uncertain, however, most likely because this requires long-term political commitment (Petrow et al., 2006). In the Netherlands, flood events in the Meuse and Rhine basins in the 1990s resulted in a similar upgrading of the flood protection programme, although uncertainty about climate change effects remains (Silva et al., 2004).

These examples illustrate that the relation between uncertainty and the timing of investments in flood protection measures presents decision-makers with a trade-off between investing in flood protection today and postponing the decision. Because the effects of climate change are uncertain, decision-makers are reluctant to invest in additional flood protection measures, especially when the costs of these measures are irreversible. When the timing of investment in flood protection measures is flexible, the investment decisions may be postponed until more information about the effects of climate change has arrived. The presence of both irreversibility and flexibility link this decision problem to the theory of investment under uncertainty (Dixit and Pindyck, 1994).

Only few studies relate the risk of flooding in river basins to the implementation of adaptive protection measures. Fankhauser et al. (1999) assess efficient adaptation to climate change-induced extreme events. Kundzewicz (2009) identifies flood protection and flood preparedness measures to avoid adverse impacts for the Baltic Sea basin. De Bruin et al. (2009) present an inventory and ranking of adaptation options for the water sector in the Netherlands. Tol et al. (2003) discuss the impacts of climate change on flood risks in the Netherlands and conclude that structural solutions that integrate land-use planning and water management are better capable of dealing with climate change than incidental solutions. The previous studies did not consider different adaptation measures under climate change uncertainty. In this paper we address flood risk in river basins and investment decisions in adaptation measures. We make a distinction between different types of protection measures and model the resolution of climate change uncertainty.

Our objective is to show how climate change uncertainty affects the decision to invest in flood protection measures. We develop a model of optimal investment in flood protection measures under climate change uncertainty. Such a model allows decision-makers to cope with the uncertain impacts of climate change on the frequency and damage of river flood events, while minimising the risk of under- or over-investment. Under-investment results in a flood damage probability that is higher than optimal, while over-investment leads to sunk costs and redundant flood protection capacity.

We adapt a model by Hennessy and Moschini (2006) on costly regulatory action under scientific uncertainty to the case of flood protection. Our simplest model specification is a discrete-state two-period model which provides a crude first decision-rule for investments. In subsequent sections, this model is extended to a continuous-state two-period and three-period model, which allows us to analyse the effects of various model elements on this decision-rule. One of these elements is the trade-off between investment in structural and non-structural measures, explained below. Another element is the resolution of climate change uncertainty, which is modeled as a gradual process over time until full resolution is reached. In the two-period model the initial investment decision can be updated when full resolution of uncertainty is reached at an unknown future moment in time. The three-period model allows for an intermediate investment decision under partial resolution of uncertainty before the adjustment of the investment decision under full resolution of climate change uncertainty, related to evidence on climate induced annual flood damage. The motivation for studying gradual resolution of uncertainty is that over time, additional evidence adds to the overall insight into these impacts, reducing their uncertainty. Our results show that the effect of uncertainty on the investment decision depends on the cost structure of the flood protection measures under consideration. To be precise, a combination of the discount rate, climate change uncertainty, and the cost structure of structural and non-structural measures determines the optimal mix of investments in these measures. A higher level of annual flood damage and later resolution of uncertainty in time increases the optimal investment decision. Furthermore, the optimal investment decision today is influenced by the possibility of the decision-maker to adjust his decision at a future moment in time.

One of the innovative elements of our paper is that we explicitly distinguish between two categories of protection measures, which vary in their cost structure. The first category, that we will refer to as *structural* measures, includes those measures that have high fixed costs relative to annual costs. Examples are dike improvement and relocation. The second category, that we will refer to as *non-structural* measures, includes those measures that have low fixed costs relative to annual costs. Examples are the creation of retention areas to accommodate peak flows, and programmes to raise public awareness on flood events. Note that our definition of structural and non-structural measures is slightly different from the one used by for instance Kundzewicz (2002, 2009), see Section 5. We will see that the inclusion of an intermediate decision moment where partial resolution is observed induces lower investments in structural measures.

The paper is structured as follows. In Section 2 we introduce the basic elements of our model to establish the optimal investment decision under uncertainty in a discrete two-period model. In Section 3 we relax the discreteness assumption as to allow for a wide range of possible climate change impacts as well as a continuous range of investment in both structural and non-structural measures. In Section 4 we introduce a three-period model, in order to analyse the effect of an intermediate investment decision under partial resolution of uncertainty. The implications of the models for flood protection are discussed in Section 5, followed by the conclusion in Section 6.

2 Discrete-state two-period model

In this section we present a simple discrete-state, two-period model, inspired by Hennessy and Moschini (2006). We assume that the world knows two possible states α; either climate change affects flood damage ($\alpha = 1$) or it does not ($\alpha = 0$). At time $t = 0$ there is uncertainty about which of the two states is the real state. State $\alpha = 1$ has probability q, and state $\alpha = 0$ has probability $1 - q$. This uncertainty will be resolved at some unknown future time $t = \kappa > 0$, where κ is exponentially distributed with $f(\kappa) = he^{-h\kappa}$, such that $E[\kappa] = 1/h$, where h is the hazard rate. A lower value of h implies that the expected resolution of uncertainty is further away in the future. An exponential distribution is often used in the R&D literature to model the expected arrival time of new information (Choi, 1991; Malueg and Tsutsui, 1997). It is a memoryless distribution, which means that the probability of arrival of new information does not depend on the arrival of past information. Following Hennessy and Moschini (2006), we further assume that new information is free and the arrival date is considered to be exogenous to the decision-maker.

The problem faced by the decision-maker is whether or not to make an irreversible and costly investment in flood protection measures m, that suffices to prevent damage in case $\alpha = 1$. Two actions are possible: $m = 1$ denotes the decision to invest and $m = 0$ the decision not to invest. In this section, we simplify matters by assuming that investment induces a fixed and irreversible investment cost C and that the flood protection measure has an infinite lifetime. Annual costs of the flood protection measure c include for instance opportunity costs (e.g. for land used as retention area) and maintenance costs (e.g. for dike maintenance). Let D_{max} denote maximum annual damage from climate change over the period up to $t = \kappa$. Damage is for instance caused by overflow, where at a certain location peak flow exceeds the critical height of the dike.

We assume that the decision-maker chooses the value of m that minimizes expected costs. The discounted realised cost is denoted as $R(m_0, \alpha, \kappa)$, where m_0 is the selected measure at time $t = 0$, α is the realized state of nature and κ is the time at which uncertainty is resolved. Costs consist of investment (C) and annual costs (c) of the implemented measure as well as damage costs D. For simplicity, α and m are the result of the normalisation of the ratio of the increase of flood damage due to climate change (A) and decrease of flood damage due to investment in flood protection measure (M), both in monetary units, with the maximum annual flood damage (D_{max}), where $\alpha = A/D_{max}$, and $m = M/D_{max}$.

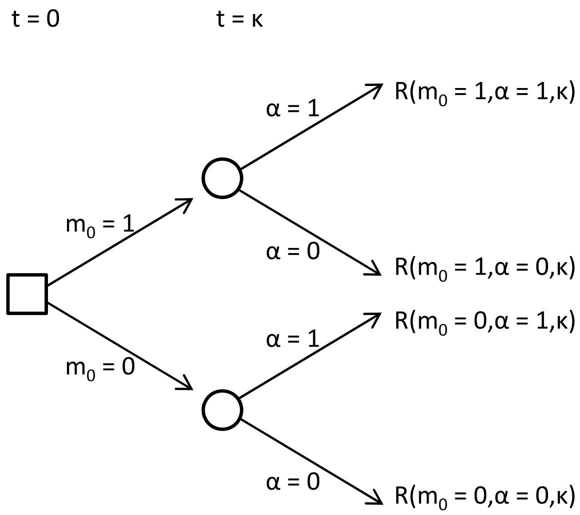


FIGURE 1: Decision tree for the discrete-state two-period model.

The decision-maker may make two erroneous decisions (Figure 1). First, if the decision-maker chooses $m_0 = 0$ and it turns out that at $t = \kappa$, $\alpha = 1$, he can revert his initial decision and invest $m_\kappa = 1$, while having incurred damage D over the period from $t = 0$ to $t = \kappa$. Second, if the decision-maker chooses $m_0 = 1$ and it turns out that at $t = \kappa$, $\alpha = 0$, he cannot retrieve his initial investment (i.e. C is irreversible), but saves annual costs c from time $t = \kappa$ onward.

Costs are evaluated at $t = 0$ present values, using the continuous-time discount rate r.

The decision node, represented as a square in Figure 1, indicates the decision to invest or not to invest at $t = 0$. The information node, shown as a circle, indicates the arrival of new information, in this situation leading to the full resolution of climate change uncertainty. The outcome of each path through the decision tree is defined is the discounted stream of costs for each specific path. The discounted realised cost is a function of m_0 and the random variables α and κ. The two random variables are independent. The outcome of each path is specified as:

$$\begin{aligned}
R(m_0 = 1, \alpha = 1, \kappa) &= C + \int_0^{\infty} ce^{-rt}dt \\
R(m_0 = 1, \alpha = 0, \kappa) &= C + \int_0^{\kappa} ce^{-rt}dt \\
R(m_0 = 0, \alpha = 1, \kappa) &= \int_0^{\kappa} D_{max}e^{-rt}dt + Ce^{-r\kappa} + \int_{\kappa}^{\infty} ce^{-rt}dt \\
R(m_0 = 0, \alpha = 0, \kappa) &= 0
\end{aligned} \tag{1}$$

The expected cost of investing, $E[R(m_0 = 1)]$, and of not investing, $E[R(m_0 = 0)]$, can be expressed as a function of the two random variables α and κ, where α is a discrete random variable, and κ a continuous random variable.

$$\begin{aligned}
E[R(m_0 = 1)] &= \int_0^{\infty} \left[qR(m_0 = 1, \alpha = 1, \kappa) + (1 - q)R(m_0 = 1, \alpha = 0, \kappa)\right] f(\kappa)d\kappa \\
&= C + q\left(\frac{c}{r}\right) + (1 - q)\left(\frac{c}{r + h}\right) \\
E[R(m_0 = 0)] &= \int_0^{\infty} \left[qR(m_0 = 0, \alpha = 1, \kappa) + (1 - q)R(m_0 = 0, \alpha = 0, \kappa)\right] f(\kappa)d\kappa \\
&= q\left(\frac{c}{r} + \frac{D_{max} - c + hC}{r + h}\right)
\end{aligned} \tag{2}$$

Comparing the expected costs, investment at $t = 0$ is optimal if $E[R(m_0 = 1)] < E[R(m_0 = 0)]$, which is equivalent to $\bar{q} < q$, where:

$$\bar{q} = \frac{c + C(r + h)}{D_{max} + Ch} \tag{3}$$

Because $\partial\bar{q}/\partial C > 0$ and $\partial\bar{q}/\partial c > 0$, investing at $t = 0$ is less likely if investment costs (fixed and/or annual) are higher. When the expected resolution of uncertainty moves closer in time (i.e. h increases) or the discount rate r increases, investing at $t = 0$ also becomes less likely, as the decision-maker prefers to postpone the uncertain decision until uncertainty is resolved. However, when the damage costs increase, investing at $t = 0$

becomes more likely; the decision-maker faces higher expected costs when postponing his investment decision. The results are intuitive and the model set-up is rather simple. For instance, the uncertainty of climate change impacts on flood damage α should preferably not be modeled as a draw from only two possible states of the world. Therefore, we introduce state-continuity of this impact and other model features in the next section, which also allows us to distinguish between investing in structural and non-structural measures.

3 Continuous-state two-period model

The continuous-state model is derived by three major adjustments to the discrete model. First, instead of the discrete set of states of nature $\alpha \in \{0, 1\}$, we now assume a continuum of states of nature $\alpha \in [0, 1]$, which has a density function $f(\alpha)$ over its domain. The interval $[0, 1]$ reflects the possible states of nature of how climate change affects expected flood damage as explained below. As before, at $t = 0$ the value of α is unknown.

Second, we introduce structural measures s and non-structural measures n. These flood protection measures serve to mitigate the increase of flood damage and thus the expected flood damage caused by climate change. Instead of the discrete investment decision $m \in \{0, 1\}$, we now assume a continuum of structural and non-structural flood protection measures with $s \in [0, 1]$ and $n \in [0, 1]$, where s and n are the result of normalisation such that $s = 0$ or $n = 0$ reflects no investment while $s = 1$ or $n = 1$ reflects maximum investment. We assume that each combination of measures suffices to adapt to the impacts of climate change if $s + n \geq \alpha$. This assumption implies that structural and non-structural measures are additive, as in the case where dike heightening (structural measure) is accompanied by an early-warning system (non-structural).

The variables α, s and n are the result of normalisation based on the variable A that denotes the increase in potential flood damage due to climate change, and S and N that denote the decrease of flood damage due to investment in structural and non-structural measures, all defined in monetary units. These variables have been normalised by taking ratios using the maximum annual flood damage (D_{max}), which leads to $s = S/D_{max}$, $n = N/D_{max}$, and $\alpha = A/D_{max}$. Thus the inequality $S + N \geq A$ is normalised by taking ratios using the maximum annual flood damage, leading to $s + n \geq \alpha$.

Costs of the measures reflect the differences between structural and non-structural measures as discussed in Section 1. Structural measures have irreversible fixed costs $C_s s$ and annual costs $c_s s$. Similarly, non-structural measures have irreversible fixed costs

$C_n n$ and annual costs $c_n n$. We assume $C_s > C_n$ but $c_s < c_n$. Structural measures have high fixed costs but low annual costs relative to non-structural measures. From this cost structure we can derive that, in absence of uncertainty and for sufficiently low discounting, structural measures are preferred over non-structural measures. Under uncertainty, however, a decision-maker may want to diversify between structural and non-structural measures in order to minimise total expected costs.

Third, instead of the fixed damage parameter D_{max}, we now assume a damage function $\mathcal{D}(\alpha, s, n)$ that maps damage as a function of uncertain climate change impact α, mitigated by flood protection measures $s+n$. Recall that we assumed that each combination of measures suffices to adapt to the impacts of climate change if $s+n \geq \alpha$, which leads to zero damage costs. This assumption allows us to use the difference between α and $s+n$ in order to account for the mitigating effect of flood protection measures on damage.

These three adjustments to the discrete model allow us to model the decision-maker's decision in a similar way as was done for the discrete case described in Section 2. Again, the decision-maker may make two erroneous decisions: First, if it turns out that at $t = \kappa$ the decision-maker has under-invested (i.e. $s_0 + n_0 < \alpha$),[1] he can upgrade his initially implemented measures to the optimal level (i.e. to $s_0+n_0+s_\kappa+n_\kappa = \alpha$), while incurring the possible additional fixed costs $C_s s_\kappa$ or $C_n n_\kappa$, and increase of annual costs by $c_s s_\kappa$ or $c_n n_\kappa$. Obviously, damage is incurred over the period from $t = 0$ to $t = \kappa$. Second, if it turns out that at $t = \kappa$ the decision-maker has over-invested (i.e. $s_0+n_0 > \alpha$), he cannot retrieve his initial investment (i.e. $C_s s_0$ and $C_n n_0$ are irreversible), but he can reduce his annual costs such that $s_0 + n_0 + s_\kappa + n_\kappa = \alpha$ from time $t = \kappa$ onward. The interval range for s_0 and n_0 is from $[0, 1]$, and the interval range for s_κ and n_κ is from $[-s_0, 1]$ and $[-n_0, 1]$. The constraints $s_\kappa \geq -s_0$ and $n_\kappa \geq -n_0$ are imposed on the interval range of s_κ and n_κ to indicate that in the case of over-investment at $t = 0$, a reduction of the annual costs at $t = \kappa$ cannot exceed the initial investment made at $t = 0$.

Figure 2 shows the decision tree for the continuous-state two-period model. The decision problem is solved backward. The decision node (square) on the right indicates the decision for s_κ and n_κ at $t = \kappa$ when a combination of s_0 and n_0 has been chosen and α is known (represented by the circular information node). We assume the optimal adjustment of the investment decision under full resolution of uncertainty at $t = \kappa$, where

$$s_\kappa + n_\kappa = \alpha - s_0 - n_0 \tag{4}$$

[1]Where necessary, we add a subscript t ($t = 0$ or $t = \kappa$) to s or n, in order to clarify the timing of the investment.

We first define the adjustment decision for the level of s_κ and then, given this choice, the investment level of n_κ, where

$$n_\kappa = \alpha - s_0 - n_0 - s_\kappa \tag{5}$$

is set. This allows us to substitute n_κ by $\alpha - s_\kappa - s_0 - n_0$, and therefore leave out the term n_κ in the decision tree and continuation of the model description. First, we solve the decision-maker's problem to choose s_κ at time $t = \kappa$, when s_0 and n_0 have been chosen and α is known. The decision for s_κ is based on the minimisation over all possible values of s_κ (represented by a range of possible values from 1 to M) given the constraint $s_\kappa \geq -s_0$. Second, given the choice at $t = \kappa$, the optimal levels of s_0 and n_0 are selected at $t = 0$. As we evaluate the costs from a $t = 0$ perspective, we consider a continuum of α, as at $t = 0$ we do not know the exact value of α at $t = \kappa$. The continuum over α is represented in Figure 2 by different regions to indicate how the combination of s_0 and n_0 and the value of α affects the optimal choice at $t = \kappa$ (see Figure 2).

The decision node on the left represents the objective of the decision-maker to choose the combination of s_0 and n_0 in order to minimise the path outcome of the decision tree, the discounted realised cost $R(s_0, n_0, s_\kappa, \kappa, \alpha)$ that consists of damage, fixed and annual costs of the flood protection measures. For each combination of s_0 and n_0 and associated choice at $t = \kappa$, the discounted realised cost is derived. The superscripts in Figure 2 and further equations indicate over which set of choices the discounted realised cost is derived; the set of $\{s_0, n_0\}$ combinations is defined from 1 to N. The set of $\{s_0, n_0\}$ combinations includes all combinations based on the interval range of s_0 and n_0. The set for $\{s_\kappa\}$ ranges from 1 to M, and is based on the interval $[-s_0, \alpha - s_0]$. The lower and upper bound of the interval are based on the constraint $s_\kappa \geq -s_0$ and $n_\kappa \geq -n_0$, where the latter constraint can be rewritten in the following way. Note that $n_\kappa \geq -n_0$, by substituting n_κ by Eq. 5, can be written as $\alpha - s_0 - s_\kappa \geq 0$, which is equal to $\alpha - s_0 \geq s_\kappa$, and presents the upper-bound of the interval for s_κ.

3.1 Specific decision path

We now highlight a specific path of the decision tree that leads to the outcome R^{ij} to indicate how the discounted realised cost is derived. The stream of costs is discounted for a specific $\{s_0, n_0\}^i$, $\{s_\kappa\}^j$, α and κ. The discounted realised cost R^{ij} is defined as

$$R^{ij} = I_0^i + D_0^i + I_\kappa^{ij} \tag{6}$$

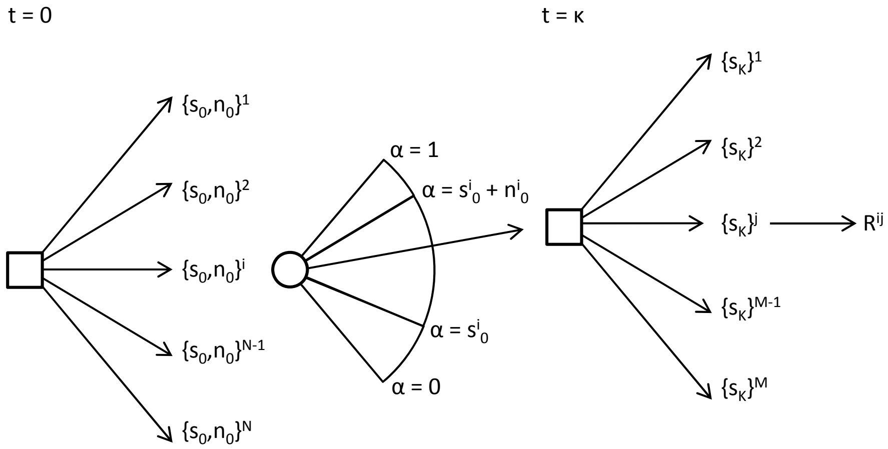


Figure 2: Decision tree for continuous-state two-period model.

which includes the discounted investment cost and discounted damage cost for the period starting at $t = 0$ (I_0^i and D_0^i) and the discounted adjustment cost for the period starting at $t = \kappa$ (I_κ^{ij}).[2] The damage cost from $t = \kappa$ onwards is zero as we assume optimal investment adjustment at $t = \kappa$. The discounted investment cost I_0^i is a function of a combination of $\{s_0, n_0\}^i$ and random variable κ:

$$\begin{aligned} I_0^i &= C_s s_0^i + C_n n_0^i + \int_0^\kappa \left(c_s s_0^i + c_n n_0^i \right) e^{-rt} dt \\ &= C_s s_0^i + C_n n_0^i + \left(\frac{c_s s_0^i + c_n n_0^i}{r} \right) (1 - e^{-r\kappa}) \end{aligned} \tag{7}$$

The discounted damage cost D_0^i is a function of a combination of $\{s_0, n_0\}^i$ and random variables κ and α:

$$\begin{aligned} D_0^i &= \int_0^\kappa \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) e^{-rt} dt \\ &= \frac{\mathcal{D}\big(\alpha, s_0^i, n_0^i\big)}{r} (1 - e^{-r\kappa}) \end{aligned} \tag{8}$$

The discounted adjustment cost I_κ^{ij} is a function of $\{s_0, n_0\}^i$, $\{s_\kappa\}^j$ and random variables

[2] The superscript ij refers to a combination of $\{s_0, n_0\}^i$ and $\{s_\kappa\}^j$ to calculate the discounted adjustment cost for the period starting at $t = \kappa$.

κ and α:

$$
\begin{aligned}
I_\kappa^{ij} &= \Big(C_s \max\big\{0, s_\kappa^j\big\} + C_n \max\big\{0, \alpha - s_0^i - n_0^i - s_\kappa^j\big\}\Big) e^{-r\kappa} \\
&\quad + \int_\kappa^\infty \Big(c_s\big(s_0^i + s_\kappa^j\big) + c_n\big(\alpha - s_0^i - s_\kappa^j\big)\Big) e^{-rt} dt \\
&= \Big(C_s \max\big\{0, s_\kappa^j\big\} + C_n \max\big\{0, \alpha - s_0^i - n_0^i - s_\kappa^j\big\}\Big) e^{-r\kappa} \\
&\quad + \Big(\frac{c_s}{r}\big(s_0^i + s_\kappa^j\big) + \frac{c_n}{r}\big(\alpha - s_0^i - s_\kappa^j\big)\Big) e^{-r\kappa} \qquad (9)
\end{aligned}
$$

Note that $s_0^i + s_\kappa^j \geq 0$ and $n_0^i + n_\kappa^j \geq 0$.

3.2 Optimal adjustment at $t = \kappa$

As we follow a backward procedure, the focus is first on the optimal adjustment decision at $t = \kappa$, denoted as $\{s_\kappa, n_\kappa\}^{j_{min}}$, which is defined as the decision where the discounted adjustment cost is minimum, i.e. $I_\kappa^{ij_{min}}$. Therefore, I_κ^{ij} is minimised over all possible values of $\{s_\kappa\}^j$ for a given $\{s_0, n_0\}^i$ and α.

We rewrite Eq. 9 as $I_\kappa^{ij} = A_\kappa^{ij} e^{-r\kappa}$, where A_κ^{ij} represents the flow of fixed and annual costs and is defined as

$$
\begin{aligned}
A_\kappa^{ij} &= C_s \max\big\{0, s_\kappa^j\big\} + C_n \max\big\{0, \alpha - s_0^i - n_0^i - s_\kappa^j\big\} \\
&\quad + \frac{c_s}{r}(s_0^i + s_\kappa^j) + \frac{c_n}{r}(\alpha - s_0^i - s_\kappa^j) \qquad (10)
\end{aligned}
$$

The minimum A_κ^{ij} can be written as a function of $\mathcal{C}_1$ and $\mathcal{C}_2$, where $\mathcal{C}_1$ and $\mathcal{C}_2$ are defined as:

$$
\begin{aligned}
\mathcal{C}_1 &= C_s + \frac{c_s}{r} - \frac{c_n}{r} \\
\mathcal{C}_2 &= C_s + \frac{c_s}{r} - C_n - \frac{c_n}{r} \qquad (11)
\end{aligned}
$$

The magnitudes of $\mathcal{C}_1$ and $\mathcal{C}_2$ are determined by the value and ratio of the fixed and annual cost elements between the structural and non-structural measure and the level of the discount rate r.[3]

There are three possible combinations for $\mathcal{C}_1$ and $\mathcal{C}_2$, namely: (1) $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$, (2) $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 < 0$ and (3) $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 \geq 0$. Note that the combination $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 \geq 0$ is not valid, as $\mathcal{C}_2$ cannot be positive if $\mathcal{C}_1$ is negative, given that $C_n > 0$.

[3] We define $C_s + c_s/r$ as the fixed plus weighted annual cost. The weighted annual cost is the present value of the infinite stream of annual costs.

For each combination of $\mathcal{C}_1$ and $\mathcal{C}_2$, the minimum A_κ^{ij} is defined by how the level of α relates to the investment decision made at $t = 0$, $\{s_0, n_0\}^i$, i.e. if the decision-maker has over- or under-invested. This can be summarized as follows:

1. $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$

$$A_\kappa^{ij_{min}} = \begin{cases} \frac{c_s}{r}\alpha & 0 \leq \alpha \leq s_0^i \\ C_s(\alpha - s_0^i) + \frac{c_s}{r}\alpha & s_0^i < \alpha \leq s_0^i + n_0^i \\ C_s(\alpha - s_0^i) + \frac{c_s}{r}\alpha & s_0^i + n_0^i < \alpha \leq 1 \end{cases} \tag{12}$$

2. $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 < 0$

$$A_\kappa^{ij_{min}} = \begin{cases} \frac{c_s}{r}\alpha & 0 \leq \alpha \leq s_0^i \\ \frac{c_s}{r}s_0^i + \frac{c_n}{r}(\alpha - s_0^i) & s_0^i < \alpha \leq s_0^i + n_0^i \\ C_s(\alpha - s_0^i - n_0^i) + \frac{c_s}{r}(\alpha - n_0^i) + \frac{c_n}{r}n_0^i & s_0^i + n_0^i < \alpha \leq 1 \end{cases} \tag{13}$$

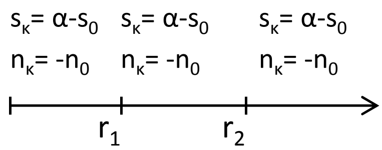
3. $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 \geq 0$

$$A_\kappa^{ij_{min}} = \begin{cases} \frac{c_s}{r}\alpha & 0 \leq \alpha \leq s_0^i \\ \frac{c_s}{r}s_0^i + \frac{c_n}{r}(\alpha - s_0^i) & s_0^i < \alpha \leq s_0^i + n_0^i \\ C_n(\alpha - s_0^i - n_0^i) + \frac{c_s}{r}s_0^i + \frac{c_n}{r}(\alpha - n_0^i) & s_0^i + n_0^i < \alpha \leq 1 \end{cases} \tag{14}$$

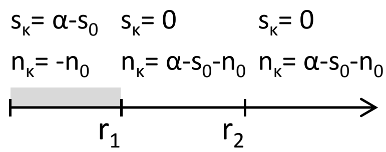
Each combination of $\mathcal{C}_1$ and $\mathcal{C}_2$ marks a different adjustment strategy. Since $\mathcal{C}_1$ and $\mathcal{C}_2$ are a function of the discount rate (r), three regions of adjustment types can be defined along the discount rate axis. This is shown in Figure 3 for the cases where the decision-maker has over- and under-invested. r_1 denotes the discount rate where $\mathcal{C}_1 = 0$, and thus if $r < r_1$ then $\mathcal{C}_1 < 0$. r_2 denotes the discount rate where $\mathcal{C}_2 = 0$, and thus if $r < r_2$ then $\mathcal{C}_2 < 0$. Investments in structural measures are indicated with a light gray bar, and non-structural measures with a dark gray bar.

If $\mathcal{C}_1 < 0$ then $C_s + \frac{c_s}{r} < \frac{c_n}{r}$, i.e. the fixed cost plus the present value of an infinite stream of the annual costs of the structural measure is smaller than the present value of an infinite stream of the annual cost of the non-structural measure. Moreover, if $\mathcal{C}_2 < 0$ then $C_s + \frac{c_s}{r} < C_n + \frac{c_n}{r}$, i.e. the fixed plus weighted annual cost of the structural measure is smaller than the fixed plus weighted annual cost of the non-structural measure. $\mathcal{C}_2$ determines the choice between structural and non-structural measures if the decision-maker has under-invested at $t = 0$ and therefore an additional investment is required at $t = \kappa$. $\mathcal{C}_1$ determines whether the non-structural measures are reduced if the decision-maker has over-invested at $t = 0$ or if they are replaced by an investment in structural measures.

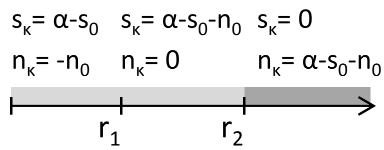
For example, if $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$ then the optimal adjustment decision at $t = \kappa$ is to reduce the investment in the non-structural measures as much as possible, i.e. $n^j_\kappa = -n^i_0$. Moreover, if $0 \leq \alpha \leq s^i_0$, the decision-maker has over-invested at $t = 0$. Even after reducing the non-structural measures at $t = \kappa$, there is still an over-investment. The structural measures are therefore reduced: $s^j_\kappa = \alpha - s^i_0$. Reducing structural measures leads to a reduction in the annual costs, but it does not imply that the initial investment is removed. If $s^i_0 < \alpha \leq s^i_0 + n^i_0$, the decision-maker has over-invested at $t = 0$. After reducing the non-structural measures, an additional investment is however required to avoid damages. He will invest in structural measures $s^j_\kappa = \alpha - s^i_0$. On the other hand, if $s^i_0 + n^i_0 < \alpha \leq 1$ the decision-maker has under-invested, there are damages incurred up to $t = \kappa$. He will further invest only in structural measures $s^j_\kappa = \alpha - s^i_0$. Inserting these conditions in Eq. 10 gives Eq. 12.



(a) Over-invested: $0 \leq \alpha \leq s^i_0$



(b) Over-invested: $s^i_0 < \alpha \leq s^i_0 + n^i_0$



(c) Under-invested $s^i_0 + n^i_0 < \alpha \leq 1$

FIGURE 3: Three regions of adjustment types along the discount rate axis defined by $\mathcal{C}_1$ and $\mathcal{C}_2$ for the cases where the decision-maker has over- and under-invested.

3.3 Optimal decision at $t = 0$

With the optimal adjustment decision at $t = \kappa$ known, the discounted realised cost in Eq. 6 is rewritten as

$$R^i = I^i_0 + D^i_0 + I^{ij_{min}}_\kappa \tag{15}$$

The discounted realised cost is a random variable as it is a function of the random variables κ and α. To derive the optimal investment decision $t = 0$ we need to first determine the expected value of R^i, which is defined as

$$E[R^i] = E[I_0^i] + E[D_0^i] + E[I_\kappa^{ij_{min}}] \tag{16}$$

We solve Eq. 16 for the defined exponential distribution of κ, however we do not yet solve for the probability distribution of α, as this probability density function may have different shapes depending on the focus of the climate change impact (i.e. peak discharge, sea-level rise, etc.). As we consider the random variables κ and α to be independent random variables, the joint probability distribution of κ and α can be written as the product of the probability distribution of κ and α ($f(\kappa, \alpha) = f(\kappa)f(\alpha)$). The expected discounted investment cost ($E[I_0^i]$) is a function of $\{s_0, n_0\}^i$:

$$\begin{aligned} E[I_0^i] &= \int_0^\infty I_0^i f(\kappa) d\kappa \\ &= C_s s_0^i + C_n n_0^i + \left(\frac{1}{h+r}\right)(c_s s_0^i + c_n n_0^i) \end{aligned} \tag{17}$$

The expected discounted damage cost ($E[D_0^i]$) is a function of $\{s_0, n_0\}^i$:

$$\begin{aligned} E[D_0^i] &= \int_0^1 \int_0^\infty D_0^i f(\kappa) f(\alpha) d\kappa d\alpha \\ &= \left(\frac{1}{h+r}\right) \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) f(\alpha) d\alpha \end{aligned} \tag{18}$$

The expected optimal discounted adjustment cost ($E[I_\kappa^{ij_{min}}]$) is a function of $\{s_0, n_0\}^i$ and the combination of $\mathcal{C}_1$ and $\mathcal{C}_2$:

$$\begin{aligned} E[I_\kappa^{ij_{min}}] &= \int_0^1 \int_0^\infty A_\kappa^{ij_{min}} e^{-r\kappa} f(\kappa) f(\alpha) d\kappa d\alpha \\ &= \left(\frac{h}{h+r}\right) \int_0^1 A_\kappa^{ij_{min}} \; f(\alpha) d\alpha \end{aligned} \tag{19}$$

With Eq. 17 to 19, we can derive the optimal investment decision at $t = 0$ for a given C_s, c_s, C_n, c_n, r, h, $\mathcal{D}$ and $f(\alpha)$. The optimal investment decision at $t = 0$ is denoted as $\{s_0, n_0\}^{i_{min}}$, and is defined as the minimisation of the expected discounted realised costs, i.e. $E[R^{i_{min}}]$,

$$E[R^{i_{min}}] = min\left\{E[R^1], ..., E[R^i], ..., E[R^N]\right\} \tag{20}$$

The decision maker will prefer an investment in structural measures to minimise the expected discounted investment costs at $t = 0$ ($E[I_0^i]$), if $\mathcal{C}_2^h < 0$, which is defined as

$$\mathcal{C}_2^h = C_s + \frac{c_s}{h+r} - C_n - \frac{c_n}{h+r} \tag{21}$$

If $\mathcal{C}_2^h < 0$, then the fixed cost plus present value of the annual costs up to the expected waiting time for resolution of uncertainty is smaller for structural measures than for non-structural measures. If the expected waiting time for resolution of uncertainty $(1/h)$ approaches infinity than, $\mathcal{C}_2^h$ approaches $\mathcal{C}_2$, defined in Eq. 11. Since $\mathcal{C}_2^h$ is a function of the discount rate (r) and the expected waiting time for resolution of uncertainty $(1/h)$, two regions of investment types at $t = 0$ that minimise $E[I_0^i]$ can be defined in the plane spanned by r and $1/h$. This is shown in Figure 4. If $1/h = 0$, only the fixed costs are relevant. Since $C_s > C_n$, non-structural measures are preferred. As $1/h$ increases, the contribution of the annual costs increases. Since $c_n > c_s$, non-structural measures become less preferable.

The optimal investment decision at $t = 0$ that minimise $E[R^i]$ will relate to the regions defined by $\mathcal{C}_2^h$ in Figure 4 and by $\mathcal{C}_1$ and $\mathcal{C}_2$ in Figure 3. This will be illustrated by numerical examples in the next section.

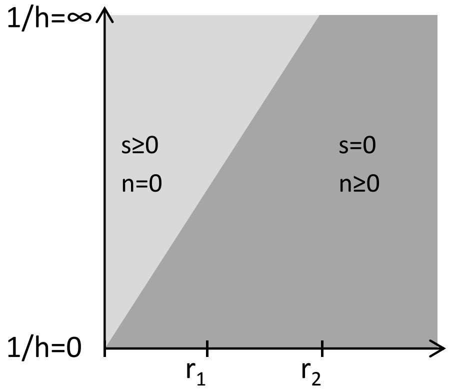


FIGURE 4: Two regions of investment types at $t = 0$ that minimise $E[I_0^i]$. Defined by $\mathcal{C}_2^h$ in the plane spanned by the discount rate (r) and the expected waiting time for resolution of uncertainty $(1/h)$.

3.4 Numerical examples

In this section we further illustrate the continuous-state two-period model. A uniform probability distribution for α and an increasing and concave damage function are applied. The damage function is given by:

$$\mathcal{D}(\alpha, s_0, n_0) = \begin{cases} D_{max}\sqrt{\alpha - s_0 - n_0} & \alpha - s_0 - n_0 > 0 \\ 0 & \alpha - s_0 - n_0 \leq 0 \end{cases}$$

If $\alpha > s_0 + n_0$, the decision-maker has under-invested and there are damage costs. A motivation for this functional form is provided in Appendix A. The resulting expressions for the expected discounted realised cost (Eq. 16, 17, 18 and 19) are programmed in MATLAB, and minimised for a range of $\{s_0, n_0\}^i$, given the constraints $0 \leq s_0^i \leq 1$, $0 \leq n_0^i \leq 1$ and $0 \leq s_0^i + n_0^i \leq 1$.

Three examples will be presented to illustrate how the combination of $\mathcal{C}_1$ and $\mathcal{C}_2$ influences the optimal investment decision at $t = 0$. The absolute value of the cost function parameters (C_s, C_n, c_s, c_n and D_{max}) used in these examples are not important. It is their relation that is of interest for this illustration. The optimal investment decision at $t = 0$ will be presented for a range of plausible parameter values for r and h. Specifically, we assess results for the intervals $r \in (0, 0.1]$ and $h \in [0.01, 1]$. The interval for r implies that we check solutions for non-negative discount rates up to 10%. The interval for h implies that we check solutions where the expected waiting time for resolution of uncertainty is between 1 year and 100 years.

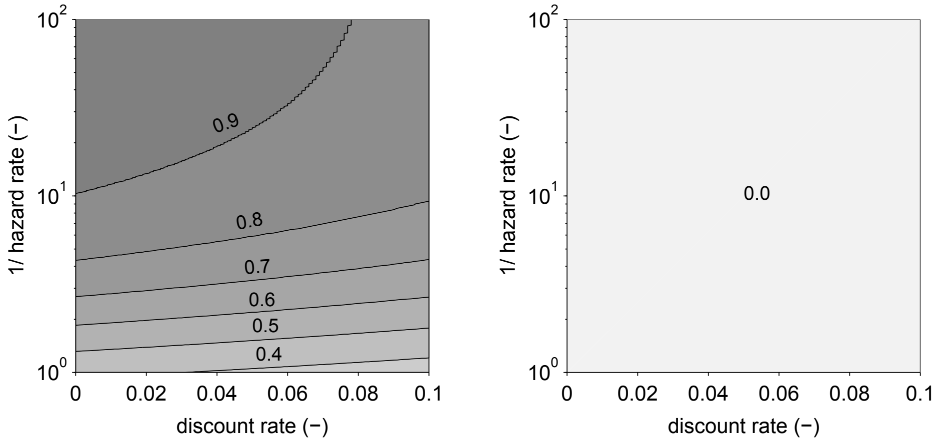
3.4.1 Example 1.

In the first example the cost function parameters are selected such that $\mathcal{C}_1 < 0$ (and thus $\mathcal{C}_2 < 0$) for the complete range of r. For illustration, we chose the following values, $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ € and $c_n = 250$ €. We consider two values of D_{max}, namely 750 € and 1500 €, to demonstrate the effect of increasing maximum annual flood damage on the optimal decision. Figure 5 and 6 present the resulting optimal investment decision at $t = 0$ as function of r and h.

Since $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$, the focus of the optimal decision at $t = 0$ will be on structural measures. This can be seen in Figure 5. No investment in non-structural measures is made at $t = 0$. Investing in non-structural measures becomes more desirable as damage costs increase, as shown by Figure 6. The relatively high non-structural costs become justifiable when the damages increase. The damage costs will be set to zero at the moment uncertainty is resolved ($t = \kappa$).

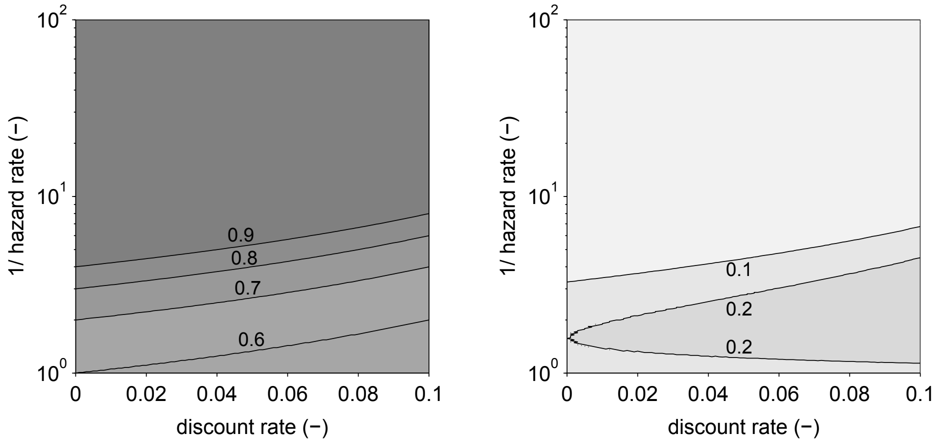
Moreover the results demonstrate that if $1/h$ increases the investment in structural measures increases. When $1/h$ increases the expected waiting time for resolution of uncertainty is longer, and accordingly the period of possible damages is longer. Therefore, the investment in structural measures will increase to avoid a long period of possible damages. This effect becomes smaller when the discount rate increases. If the discount rate increases, future costs receive less weight, therefore the stream of damage costs receives less weight, and the investment in structural measures will increase less. Investment in structural measures increases stronger with lower discount rates and longer expected waiting time for resolution of uncertainty.

The non-structural measures, on the other hand, increase first and then decrease again if $1/h$ increases. This is related to the period of possible damages and the fact that non-structural measures become optimal to minimize $E[I_0^i]$ for small $1/h$. As the period of possible damage increases, the relatively high non-structural annual costs become justifiable. However, if this period further increases, the relative high annual non-structural costs are no longer justifiable. It is better to increase the structural measures. If the damages costs decreases it is not justifiable to invest first in non-structural measures - although it is optimal to minimize $E[I_0^i]$ for small $1/h$ - as they will be reduced at the moment uncertainty is resolved. This is reflected in Figure 5 and 6.



Structural measures   Non-structural measures

FIGURE 5: Example 1. Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 250$ € and $D_{max} = 750$ €. (Calculation based on step-size 0.001 for interval α.)



Structural measures   Non-structural measures

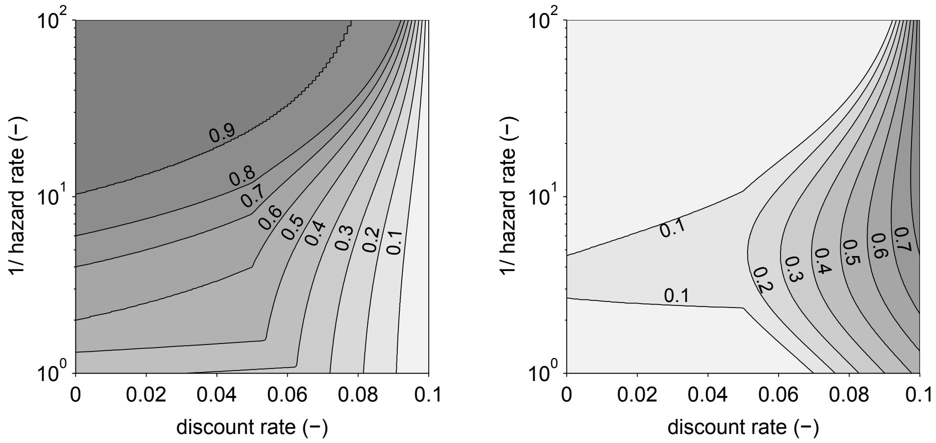
FIGURE 6: Example 1. Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 250$ € and $D_{max} = 1500$ €. (Calculation based on step-size 0.001 for interval α.)

3.4.2 Example 2.

In the second example the cost function parameters are selected such that $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$ for $r \in (0, 0.05]$, defined as region 1 and $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 < 0$ for $r \in [0.05, 0.1]$, defined as region 2. For illustration, we chose the following values, $C_s =$ 1000 €, $C_n = 500$ €, $c_s = 150$ € and $c_n = 200$ €. Two values of D_{max} are considered: 750 € and 1500 €. Figure 7 and 8 present the resulting optimal investment decision at $t = 0$ as function of r and h.

The results demonstrate that the optimal investment decision at $t = 0$ is differently related to r and h for the two regions. Similar characteristics as discussed in the first example, are present for $r \in (0, 0.05]$. For $r \in [0.05, 0.1]$, it can be observed that it becomes more favorable to invest in non-structural measures as r increases. Moreover, the optimal investment decision at $t = 0$ depends less on the $1/h$ as r increases.

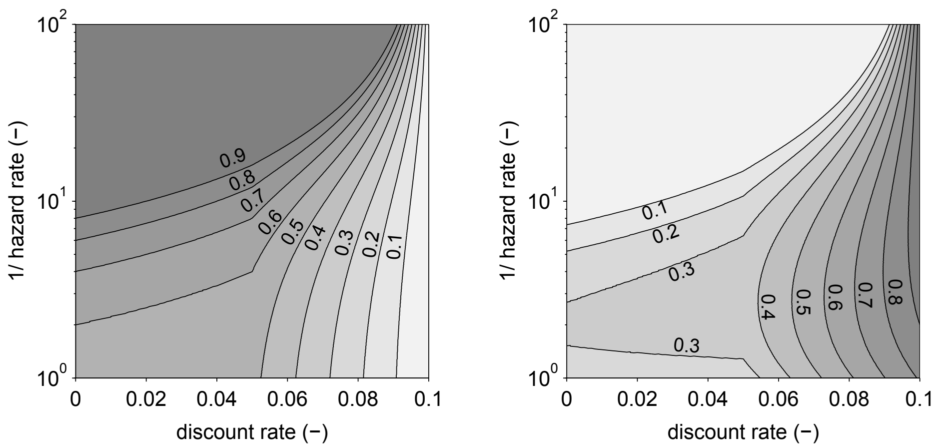
If $\mathcal{C}_1 \geq 0$ then $C_s + \frac{c_s}{r} \geq \frac{c_n}{r}$, i.e. the fixed plus weighted annual costs of structural measures are greater than or equal to the weighted annual costs of non-structural measures. If $\mathcal{C}_2 < 0$ then $C_s + \frac{c_s}{r} < C_n + \frac{c_n}{r}$, i.e. the fixed plus weighted annual costs of structural measures are smaller than those of non-structural measures. For the optimal decision at $t = 0$, these conditions imply that it is still favorable to invest in structural measures. However, as the discount rate increases, the difference between the fixed plus weighted annual costs of the structural and non-structural measures becomes smaller, making non-structural measures justifiable to reduce the damages. Especially for shorter periods of possible damages (smaller $1/h$) this becomes justifiable (see Figure 4). If the decision-maker has over-invested, the best is to reduce the non-structural measures (as $c_n > c_s$) and let the structural measures unchanged (see Figure 3). Therefore, non-structural measures at $t = 0$ are justifiable if the damage costs increase and the period of possible damages is smaller, such that the annual costs can be limited. The second region can be considered as a transition zone. This is illustrated by the next example.



Structural measures

Non-structural measures

FIGURE 7: Example 2. Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 200$ € and $D_{max} = 750$ €. (Calculation based on step-size 0.001 for interval α.)



Structural measures

Non-structural measures

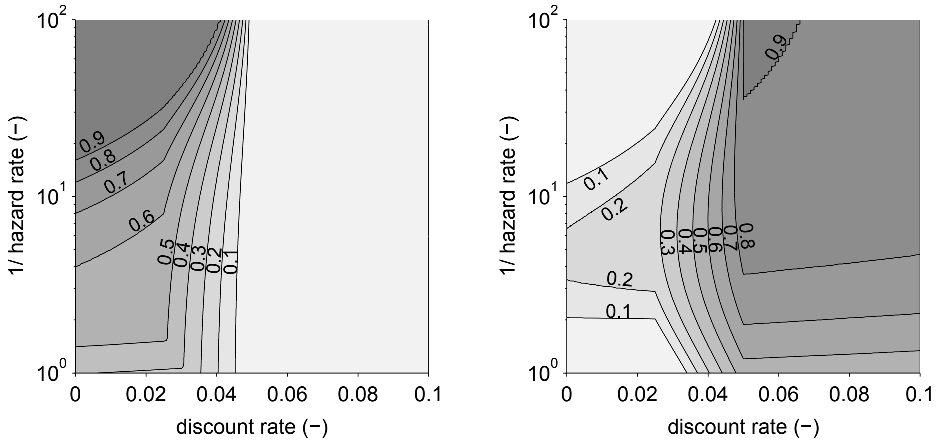
FIGURE 8: Example 2. Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 200$ € and $D_{max} = 1500$ €. (Calculation based on step-size 0.001 for interval α.)

3.4.3 Example 3.

In the third example the cost function parameters are selected such that $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$ for $r \in (0, 0.025]$ (region 1), $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 < 0$ for $r \in [0.025, 0.05]$ (region 2) and $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 \geq 0$ for $r \in [0.05, 0.1]$ (region 3). For illustration, we chose the following values, $C_s = 1000$ €, $C_n = 500$ €, $c_s = 175$ € and $c_n = 200$ €. Two values of D_{max} are considered: 750 € and 1500 €. Figure 9 and 10 present the resulting optimal investment decision at $t = 0$ as function of r and h.

The results demonstrate that the optimal investment decision at $t = 0$ is different related to r and h for these three regions. Similar characteristics discussed in the first and second example, are present for $r \in (0, 0.025]$ and $r \in [0.025, 0.05]$, respectively. For $r \in [0.05, 0.1]$, it can be observed that it is favorable to invest in non-structural measures.

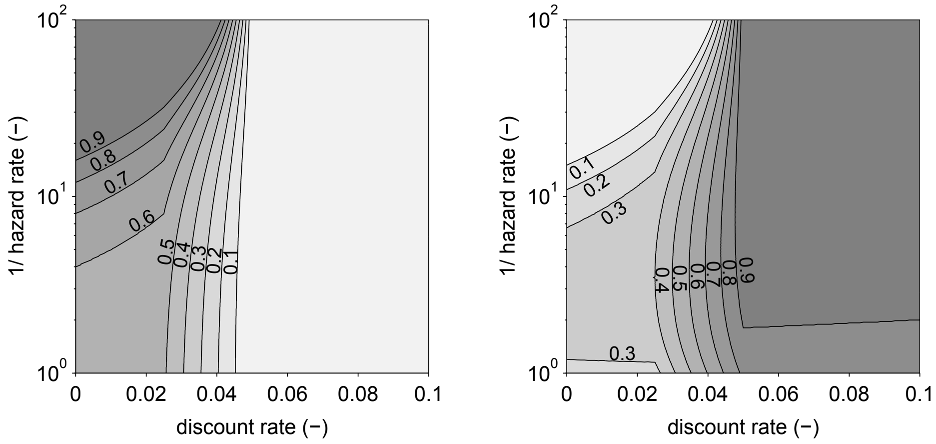
If $\mathcal{C}_1 \geq 0$ then $C_s + \frac{c_s}{r} \geq \frac{c_n}{r}$, i.e. the fixed plus weighted annual costs of structural measures are greater than or equal to the weighted annual costs of non-structural measures. If $\mathcal{C}_2 \geq 0$ then $C_s + \frac{c_s}{r} \geq C_n + \frac{c_n}{r}$, i.e. the fixed plus weighted annual costs of structural measures are greater than or equal to those of non-structural measures. For the optimal decision at $t = 0$, these conditions imply that it is favorable to invest in non-structural measures. The relatively high structural costs become unjustifiable. If the damages increase, the non-structural measures will further increase.



Structural measures

Non-structural measures

FIGURE 9: Example 3. Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 175$ €, $c_n = 200$ € and $D_{max} = 750$ €. (Calculation based on step-size 0.001 for interval α.)



Structural measures

Non-structural measures

FIGURE 10: Example 3. Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 175$ €, $c_n = 200$ € and $D_{max} = 1500$ €. (Calculation based on step-size 0.001 for interval α.)

4 Continuous-state three-period model

In this section we expand upon the two-period model by considering an intermediate decision moment at which there is partial resolution of climate change uncertainty. This is a natural extension of our analysis given that the resolution of the uncertainty of climate change impacts on river flow is a gradual process and the decision-maker will have additional opportunities to adjust his initial investment decision. For the three-period model, an investment decision is made at $t = 0$ and an adjustment decision, under full resolution of climate change uncertainty, is made at an unknown future time $t = \kappa$. This unknown future moment is equal to the full resolution moment in the continuous-state two-period model, therefore κ has the same probability distribution, i.e. an exponentially distributed with $f(\kappa) = he^{-h\kappa}$, such that $E[\kappa] = 1/h$, where h is denoted as the hazard rate. At an intermediate decision moment, defined as $t = x\kappa$ partial resolution of uncertainty is used to make an additional investment decision. Note that x is a fraction, where $x \in (0, 1)$. From todays perspective, this moment is unknown as also $t = \kappa$ is unknown. The decision-maker defines his investment strategy based on the expected value $t = \kappa$ and thus $t = x\kappa$, i.e. $1/h$ and x/h, respectively. If the decision-maker sets x equal to 0.2 today, he will use the information about the partial resolution of climate change uncertainty at 20% of the expected waiting time of full resolution $1/h$. Therefore, this model focuses on the effect of the use of intermediate information.[4]

Figure 11 shows the decision tree for the continuous-state three-period model. The decision problem is solved backward. The decision node on the far right indicates the decision for s_κ and n_κ at $t = \kappa$ when a combination of s_0, n_0, $s_{x\kappa}$ and $n_{x\kappa}$ has been chosen and α is known, based on the reduced range of α resulting from partial resolution of uncertainty (circular information node on the right). The decision node in the middle of the decision tree indicates the decision for $s_{x\kappa}$ and $n_{x\kappa}$ at $t = x\kappa$ when a combination of s_0 and n_0 has been chosen and the probability distribution of α is updated based on the received evidence range w (indicated by the two circular information nodes on the left). The decision node on the left represents the objective of the decision-maker to choose the combination of s_0 and n_0 in order to minimise the path outcome of the decision tree.

In the following subsections, we will present a specific decision path of the decision tree following a backward procedure. This includes the optimal adjustment at $t = \kappa$, the

[4] Note that the intermediate information is not used to update the expected time of full resolution.

optimal decision at $t = x\kappa$ and the optimal decision at $t = 0$. Next, the process of gradual resolution of uncertainty that leads to an update of the prior distribution of α for partial resolution of climate change uncertainty is explained. Furthermore, we show that the continuous-state two-period model is a special case of the continuous-state three-period model and present numerical examples.

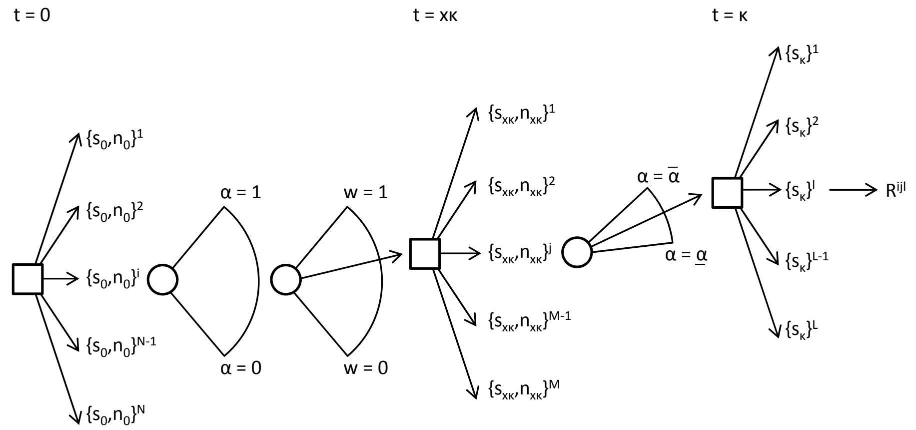


FIGURE 11: Decision tree for continuous-state three-period model.

4.1 Specific decision path

We now highlight a specific path of the decision tree that leads to the outcome R^{ijl} to indicate how the discounted realised cost is derived. The stream of costs is discounted for a specific $\{s_0, n_0\}^i$, $\{s_{x\kappa}, n_{x\kappa}\}^j$, $\{s_\kappa\}^l$, α, x and κ. The discounted realised cost R^{ijl} is defined as

$$R^{ijl} = I_0^i + D_0^i + I_{x\kappa}^{ij} + D_{x\kappa}^{ij} + I_\kappa^{ijl} \tag{22}$$

which includes the discounted investment and damage cost from $t = 0$ up to $t = x\kappa$ (I_0^i and D_0^i), the discounted adjustment and damage cost from $t = x\kappa$ up to $t = \kappa$ ($I_{x\kappa}^{ij}$ and $D_{x\kappa}^{ij}$). Further, it includes the discounted adjustment cost for the period starting at $t = \kappa$ (I_κ^{ijl}). The damage cost from $t = \kappa$ onwards is zero as we assume optimal adjustment at $t = \kappa$.

The discounted investment cost I_0^i is a function of $\{s_0, n_0\}^i$ and the random variable κ:

$$\begin{aligned} I_0^i &= C_s s_0^i + C_n n_0^i + \int_0^{x\kappa} \Big(c_s s_0^i + c_n n_0^i \Big) e^{-rt} dt \\ &= C_s s_0^i + C_n n_0^i + \left(\frac{c_s s_0^i + c_n n_0^i}{r} \right) (1 - e^{-rx\kappa}) \end{aligned} \tag{23}$$

The discounted damage cost D_0^i is a function of $\{s_0, n_0\}^i$ and the random variables κ and α:

$$\begin{aligned} D_0^i &= \int_0^{x\kappa} \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) e^{-rt} dt \\ &= \frac{\mathcal{D}\big(\alpha, s_0^i, n_0^i\big)}{r}(1 - e^{-rx\kappa}) \end{aligned} \tag{24}$$

The discounted adjustment cost $I_{x\kappa}^{ij}$ is a function of $\{s_0, n_0\}^i$, $\{s_{x\kappa}, n_{x\kappa}\}^j$ and the random variable κ:

$$\begin{aligned} I_{x\kappa}^{ij} &= \Big(C_s \max\big\{0, s_{x\kappa}^j\big\} + C_n \max\big\{0, n_{x\kappa}^j\big\}\Big) e^{-rx\kappa} \\ &+ \int_{x\kappa}^{\kappa} \Big(c_s(s_0^i + s_{x\kappa}^j) + c_n(n_0^i + n_{x\kappa}^j)\Big) e^{-rt} dt \\ &= \Big(C_s \max\big\{0, s_{x\kappa}^j\big\} + C_n \max\big\{0, n_{x\kappa}^j\big\}\Big) e^{-rx\kappa} \\ &+ \Big(\frac{c_s}{r}(s_0^i + s_{x\kappa}^j) + \frac{c_n}{r}(n_0^i + n_{x\kappa}^j)\Big)(e^{-rx\kappa} - e^{-r\kappa}) \end{aligned} \tag{25}$$

Note that $s_0^i + s_{x\kappa}^j \geq 0$ and $n_0^i + n_{x\kappa}^j \geq 0$. The discounted damage cost $D_{x\kappa}^{ij}$ is a function of $\{s_0, n_0\}^i$, $\{s_{x\kappa}, n_{x\kappa}\}^j$ and the random variables κ and α:

$$\begin{aligned} D_{x\kappa}^{ij} &= \int_{x\kappa}^{\kappa} \mathcal{D}\big(\alpha, s_0^i, n_0^i, s_{x\kappa}^j, n_{x\kappa}^j\big) e^{-rt} dt \\ &= \frac{\mathcal{D}\big(\alpha, s_0^i, n_0^i, s_{x\kappa}^j, n_{x\kappa}^j\big)}{r}(e^{-rx\kappa} - e^{-r\kappa}) \end{aligned} \tag{26}$$

Finally, the discounted adjustment cost I_{κ}^{ijl} is a function of $\{s_0, n_0\}^i$, $\{s_{x\kappa}, n_{x\kappa}\}^j$, $\{s_\kappa\}^l$ and the random variables κ and α:

$$\begin{aligned} I_{\kappa}^{ijl} &= \Big(C_s \max\big\{0, s_\kappa^l\big\} + C_n \max\big\{0, n_\kappa^l\big\}\Big) e^{-r\kappa} \\ &+ \int_{\kappa}^{\infty} \Big(c_s\big(s_0^i + s_{x\kappa}^j + s_\kappa^l\big) + c_n\big(n_0^i + n_{x\kappa}^j + n_\kappa^l\big)\Big) e^{-rt} dt \\ &= \Big(C_s \max\big\{0, s_\kappa^l\big\} + C_n \max\big\{0, \alpha - s_0^i - n_0^i - s_{x\kappa}^j - n_{x\kappa}^j - s_\kappa^l\big\}\Big) e^{-r\kappa} \\ &+ \Big(\frac{c_s}{r}\big(s_0^i + s_{x\kappa}^j + s_\kappa^l\big) + \frac{c_n}{r}\big(\alpha - s_0^i - s_{x\kappa}^j - s_\kappa^l\big)\Big) e^{-r\kappa} \end{aligned} \tag{27}$$

Note that $s_0^i + s_{x\kappa}^j + s_\kappa^l \geq 0$ and $n_0^i + n_{x\kappa}^j + n_\kappa^l \geq 0$.

4.2 Optimal adjustment at $t = \kappa$

As we follow a backward procedure, similar to the continuous-state two-period model, the focus is first on the optimal decision at $t = \kappa$, denoted as $\{s_\kappa\}^{l_{min}}$ and is defined as the minimisation of the discounted adjustment cost, i.e. $I_\kappa^{ijl_{min}}$. Therefore, I_κ^{ijl} is minimised over all possible values of $\{s_\kappa\}^l$ for a given $\{s_0, n_0\}^i$, $\{s_{x\kappa}, n_{x\kappa}\}^j$ and α.

Eq. 27 is rewritten as $I_\kappa^{ijl} = A_\kappa^{ijl} e^{-r\kappa}$, where A_κ^{ijl} is a function of $\mathcal{C}_1 = C_s + \frac{c_s}{r} - \frac{c_n}{r}$ and $\mathcal{C}_2 = C_s + \frac{c_s}{r} - C_n - \frac{c_n}{r}$. For each combination of $\mathcal{C}_1$ and $\mathcal{C}_2$, the minimum A_κ^{ijl} is defined by how the level of α relates to the investment decision made at $t = 0$ and $t = x\kappa$, $\{s_0 + s_{x\kappa}, n_0 + n_{x\kappa}\}^{ij}$, i.e. if the decision-maker has over- or under-invested. This can be summarized as follows:

1. $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$

$$A_\kappa^{ijl_{min}} = \begin{cases} \frac{c_s}{r}\alpha & 0 \leq \alpha \leq s^{ij} \\ C_s(\alpha - s^{ij}) + \frac{c_s}{r}\alpha & s^{ij} < \alpha \leq s^{ij} + n^{ij} \\ C_s(\alpha - s^{ij}) + \frac{c_s}{r}\alpha & s^{ij} + n^{ij} < \alpha \leq 1 \end{cases} \tag{28}$$

2. $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 < 0$

$$A_\kappa^{ijl_{min}} = \begin{cases} \frac{c_s}{r}\alpha & 0 \leq \alpha \leq s^{ij} \\ \frac{c_s}{r}s^{ij} + \frac{c_n}{r}(\alpha - s^{ij}) & s^{ij} < \alpha \leq s^{ij} + n^{ij} \\ C_s(\alpha - s^{ij} - n^{ij}) + \frac{c_s}{r}(\alpha - n^{ij}) + \frac{c_n}{r}n^{ij} & s^{ij} + n^{ij} < \alpha \leq 1 \end{cases} \tag{29}$$

3. $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 \geq 0$

$$A_\kappa^{ijl_{min}} = \begin{cases} \frac{c_s}{r}\alpha & 0 \leq \alpha \leq s^{ij} \\ \frac{c_s}{r}s^{ij} + \frac{c_n}{r}(\alpha - s^{ij}) & s^{ij} < \alpha \leq s^{ij} + n^{ij} \\ C_n(\alpha - s^{ij} - n^{ij}) + \frac{c_s}{r}s^{ij} + \frac{c_n}{r}(\alpha - n^{ij}) & s^{ij} + n^{ij} < \alpha \leq 1 \end{cases} \tag{30}$$

where $s^{ij} = s_0^i + s_{x\kappa}^j$ and $n^{ij} = n_0^i + n_{x\kappa}^j$

If $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$ then the optimal adjustment decision at $t = \kappa$ is to reduce the investment in the non-structural measures as much as possible, i.e. $n_\kappa^l = -n^{ij}$ and invest only in structural measures. Moreover, if $0 \leq \alpha \leq s^{ij}$ the decision-maker has over-invested and the structural measures are therefore reduced: $s_\kappa^l = \alpha - s^{ij}$. Reducing structural measures leads to a reduction in the annual costs, but, it does not imply that the initial investment is removed. On the other hand, if $s^{ij} < \alpha \leq 1$ the decision-maker has under-invested, there are damages incurred and at $t = \kappa$, he will further invest only in structural measures $s_\kappa^l = \alpha - s^{ij}$. Note that in all cases it is required that $-s^{ij} \leq s_\kappa^l \leq \alpha - s^{ij}$ because $s^{ij} + s_\kappa^l \geq 0$ and $n^{ij} + n_\kappa^l \geq 0$.

4.3 Optimal decision at $t = x\kappa$

With the optimal adjustment decision at $t = \kappa$ known, the discounted realised cost in Eq. 22 is rewritten as

$$\begin{aligned} R^{ij} &= I_0^i + D_0^i + I_{x\kappa}^{ij} + D_{x\kappa}^{ij} + I_\kappa^{ijl_{min}} \\ &= I_0^i + D_0^i + R^{j|i} \end{aligned} \tag{31}$$

The focus is now on the optimal discounted cost at $t = x\kappa$ ($R^{j|i}$), which is a random variable as it is a function of the random variables κ and α. To derive the optimal investment decision $t = x\kappa$ we need to first determine the expected value of $R^{j|i}$, which is defined as

$$E[R^{j|i}] = E[I_{x\kappa}^{ij}] + E[D_{x\kappa}^{ij}] + E[I_\kappa^{ijl_{min}}] \tag{32}$$

We consider the random variables κ and α to be independent random variable. The random variable α is conditioned on evidence for α, i.e. w. The joint probability function of $R^{j|i}$ is therefore given by: $f(\kappa, \alpha|w) = f(\kappa)f(\alpha|w)$.

The expected discounted investment cost ($E[I_{x\kappa}^{ij}]$) is a function of $\{s_0, n_0\}^i$ and $\{s_{x\kappa}, n_{x\kappa}\}^j$:

$$\begin{aligned} E[I_{x\kappa}^{ij}] &= \int_0^\infty I_{x\kappa}^{ij} f(\kappa) d\kappa \\ &= \frac{h}{(h + xr)} \Big(C_s \max\big\{0, s_{x\kappa}^j\big\} + C_n \max\big\{0, n_{x\kappa}^j\big\}\Big) \\ &+ \frac{(1-x)h}{(h+xr)(h+r)} \Big(c_s(s_0^i + s_{x\kappa}^j) + c_n(n_0^i + n_{x\kappa}^j)\Big) \end{aligned} \tag{33}$$

The expected discounted damage cost ($E[D_{x\kappa}^{ij}]$) is a function of $\{s_0, n_0\}^i$, $\{s_{x\kappa}, n_{x\kappa}\}^j$ and evidence for α, i.e. w:

$$\begin{aligned} E[D_{x\kappa}^{ij}] &= \int_0^1 \int_0^\infty D_{x\kappa}^{ij} f(\kappa) f(\alpha|w) d\kappa d\alpha \\ &= \frac{(1-x)h}{(h+xr)(h+r)} \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i, s_{x\kappa}^j, n_{x\kappa}^j\big) f(\alpha|w) d\alpha \end{aligned} \tag{34}$$

The expected optimal discounted adjustment cost ($E[I_\kappa^{ijl_{min}}]$) is a function of $\{s_0, n_0\}^i$,

$\{s_{x\kappa}, n_{x\kappa}\}^j$, evidence for α, i.e. w, and the combination of $\mathcal{C}_1$ and $\mathcal{C}_2$:

$$E[I_\kappa^{ijl_{min}}] = \int_0^1 \int_0^\infty A_\kappa^{ijl_{min}} e^{-r\kappa} f(\kappa) f(\alpha|w) d\kappa d\alpha$$
$$= \frac{h}{h+r} \int_0^1 A_\kappa^{ijl_{min}} f(\alpha|w) d\alpha \tag{35}$$

With Eq. 33 to 35, we can derive the optimal adjustment decision at $t = x\kappa$ for a given $\{s_0, n_0\}^i$, C_s, c_s, C_n, c_n, r, h, $\mathcal{D}$, x, w and $f(\alpha|w)$. The optimal adjustment decision at $t = x\kappa$ is denoted as $\{s_{x\kappa}, n_{x\kappa}\}^{ij_{min}}$, and is defined as the minimisation of the expected discounted costs at $t = x\kappa$, i.e. $E[R^{j_{min}|i}]$,

$$E[R^{j_{min}|i}] = min\left\{E[R^{1|i}], ..., E[R^{j|i}], ..., E[R^{M|i}]\right\} \tag{36}$$

4.4 Optimal decision at $t = 0$

With the optimal decisions at $t = \kappa$ and $t = x\kappa$ known, the discounted realised cost in Eq. 31 is rewritten as

$$R^i = I_0^i + D_0^i + E[R^{j_{min}|i}] \tag{37}$$

The discounted realised costs is a random variable as I_0^i is a function of the random variable $x\kappa$, D_0^i is a function of the random variables $x\kappa$ and α and $E[R^{j_{min}|i}]$ is a function of the random variable w, as there is not one evidence, but a range of evidence of α at $t = x\kappa$ possible. To derive the optimal investment decision $t = 0$, we first determine the expected value of R^i, which is defined as:

$$E[R^i] = E[I_0^i] + E[D_0^i] + E\Big[E[R^{j_{min}|i}]\Big] \tag{38}$$

The expected discounted investment cost ($E[I_0^i]$) is a function of $\{s_0, n_0\}^i$:

$$E[I_0^i] = \int_0^\infty I_0^i f(\kappa) d\kappa$$
$$= C_s s_0^i + C_n n_0^i + \frac{x}{h+xr}(c_s s_0^i + c_n n_0^i) \tag{39}$$

The expected discounted damage cost ($E[D_0^i]$) is a function of $\{s_0, n_0\}^i$:

$$E[D_0^i] = \int_0^1 \int_0^\infty D_0^i f(\kappa) f(\alpha) d\kappa d\alpha$$
$$= \frac{x}{h+xr} \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) f(\alpha) d\alpha \tag{40}$$

The expected value of the minimum expected discounted costs at $t = x\kappa$ is a function of $\{s_0, n_0\}^i$:

$$E\Big[E[R^{j_{min}|i}]\Big] = \int_0^1 E[R^{j_{min}|i}]f(w)dw \tag{41}$$

With Eq. 39 to 41, we can derive the optimal investment decision at $t = 0$ for a given C_s, c_s, C_n, c_n, r, h, $\mathcal{D}$, x, $f(w)$ and $f(\alpha)$. The decision is denoted as $\{s_0, n_0\}^{i_{min}}$, and is defined as the minimisation of the expected discounted costs at $t = 0$, i.e. $E[R^{i_{min}}]$,

$$E[R^{i_{min}}] = min\left\{E[R^1], ..., E[R^i], ..., E[R^N]\right\} \tag{42}$$

4.5 Gradual resolution of uncertainty

We further examine the probability functions $f(\alpha)$, $f(w)$ and $f(\alpha|w)$, which are required to define the optimal investment at $t = 0$. The conditional function of α, given a specific value for the evidence w equals:

$$f(\alpha|w) = \frac{f(\alpha, w)}{f(w)} \text{ , where } \alpha \in [0, 1] \text{ and } w \text{ is a constant} \tag{43}$$

The conditional probability is proportional to the joint probability function of α and w, where evidence w is fixed to a specific value. Given this evidence w, α is more likely to occur, i.e. the universe is reduced. Therefore the joint probability function is divided by $f(w)$, the probability of this specific evidence. $f(w)$, also denoted as the marginal distribution, is found by integrating the joint probability function over the whole range of α:

$$f(w) = \int_0^1 f(\alpha, w)d\alpha \tag{44}$$

We could reverse the role of α and w in Eq. 43. The conditional probability of w given a specific value of α would be

$$f(w|\alpha) = \frac{f(\alpha, w)}{f(\alpha)}$$

$$\text{which can be rewritten as: } f(\alpha, w) = f(w|\alpha)f(\alpha) \tag{45}$$

Substituting Eq. 44 and 45 into the definition of the conditional function of α, given a specific value for evidence w, gives:

$$f(\alpha|w) = \frac{f(w|\alpha)f(\alpha)}{\int_0^1 f(w|\alpha)f(\alpha)d\alpha} \text{ , } \alpha \in [0,1] \text{ and } w \text{ is a constant} \tag{46}$$

Eq. 46 is known as Bayes' theorem (Bolstad, 2007). Bayes' theorem is used to revise our beliefs of α on the basis of evidence w. $f(\alpha)$ is the prior distribution for α. It gives the weight we attach to each value of α from our prior belief. $f(w|\alpha)$ is the likelihood for α and is the conditional probability that a specific evidence w has occurred given each value of α. Finally, $f(\alpha|w)$ is the posterior distribution for α. It gives the weight we attach to each value of α after we have observed a specific evidence w. The posterior thus combines our prior beliefs with the evidence given by the occurrence of w:

$$posterior = \frac{likelihood \times prior}{\int(likelihood \times prior)} \tag{47}$$

4.5.1 Likelihood

The likelihood function that needs to be defined in a Bayesian framework, is based upon an understanding of the evidence-generating process (Patwardhan and Small, 1992). We need to evaluate the likelihood of a stream of evidence of climate induced annual flood damages given a true state of climate induced annual flood damages. In general, we do not directly obtain the stream of evidence of increased annual flood damages, because we make associated observations, for example the annual peak discharges measured at different measuring stations along rivers. The relationship between the associated observations and climate induced annual flood damages is defined by parametric models, as shown in Figure 12. These parameters reflect the true state of the climate. Patwardhan and Small (1992) explain the case of sea-level rise, where a relation is defined between the long-term variation in global mean sea level change relative to a base year and observations of relative sea level at different tide gauges stations around the world.

If such models were formulated, the likelihood of a stream of evidence of climate induced annual flood damages given a true state of climate induced annual flood damages would be defined by a Monte Carlo simulation of the model while varying the model parameters for each possible true state of the climate.

For simplification we assume that the stream of evidence of climate induced annual flood damages can directly be determined. The associated observations and the models are therefore omitted. We denote the yearly determined evidence of climate induced annual

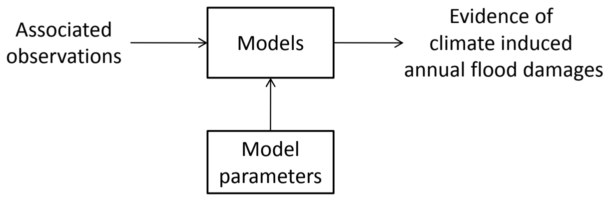


Figure 12: Relationship between the associated observations and climate induced annual flood damages

flood damages as y_ρ. This evidence has a normal distribution with mean α (the true state) and variance σ^2, i.e.

$$f(y_\rho|\alpha) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(y_\rho - \alpha)^2} \tag{48}$$

The variance reflects the degree to which we are able to determine climate induced annual flood damage. This variance is in expected terms proportional to the expected arrival time of full information, i.e. $1/h$, and can be compared to a measurement error of an instrument. The smaller the variance, the more accurate the instrument. Likewise, the better our capabilities to determine the climate induced annual flood damage, the shorter the expected arrival time of full information. Through investment in research we can enhance our capabilities to reduce the variance, however in this model the variance is constant thus no additional research costs are specified.

Every year we determine evidence of climate induced annual flood damages in a similar way, but independently from each other. This results in a sample (database) $y_1, \ldots, y_P$, after P years. Each evidence has the same normal distribution with mean α and variance σ^2, because the true state and our capabilities to determine evidence are considered to be constant over time. The joint likelihood of this sample after P years is the product of the individual likelihoods, because each evidence is independent of the other evidence. Using Eq. 48 and introducing the mean of the sample $\bar{y} = \frac{1}{P}\sum_{\rho=1}^{P} y_\rho$, gives:

$$\begin{aligned} f(y_1, \ldots, y_P|\alpha) &= \frac{1}{\left(\sqrt{2\pi}\sigma\right)^P} \prod_{\rho=1}^{P} e^{-\frac{1}{2\sigma^2}(y_\rho - \alpha)^2} \\ &= \frac{1}{\left(\sqrt{2\pi}\sigma\right)^P} e^{-\frac{P}{2\sigma^2}\left(\frac{1}{P}\sum y_\rho^2 - \bar{y}^2\right)} e^{-\frac{P}{2\sigma^2}(\alpha - \bar{y})^2} \\ &= K(y_1, \ldots, y_P) e^{-\frac{P}{2\sigma^2}(\bar{y} - \alpha)^2} \end{aligned} \tag{49}$$

The posterior of α given the stream of evidence $y_1, \ldots, y_P$, is accordingly (see Eq. 47):

$$f(\alpha|y_1, \ldots, y_P) = \frac{e^{-\frac{P}{2\sigma^2}(\bar{y}-\alpha)^2} f(\alpha)}{\int_0^1 e^{-\frac{P}{2\sigma^2}(\bar{y}-\alpha)^2} f(\alpha) d\alpha} \tag{50}$$

Note that the constant K drops out of the equation. Further, it is noted that the likelihood of α is proportional to the distribution of the sample mean ($\bar{y}$). The sample mean (the sum of P independent normal distribution with mean α and variance σ^2) itself has a normal distribution with mean α but with variance $\tilde{\sigma}^2 = \frac{\sigma^2}{P}$. Therefore, the posterior of α given the evidence $\bar{y}$ after P years follows the same equation as the posterior of α given the stream of evidence $y_1, \ldots, y_P$.

Finally, the sample mean after $x\kappa$ years is denoted as w. Therefore, Eq. 46 becomes:

$$f(\alpha|w) = \frac{e^{-\frac{1}{2\tilde{\sigma}^2}(w-\alpha)^2} f(\alpha)}{\int_0^1 e^{-\frac{1}{2\tilde{\sigma}^2}(w-\alpha)^2} f(\alpha) d\alpha} \text{ , } \alpha \in [0,1] \text{ and } w \text{ is a constant} \tag{51}$$

Full resolution of uncertainty is obtained when $\tilde{\sigma}^2 \mapsto 0$. This would happen when we determine evidence for infinity, $P \mapsto \infty$. However, it is considered that at $t = \kappa$, $\tilde{\sigma}$ becomes small enough in order to be considered as full resolution. This threshold is indicated by δ, thus at $t = \kappa$, $\tilde{\sigma} = \delta$. The variance of the sample mean after $x\kappa$ years, can therefore be defined as

$$\begin{aligned} \tilde{\sigma}^2 &= \frac{\sigma^2}{x\kappa} \\ &= \frac{\delta^2\kappa}{x\kappa} \text{ , } x \in [0,1] \end{aligned} \tag{52}$$

As mentioned before, the variance of the evidence determined in year ρ, is proportional to the year in which full resolution is considered (up to δ), i.e. $\sigma^2 \propto \kappa$. In expected terms this implies that $E[\sigma^2] \propto E[\kappa]$, which equals $1/h$. This reflects the degree to which we are able to determine evidence of climate induced annual flood damage.

The posterior of α as function of three different observation moments $x\kappa$ is illustrated in Figure 13 for evidence $w = 0.3$. Note that the prior of α is uniformly distributed, where no value is favored over any other. The posterior is shown for $x = 0.1$, $x = 0.5$ and $x = 0.9$, and $\delta = 0.025$. It shows that the posterior of α gradually reduces. For example, if the expected arrival time of full information ($1/h$) is set at 50 years, then already after 5 years there is a considerable reduction in uncertainty. In the decision tree (Figure 11) this reduced area is indicated by $\alpha \in [\underline{\alpha}, \overline{\alpha}]$.

In this study, one intermediate decision moment, $t = x\kappa$ is considered where $0 < x < 1$. At $t = 0$, the distribution of α is given by the prior. At $t = \kappa$, the distribution of α is not given by the posterior defined in Eq. 51 because full resolution is assumed in the model. This is represented by the assumption that $n_\kappa = \alpha - s_0 - n_0 - s_{x\kappa} - n_{x\kappa} - s_\kappa$ at full resolution of uncertainty. The posterior formulated in Eq. 51 is therefore only used for the intermediate decision moment. As a consequence δ can be increased in order to obtain a posterior with higher variance at $t = x\kappa$, without affecting the distribution of α at $t = \kappa$.

The evidence w at $t = x\kappa$ is used to update our prior belief about α and to make an intermediate adjustment decision. This procedure at $t = x\kappa$ is called a posterior analysis. However, from the viewpoint at $t = 0$ different observations are possible at $t = x\kappa$. Therefore, $E[R^{j_{min}|i}]$ becomes a random variable as w becomes a random variable from our viewpoint $t = 0$. This procedure at $t = 0$ is called the pre-posterior analysis.

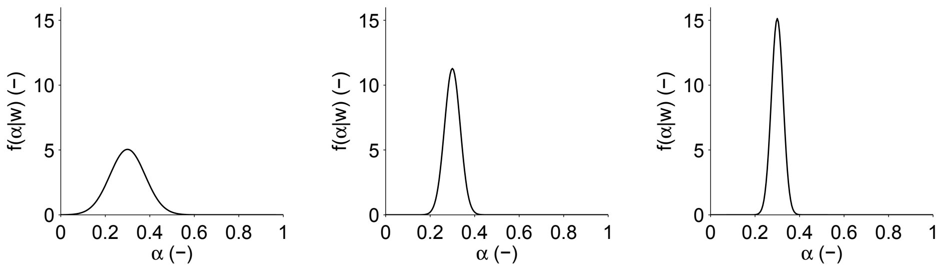


FIGURE 13: Posterior of α as function of three observation moments $x\kappa$ for evidence $w = 0.3$

4.5.2 The continuous-state two-period model, a special case

There is a gradual resolution of uncertainty both in the two-period and the three-period model. The difference between the two models is that in case of the three-period model, we make use of the partial resolution of uncertainty at $t = x\kappa$ to adjust the investment decision. Therefore, forcing $\{s_{x\kappa} = 0, n_{x\kappa} = 0\}$ in the three-period model, i.e. making no investment at $t = x\kappa$, should result in the same optimal investment at $t = 0$ as for the two-period model. This can be shown in the following way.

It is shown that Eq. 38 equals Eq. 16 if $\{s_{x\kappa} = 0, n_{x\kappa} = 0\}$. The expected value of the minimum expected discounted cost at $t = x\kappa$ is first specified using Eq. 41, 33, 34 and 35.

Note that superscript j_0 indicates that no investment is made at $t = x\kappa$.

$$
\begin{aligned}
E\Big[E[R^{j_{min}|i}]\Big] &= \int_0^1 \Big(E[I_{x\kappa}^{ij_0}] + E[D_{x\kappa}^{ij_0}] + E[I_{\kappa}^{ij_0 l_{min}}]\Big) f(w)dw \\
&= \frac{(1-x)h}{(h+xr)(h+r)} \Big(c_s s_0^i + c_n n_0^i\Big) \int_0^1 f(w)dw \\
&+ \frac{(1-x)h}{(h+xr)(h+r)} \int_0^1 \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) f(\alpha|w) f(w) d\alpha dw \\
&+ \frac{h}{h+r} \int_0^1 \int_0^1 A_{\kappa}^{ij_0 l_{min}} f(\alpha|w) f(w) d\alpha dw
\end{aligned} \tag{53}
$$

Next we change the order of integration in Eq. 53. First we integrate to w for a fixed α, resulting in the marginal distribution of α i.e.

$$
\begin{aligned}
\int_0^1 \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) f(\alpha|w) f(w) d\alpha dw &= \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) \int_0^1 f(\alpha|w) f(w) dw d\alpha \\
&= \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) f(\alpha) d\alpha
\end{aligned}
$$

Further, note that $\int_0^1 f(w)dw = 1$ and that $A_{\kappa}^{ij_0 l_{min}}$ is equal to $A_{\kappa}^{ij_{min}}$ in Eq. 12 to 14 of the two-period model. Therefore, Eq. 53 becomes:

$$
\begin{aligned}
E\Big[E[R^{j_{min}|i}]\Big] &= \frac{(1-x)h}{(h+xr)(h+r)} \Big(c_s s_0^i + c_n n_0^i\Big) \\
&+ \frac{(1-x)h}{(h+xr)(h+r)} \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) f(\alpha) d\alpha \\
&+ \frac{h}{h+r} \int_0^1 A_{\kappa}^{ij_{min}} f(\alpha) d\alpha
\end{aligned} \tag{54}
$$

Substituting this back in Eq. 38 and combining this with Eq. 39 and 40, gives the following equation for the expected discounted realised cost R^i:

$$
\begin{aligned}
E[R^i] = C_s s_0^i + C_n n_0^i &+ \Big(\frac{x}{h+xr} + \frac{(1-x)h}{(h+xr)(h+r)}\Big)\Big(c_s s_0^i + c_n n_0^i\Big) \\
&+ \Big(\frac{x}{h+xr} + \frac{(1-x)h}{(h+xr)(h+r)}\Big) \int_0^1 \mathcal{D}\big(\alpha, s_0^i, n_0^i\big) f(\alpha) d\alpha \\
&+ \frac{h}{h+r} \int_0^1 A_{\kappa}^{ij_{min}} f(\alpha) d\alpha
\end{aligned} \tag{55}
$$

As $\frac{x}{h+xr} + \frac{(1-x)h}{(h+xr)(h+r)} = \frac{1}{h+r}$, the expected discounted realised cost in the three-period model with $\{s_{x\kappa} = 0, n_{x\kappa} = 0\}$, is equal to the expected discounted realised cost in two-period model, given in Eq. 16 with Eq. 17, 18 and 19.

4.6 Numerical examples

We illustrate the three-period model using the same cost function parameters as in example 2 in Section 3.4 (Figure 7 and 8), where $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 200$ € and two values of D_{max}, 750 € and 1500 €. Due to the similar cost structure, similar regions are present in Figure 14 to 17, where $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$ for $r \in (0, 0.05]$ is defined as region 1 and $\mathcal{C}_1 \geq 0$ and $\mathcal{C}_2 < 0$ for $r \in [0.05, 0.1]$ is defined as region 2. For region 1, this implies that structural measures are preferred over non-structural measures. The non-structural measures first increase and then decrease if $1/h$ increases. As the period of possible damages increases, the relatively high annual non-structural costs become justifiable. However, as this period further increases, the relative high annual non-structural costs are no longer justifiable. It is better to increase the structural measures. In region 2 it is still favorable to invest in structural measures. However, non-structural measures are justifiable to reduce the damages if the discount rate increases. The optimal investment decision depends less on the $1/h$ as r increases.

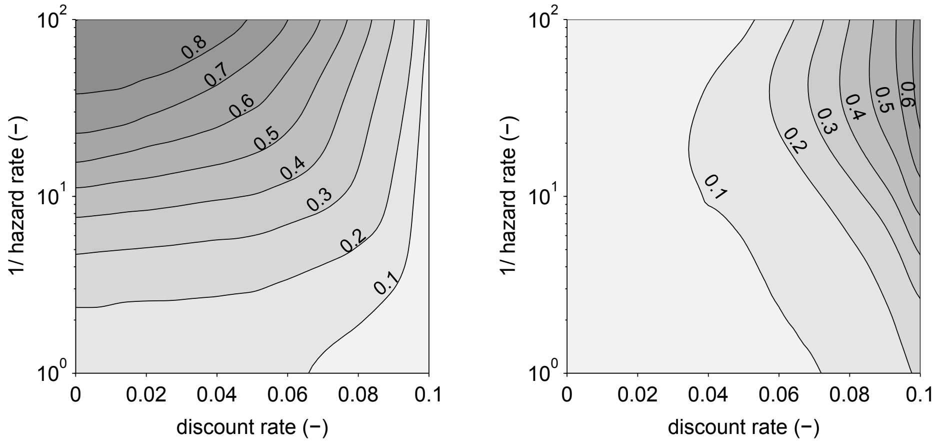
In the three-period model we introduce an intermediate decision moment ($t = x\kappa$), where partial resolution of uncertainty is used. The decision-maker sets the fraction x, which sets the moment at which partial resolution is used, relative to the moment of full resolution of uncertainty. We illustrate two different intermediate decision moments, namely $x = 0.1$ and $x = 0.5$ for a given expected value of the variance σ^2, i.e. δ^2/h. The variance reflects the capacity to collect evidence to reduce the domain of the prior distribution and is therefore proportional to the moment of full resolution of uncertainty. With $x = 0.1$ the moment at which partial resolution of uncertainty is used, is set at 10% of full resolution of uncertainty and with $x = 0.5$ at 50%. Figure 13 shows the posterior distribution of α as a function of three different observation moments with $\delta = 0.025$ for a constant evidence w. From the $t = 0$ perspective we consider a continuum of w, as at $t = 0$ we do not know the exact value of the evidence that will be made at this future time instant.

Figure 14 and 15 present the resulting optimal investment decision at $t = 0$, for $x = 0.1$ as function of r and h. When compared to Figure 7 and 8 of the two-period model it is noted that the total investment in structural and non-structural measures decreases. The intermediate adjustment of the initial investment decision based on reduced area $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, reduces the stream of possible future damages, therefore at $t = 0$ with full uncertainty, there is no need to over-invest as in the near future information becomes available. With low discount rates and smaller $1/h$, there is less investment in non-structural measures as future annual costs receive more weight. With high discount rates and larger $1/h$, investment in non-structural measures increases and structural measures decreases. This is due to the high fixed investment costs of the structural measures compared to the non-structural fixed investment costs, and future annual costs receive less weight, which makes investment in non-structural measures more justifiable.

When the damage costs increase, with a low discount rate structural measures are still preferred over non-structural measures. However, when the expected resolution of uncertainty increases, investment in non-structural measures becomes justifiable as the period of possible damage increases. When the period of possible damages further increases (larger $1/h$), the high annual costs of the non-structural measures receive more weight, this leads to a reduction of non-structural measures and an increase in investment in structural measures. When the damage costs increase, the transition of investment in non-structural measures, at low discount rates, shifts upwards as the level of damages does not justify investment in non-structural measures.

When we consider the intermediate decision moment at $t = 0.5\kappa$, and compare Figure 16 and 17 with Figure 7 and 8, there is considerably less difference. With $x = 0.5$ partial resolution of uncertainty is used at 50% of the expected full resolution of uncertainty. At this intermediate time instant we have a posterior distribution that has a smaller reduced area $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, indicating less uncertainty, thus we have a more optimal adjustment of the initial investment decision than when $x = 0.1$. However, we only benefit from this adjustment for a short period, the period of reduced damages is shorter. Figure 16 and 17 show that the latter effect dominates, as the shrinking domain is less important than the moment at which the partial resolution of uncertainty is used. The time to the moment at which the partial resolution is used, is much longer, therefore the initial investment decision does not differ much from the initial investment decision in the two-period model. For a given σ (and thus δ and $1/h$), which relates to the capacity to collect evidence to reduce the domain of the prior distribution, the decision-maker can influence the timing of the intermediate investment decision through the selection of x. This choice impacts the level of reduced damages, which depends on the timing of the intermediate decision and associated level of partial resolution. When the decision-maker can influence the level of σ through additional research, there is a trade-off between the cost of additional research and the level of reduced damages.

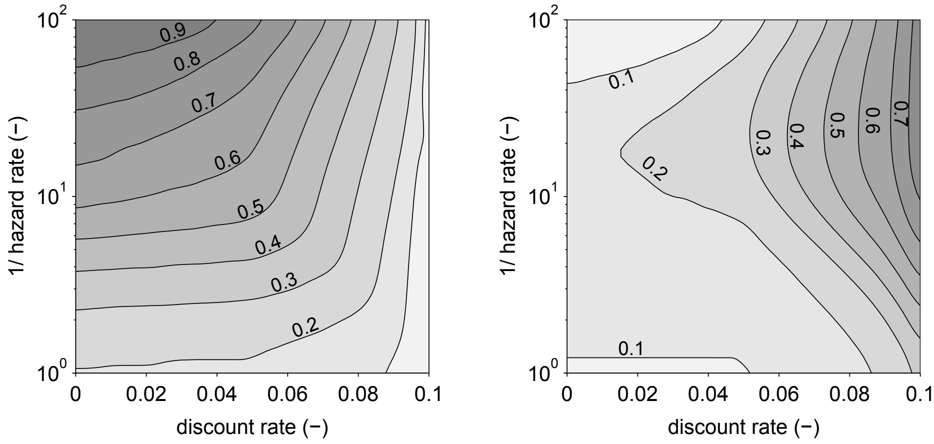
In this particular illustration we note that using the partial resolution at 10% of $1/h$ already leads to a considerable reduced domain $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. This is due to the value of δ, where $\delta = 0.025$ means that already at 10% of $1/h$ a large part of uncertainty is resolved. This is shown in Figure 13, where the prior distribution of α is uniformly distributed and the posterior of α for $x = 0.1$ already has a considerable reduced domain.



Structural measures

Non-structural measures

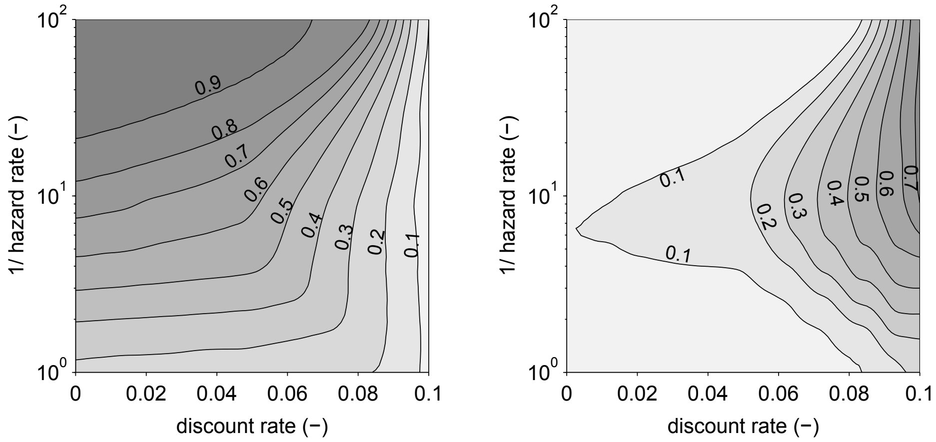
FIGURE 14: Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 200$ €, $D_{max} = 750$ € and $x = 0.1$, the calculation is based on step-size 0.01 for interval α.



Structural measures

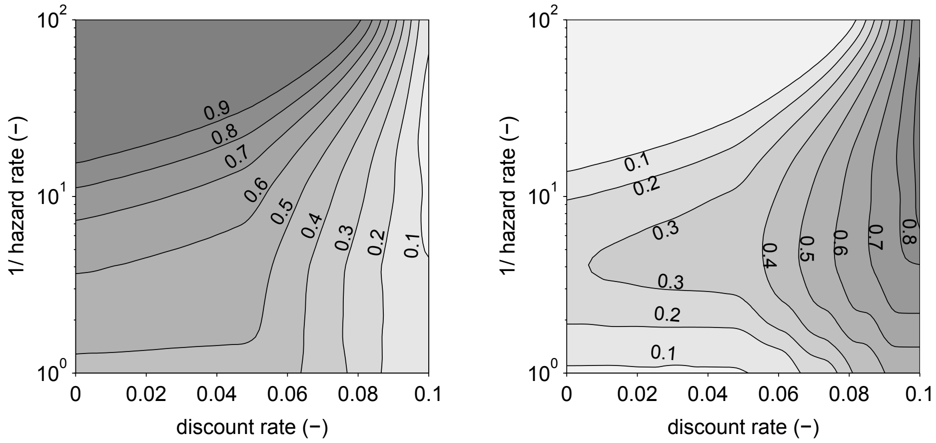
Non-structural measures

FIGURE 15: Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 200$ €, $D_{max} = 1500$ € and $x = 0.1$, the calculation is based on step-size 0.01 for interval α.)



Structural measures | Non-structural measures

FIGURE 16: Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 200$ €, $D_{max} = 750$ € and $x = 0.5$, the calculation is based on step-size 0.01 for interval α.



Structural measures | Non-structural measures

FIGURE 17: Optimal investment decision at $t = 0$ as a function of discount rate r and hazard rate $(1/h)$. $C_s = 1000$ €, $C_n = 500$ €, $c_s = 150$ €, $c_n = 200$ €, $D_{max} = 1500$ € and $x = 0.5$, the calculation is based on step-size 0.01 for interval α.

5 Implications for flood management

In this section we discuss the implications of the model outcome for decision-making in flood management. We first relate our model results to real-world decision making in flood protection. In the second part of this section we discuss four biases that could occur in flood protection decision-making and show their implications in the context of our model results.

5.1 Decision-making

Our definition of structural and non-structural measures—based on their ratio of fixed costs relative to annual costs—is slightly different from the one used by for instance Kundzewicz (2002, 2009). In this more general interpretation structural measures refer to engineering solutions (e.g. dikes, dams, reservoirs, diversions, channels, flood-ways), while non-structural measures refer to legislation, regulatory, and institutional solutions (e.g. watershed and landscape management, laws and regulations, zoning, economic instruments, and early warning systems). Although engineering solutions often induce relatively large fixed costs, this may not hold for all engineering solutions. A similar observation can be made with respect to regulatory and institutional solutions. Hence, while largely overlapping, the two definitions are not identical. Our definition of structural measures makes it possible to rank any set of measures according to their cost structure, where one engineering solution can be considered more structural than another (implying that it has a higher fixed-to-annual-costs ratio).

Over the last decades, flood management has shown a shift from structural to non-structural approaches. In many cases, decision-makers have decided to invest in a mix of both structural and non-structural measures. In the UK case, for instance, Penning-Rowsell et al. (2006) discuss such a gradual shift in flood management policy through the 20^{th} century. This shift from structural flood defense to flood risk management was stimulated by two major flood events in 1998 and 2000 (Tunstall et al., 2009). Although it is impossible to foresee future policy changes, Penning-Rowsell et al. (2006) predict a "greater reliance on a location-specific mix of non-structural and people-centred flood mitigation actions, and lessening of the influence of traditional approaches".

A similar shift has occurred in many other countries and basins, of which we mention two. In Germany, after the 2002 floods in the Elbe, new concepts for flood protection measures were developed, which included a combination of structural and non-structural measures. These non-structural measures were said to focus on the prevention and mitigation of the impact of floods (Petrow et al., 2006). In Canada, flood management reform was also triggered by major flood events. In this context, De Loë (2000) states that "responses to the flooding problem have evolved in Canada from an emphasis on controlling 'water out of place' through structural measures such as dams and dikes, to managing human behaviour using zoning to keep development away from hazardous areas."

The model shows that if climate change uncertainty is present, then non-structural measures become a more attractive option for flood protection. This shift in flood management is ongoing in many countries. There is increasing attention for non-structural measures in flood protection policy. Non-structural measures are often more flexible, less committing, and more sustainable (Kundzewicz, 2002). These characteristics are important, especially in the current context of uncertain impacts of climate change on flood damage, as discussed in Section 1. Both the flexibility and the commitment argument put forward by Kundzewicz (2002) show up as the key factors for investment decisions in our model setting. In the examples listed above (Germany, UK, Canada), the effects of climate change have entered the discussion on flood protection decision-making.

In conventional analyses of decision-making on flood protection, the role of uncertainty has often been ignored. Brouwer and Van Ek (2004), for instance, analyse several flood protection options in the context of a Dutch case study. They assess the trade-offs between costs and benefits of three policy measures (dike heightening, land use change and floodplain restoration), and conclude that the preference for one of these measures depends on the value attached to future ecological and socio-economic benefits. They do not, however, consider how uncertainty would affect the attractiveness of these measures. Current Dutch flood protection policies encompass a mix of structural and non-structural measures, and the public debate on protection from flood events revolves around the uncertainty of future flood events due to climate change. Without a doubt this uncertainty affects the optimal investment decision. Rosenberg et al. (2010) argue that for investments in storm-water infrastructure "the range of predicted change...is much too large to provide a basis for engineering design". This statement implies that when the impacts of climate change are uncertain, no sound investment decision in flood protection can be made. The results from our model, however, show that in the presence of uncertainty about the range of predicted climate change, it is possible to determine the optimal investment decision, where the prior distribution of α represents the prior knowledge of the decision-maker. A uniform distribution was implemented to indicate that no value is favored over any other. Furthermore, waiting for new information about climate change impacts, which reduces uncertainty, does not imply a complete postponement of the investment decision today. Depending on the level of uncertainty, though, structural or non-structural measures may be preferred. Specifically, our model results show that the decision-maker's preference for structural and non-structural measures depends on the combination of the cost structure of these measures, the level of the discount rate and uncertainty due to climate change.

An additional factor that may lead to a preference for non-structural measures is the short horizon of decision-makers in many institutional contexts. There is a disincentive to invest in structural measures if most benefits of these investments will only occur over a very long time horizon. Non-structural measures are more profitable in the short run, in terms of lower investment costs and may therefore provide a politically feasible alternative to structural measures.

5.2 Possible biases of decision-makers

The results of the three models developed in Sections 2–4 suggest that decision-makers may be biased in four ways. A first bias is that decision-makers could mistakenly assume a discrete set of states of nature (true or false) and ignore the continuous character of these impacts. The consequences of this bias can be analysed using the models of Sections 2 and 3. Clearly, if climate change is either true or false, this implies that the decision-maker assumes that either $\alpha = 0$ or $\alpha = 1$, while in fact the full range of values $\alpha \in [0, 1]$ is possible. This constraint on values of α gives more weight to the two extreme values in the initial decision to invest. Due to concavity of the damage function this leads to lower expected damages. Hence, this bias causes decision-makers to under-invest.

A second bias is related to the damage function and is discussed by Petrow et al. (2006) in the context of flood protection in the Elbe basin. They find that decision-makers in the Elbe basin have focused too much on one possible flood scenario, corresponding to the area affected by a 100-year return period flood. In the context of our model, decision-makers may assume one damage estimate corresponding to one particular value of α, instead of considering the full range of possible damage depending on the full range of possible climate change impacts. The result of this bias is similar; it leads to under-investment in flood protection measures.

A third bias occurs when decision-makers consider only one measure instead of a set of possible structural and non-structural measures. Each combination of the discount rate and climate change uncertainty implies a different optimal investment combination, as can be seen from the continuous models in Sections 3 and 4. Ignoring one (or more) measures would yield a sub-optimal investment decision. Whether this implies too much or too little investment in structural or non-structural measures depends on the set of measures considered, as well as their cost structure. Our model results show that ignoring one or more measures distorts the interplay between structural and non-structural measures that is required for optimal investment.

A fourth and last bias is the incorrect assumption that uncertainty will be resolved at once at a future date, while in reality this resolution is likely to be gradual. The difference in results between Sections 3 and 4 illustrates the implications of this bias. The optimal mix of structural and non-structural measures is affected by this bias. The option of adjusting the investment decision when more information arrives induces lower initial investments, however this depends on the timing of the intermediate investment moment and the level of partial resolution revealed.

6 Conclusion

Climate change uncertainty affects the decision to invest in flood protection measures. The model developed in this paper shows how an optimal investment strategy in flood protection measures reduces the risk of under- or over-investment to the decision-maker. Our results confirm the argument of Kundzewicz et al. (2010), who state that "flood preparedness (adaptation) measures should consist of an optimal, site-specific, mix from the menu of structural and non-structural measures". We provide a theoretical foundation for this argument using a model of decision-making under uncertainty. A combination of the discount rate, climate change uncertainty, and the cost structure of structural and non-structural measures determines the optimal mix of investments in these measures. Our model results predict that if climate change uncertainty is present, then non-structural measures become a more attractive option for flood protection.

The results from our continuous-state two-period model show that the level of the optimal mix of the structural and non-structural measures is affected by the level of the maximum annual flood damage and the expected arrival time of full resolution. If maximum annual flood damage and the expected arrival time of full resolution of uncertainty increases, this leads to longer periods of possible damages, which increases the level of the optimal mix of flood protection measures. The proportion of structural and non-structural measures in the optimal policy is affected by the cost structure, discount rate and expected arrival time of full resolution of climate change uncertainty. If the discount rate increases, this puts less weight on future costs, which decreases the investment in structural measures and increases in non-structural measures.

In the three-period model, the inclusion of an intermediate decision moment, where partial resolution of uncertainty is observed, leads to lower investments in structural and non-structural measures. However, the level of reduced damages is impacted by the timing of the intermediate investment moment and the level of partial resolution revealed. If the intermediate investment decision is in the near future, only little evidence is collected to update the prior distribution, thus only little knowledge about the true state is revealed. Still the investment decision can be updated to reduce the stream of possible future damages, therefore the initial investment decision is lower in anticipation of the intermediate decision moment. If the intermediate investment decision is later in time, more evidence is collected and the posterior distribution has a smaller range, which comes closer to full resolution of uncertainty. However, as the period between the initial investment moment and the intermediate moment is longer, the period of possible damages is longer, thus a higher level of initial investment will be justified compared to early partial resolution of uncertainty. When the decision-maker is able to increase the capacity to reduce climate change uncertainty through additional research, there is a trade-off between the cost of research and the level of reduced damages.

Thus, we conclude that the optimal investment decision today depends strongly on the cost structure of the adaptation measures and the discount rate, especially the ratio of fixed and weighted annual costs of the measures. We define the optimal investment decision today as a specific mix of measures that minimizes the total expected net cost. A higher level of annual flood damage and later resolution of uncertainty in time increases the optimal investment decision. Furthermore, the optimal investment decision today is influenced by the possibility of the decision-maker to adjust his decision at a future moment in time.

Although we have used river flooding as our motivating example, the results of this paper may apply more widely. A relevant application is coastal areas where climate change induces uncertain sea-level rise and related flood events. Again, different types of measures can be considered, that vary in their cost structure. Examples are dike heightening, beach nourishment, and restrictions on development and land use.

One possible extension to our model relates to the distinction between structural and non-structural flood protection measures. In this paper, we distinguished between the two based on their cost structure only, so that the measures are perfect substitutes.

Alternatively, the measures can be modeled as imperfect substitutes or as partly complementary, so that the interplay between the two measures is taken into account.

Appendix A

A motivation for the functional form of the function $\mathcal{D}(\alpha, s, n)$, introduced in Section 3 is the following. Consider the capacity of existing flood protection measures to be based on a known Gumbel distribution for peak flow discharge. The Gumbel distribution is a commonly used distribution for the modeling of peak river flow. Its probability density function is

$$g(w; \mu, \beta) = \frac{z \exp(-z)}{\beta} \quad \text{with} \quad z = \exp\left[\frac{w-\mu}{\beta}\right], \tag{56}$$

where w denotes peak discharge, β is the scale parameter and μ is the location parameter. Without loss of generality, we assume that climate change affects the scale parameter only, by scaling β by $(1 + \alpha\gamma)$, with $\gamma > 0$, such that $(1 + \alpha\gamma)\beta \geq \beta$.[5] Climate change leads to an increase of the scale parameter, implying 'fatter tails' in the distribution of peak discharge. This corresponds to evidence on increased variance of peak river flow (IPCC, 2007). The maximum increase in the scale parameter depends on the level of γ, i.e. if $\gamma = 1$ the scale parameter is doubled at maximum. We assume that there is at maximum one flood per year, correlating to that year's peak discharge. In year t, peak discharge w causes a flood if $w > \overline{w}$, where $\overline{w}$ denotes the maximum capacity provided by current protection measures. Damage from floods is increasing and concave in $w - \overline{w}$, so that we have the following damage function:

$$h(w, \overline{w}) = \begin{cases} \lambda(w - \overline{w}) & \text{if } w > \overline{w}\ , \\ 0 & \text{otherwise,} \end{cases}$$

where $\lambda(w - \overline{w})$ is increasing and concave. Expected damage D in a given year can now be calculated as the integral of the damage function over the Gumbel distribution of w (whose scale parameter is affected by α):

$$D = \int_0^\infty [h(w, \overline{w})]\ g(w) dw. \tag{57}$$

[5] Alternatively we could have modified the location parameter μ so that average peak discharge increases and thereby the probability of extreme peak discharges. Under our model assumptions, both methods lead to a similar—concave—relation between α and D.

Recall that we assumed that each combination of measures suffices to adapt to the impacts of climate change if $s + n \geq \alpha$. This assumption allows us to use the difference of α and $s + n$ in order to account for the mitigating effect of flood protection measures on damage. Hence we summarise the relation between climate change impact α, flood protection measures $s + n$, and expected damage D in the function $\mathcal{D}(\alpha, s, n) = D_{max}\sqrt{\alpha - s - n}$, that is increasing and concave in $(\alpha - s - n)$, with $\mathcal{D}(\alpha - s - n \leq 0) = 0$.[6]

[6]This requires that the damage function $\lambda(w - \overline{w})$ is sufficiently concave.

References

Becker, A. and U. Grünewald (2003). Disaster management: flood risk in central Europe. *Science 300*(5622), 1099.

Bolstad, W. M. (2007). *Introduction to Bayesian statistics.* John Wiley & Sons, Inc., Hoboken, New Jersey, USA.

Brouwer, R. and R. Van Ek (2004). Integrated ecological, economic and social impact assessment of alternative flood control policies in the Netherlands. *Ecological Economics 50*(1-2), 1–21.

Choi, J. P. (1991). Dynamic R&D competition under "hazard rate" uncertainty. *The RAND Journal of Economics 22*(4), 596–610.

Christensen, J. H. and O. B. Christensen (2003). Severe summertime flooding in Europe. *Nature 421*(6925), 805–806.

De Bruin, K., R. B. Dellink, A. Ruijs, L. Bolwidt, A. Van Buuren, J. Graveland, R. S. De Groot, P. J. Kuikman, S. Reinhard, R. P. Roetter, V. C. Tassone, A. Verhagen, and E. C. Van Ierland (2009). Adapting to climate change in the Netherlands: An inventory of climate adaptation options and ranking of alternatives. *Climatic Change 95*(1-2), 23–45.

De Kok, J.-L. and M. Grossmann (2010). Large-scale assessment of flood risk and the effects of mitigation measures along the Elbe River. *Natural Hazards 52*(1), 143–166.

De Loë, R. (2000). Floodplain management in Canada: overview and prospects. *Canadian Geographer 44*(4), 355–368.

Dixit, A. K. and R. S. Pindyck (1994). *Investment under uncertainty.* Princeton, New Jersey: Princeton University Press, USA.

Fankhauser, S., J. B. Smith, and R. S. J. Tol (1999). Weathering climate change: some simple rules to guide adaptation decisions. *Ecological Economics 30*(1), 67–78.

Helms, M., B. Büchele, U. Merkel, and J. Ihringer (2002). Statistical analysis of the flood situation and assessment of the impact of diking measures along the Elbe (Labe) river. *Journal of Hydrology 267*(1-2), 94–114.

Hennessy, D. A. and G. Moschini (2006). Regulatory actions under adjustment costs and the resolution of scientific uncertainty. *American Journal of Agricultural Economics 88*(2), 308–323.

IPCC (2007). Climate Change 2007: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Fourth Assessment Report of the International Panel on Climate Change [M. L. Parry, O. F. Canziani, J. P. Palutikof, P. J. van der Linden and C. E. Hanson (eds.)]. Cambridge University Press, Cambridge, UK, 976pp.

Kundzewicz, Z. W. (2002). Non-structural flood protection and sustainability. *Water International 27*(1), 3–13.

Kundzewicz, Z. W. (2005). Is the frequency and intensity of flooding changing in Europe? *In book: Extreme weather events and public health responses. Springer Berlin, Heidelberg, Germany*, 25–32.

Kundzewicz, Z. W. (2009). Adaptation to floods and droughts in the Baltic Sea basin under climate change. *Boreal Environment Research 14*, 193–203.

Kundzewicz, Z. W., D. Graczyk, T. Maurer, I. Pińskwar, M. Radziejewski, C. Svensson, and M. Szwed (2005). Trend detection in river flow series: 1. Annual maximum flow. *Hydrological Sciences Journal 50*(5), 797–810.

Kundzewicz, Z. W., Y. Hirabayashi, and S. Kanae (2010). River floods in the changing climate - observations and projections. *Water Resources Management 24*(11), 2633–2646.

Malueg, D. A. and S. O. Tsutsui (1997). Dynamic R&D competition with learning. *The RAND Journal of Economics 28*(4), 751–772.

Milly, P. C. D., R. T. Wetherald, K. A. Dunne, and T. L. Delworth (2002). Increasing risk of great floods in a changing climate. *Nature 415*(6871), 514–517.

Mudelsee, M., M. Borngen, G. Tetzlaff, and U. Grunewald (2003). No upward trends in the occurrence of extreme floods in central Europe. *Nature 425*(6954), 166–169.

Patwardhan, A. and M. J. Small (1992). Bayesian methods for model uncertainty analysis with application to future sea level rise. *Risk Analysis 12*(4), 513–523.

Penning-Rowsell, E., C. Johnson, and S. Tunstall (2006). Signals' from pre-crisis discourse: lessons from UK flooding for global environmental policy change? *Global Environmental Change 16*(4), 323–339.

Petrow, T. and B. Merz (2009). Trends in flood magnitude, frequency and seasonality in Germany in the period 1951–2002. *Journal of Hydrology 371*(1-4), 129–141.

Petrow, T., A. H. Thieken, H. Kreibich, B. Merz, and C. H. Bahlburg (2006). Improvements on flood alleviation in Germany: lessons learned from the Elbe flood in August 2002. *Environmental Management 38*(5), 717–732.

Rosenberg, E. A., P. W. Keys, D. B. Booth, D. Hartley, J. Burkey, A. C. Steinemann, and D. Lettenmaier (2010). Precipitation extremes and the impacts of climate change on stormwater infrastructure in Washington State. *Climatic Change 102*(1-2), 319–349.

Silva, W., J. P. M. Dijkman, and D. P. Loucks (2004). Flood management options for The Netherlands. *International Journal of River Basin Management 2*, 101–112.

Te Linde, A. H., J. Aerts, R. Hurkmans, and M. Eberle (2008). Comparing model performance of two rainfall-runoff models in the Rhine basin using different atmospheric forcing data sets. *Hydrology and Earth System Sciences 12*, 943–957.

Tol, R. S. J., N. Van der Grijp, A. A. Olsthoorn, and P. E. Van der Werff (2003). Adapting to climate: a case study on riverine flood risks in the Netherlands. *Risk Analysis 23*(3), 575–583.

Towler, E., B. Rajagopalan, E. Gilleland, R. S. Summers, D. Yates, and R. W. Katz (2010). Modeling hydrologic and water quality extremes in a changing climate: A statistical approach based on extreme value theory. *Water Resources Research 46*(11), W11504.

Trenberth, K. E., P. D. Jones, P. Ambenje, R. Bojariu, D. Easterling, A. Klein Tank, D. Parker, F. Rahimzadeh, J. A. Renwick, M. Rusticucci, B. Soden, and P. Zhai (2007). Observations: Surface and Atmospheric Climate Change. In: Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the International Panel on Climate Change [S. Solomon, D. Qin, M. Manning, Z. Chen, M. Marquis, K.B. Averyt, M. Tignor and H.L. Miller (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Tunstall, S., S. McCarthy, and H. Faulkner (2009). Flood risk management and planning policy in a time of policy transition: the case of the Wapshott Road Planning Inquiry, Surrey, England. *Journal of Flood Risk Management 2*(3), 159–169.