# **Costs Savings of a Flexible Multi-Gas Climate Policy**

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#### Sammendrag:

Gjeldene klimapolitikk er basert på bruken av globale oppvarmingspotensialer (GWPer) for å sammenligne utslipp av ulike drivhusgasser. I et økonomisk perspektiv fins det imidlertid mer effektive måter å sammenligne ulike gasser på. Formålet med denne studien er å undersøke mulige kostnadsbesparelser ved å bruke en mer effektiv og fleksibel metodikk knyttet til vekting av ulike gasser, når stabilisering av klimaet på lang sikt er målet. Kostnadsbesparelsene blir også beregnet i forhold til en strategi der kun CO<sub>2</sub> utslipp reduseres. Sammenlignet med en strategi basert på fleksible vekter, finner vi at de globale kostnadene ved klimapolitikk øker med omtrent 2% ved bruk av GWP. Dette tilsvarer mellom 16 og 106 Mrd \$ per år avhengig av hva det langsiktige stabiliseringsmålet er. Hvis målet skal nås kun gjennom reduksjon av CO<sub>2</sub> utslipp øker kostnadene med ca 11%. Basert på disse estimatene konkluderer vi med at de største kostnadsbesparelsene knyttet til det å inkludere flere gasser i en langsiktig klimapolitikk, kan bli realisert med bruk av GWPer selv om disse vektene er svært forskjellige fra de som er mest økonomisk effektive.

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#### Abstract:

Current climate policies are based on the use of the Global Warming Potential (GWP) index to compare various greenhouse gases. Yet, from an economic point of view, more efficient methods exist. The purpose of this paper is to examine the potential cost savings from applying an efficient and more flexible metric as compared to using GWPs, given some longterm goal for stabilization of the climate. We also calculate the costs when only emissions of carbon dioxide  $(CO_2)$  are targeted. As compared to the least cost multi-gas flexible case, we estimate that the mitigation costs are increased by about 2% by using GWPs, which amounts to about 16-106 Billion US \$ per year depending on the stabilization goal. If only CO<sub>2</sub> emissions are targeted, costs increase by about 11%. Given our assumptions we conclude from this that most cost savings that stem from including non- $CO_2$  greenhouse gases in climate policy may be realized when applying GWPs, even though these gas tradeoffs are rather different from the efficient ones.

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# **1** Introduction

Many of the gases emitted from human activities enhance the greenhouse effect, yet they do so in quite different manners. The responses to a perturbation of some gases, like carbon dioxide ( $CO_2$ ), lasts for hundreds of years, while for others, like methane ( $CH_4$ ), the perturbation degrades within a decade or two. When designing policies to stabilize the climate, it is important to ask not only how much the various GHGs should be reduced, but also *when* it should be done.

These questions have puzzled many researchers from different disciplines the last couple of decades. In an attempt to give some guidance to the first question, Lashof and Ahuja (1990) and Rohde (1990) provided a first order assessment of the total impact on climate from emissions of the various gases. This resulted in the Global Warming Potential (GWP) index (IPCC, 1990), which has been adopted in the Kyoto Protocol. The index compares the radiative forcing from a pulse emission of a gas integrated over a given time horizon to that of  $CO_2$ . Multiplication of the emissions of various non- $CO_2$  gases by their respective GWP values translate these into a common (and perhaps somewhat misleading) unit: namely "CO<sub>2</sub> equivalents".

Subsequently, various studies of the importance of the various gases from an economic perspective that recognized the dynamic nature of the climate problem started to appear in the literature. By now, this collection of studies is quite extensive, and they all implicitly or explicitly criticize the GWP approach. The methodology common in these studies is to estimate the optimal mix of abatement of various gases, given some economic criterion. From this, the efficient prices, and hence the efficient gas tradeoffs or weights are derived. Two approaches to this problem prevail in the economic literature. One, which we shall adopt, is the *cost-effective approach*, where some pre-specified climate constraint is taken as given, and costs subject to that constraint are minimized (Eckaus, 1992; Michaelis, 1992; 1999; Aaheim, 1999; Manne and Richels, 2001; and O'Neill, 2003). The other, the so-called *cost-benefit approach*, determines the optimal mix of abatement of various gases based on the balancing of climate damages and abatement costs (see Fuglestvedt et al., 2003 for an overview of relevant studies).<sup>1</sup>

Two conclusions can be drawn from the economic analysis of efficient greenhouse gas tradeoffs. First, the weights for how emissions of various GHGs could be guided efficiently today may be quite different from those given by the GWP index. However, whether a particular gas is given a weight that is too small or too large in the GWP index seems to depend particularly on the assumptions made in the economic model, where the chosen objective for reducing emissions from a general perspective typically plays a key role. The second finding, which is only evident from those studies analyzing the intertemporal optimization problem, is that the weights change over time, not only for climatic reasons as is true also for the GWP index, but also for economic reasons. For instance, it follows from Manne and Richels (2001) that if the objective is to stabilize the long-term climate, reducing emissions of methane today is practically worthless from an economic point of view. This is reflected in a weight close to zero for this gas for several of the upcoming decades. Towards the end of the century, however, methane emissions reductions are much more valuable than reflected by the GWP index. However, if the rate of climate change is of greater concern, these results may be at least in part reversed, illustrating the importance of the relationship

<sup>&</sup>lt;sup>1</sup> It may be added that the cost-effective approach essentially is a specific case of the cost-benefit approach, namely the one where damages are assumed to be zero below a specific level of climate change, and infinitely high above this level.

between how gases can be compared efficiently and what the purpose of emissions reductions generally are.

It is clear that the use of the GWPs for policy making is fixed for the period of the Kyoto Protocol, 2008-2012. And, even though the Intergovernmental Panel on Climate Change (IPCC) so far has not assessed other more promising ways to compare GHGs (Godal, 2003), that issue may appear on the agenda when designing policies for combating climate change after the year 2012, especially as the principle of comprehensiveness is embedded in the UNFCCC.<sup>2</sup>

However, although a wide range of studies conclude that using the GWP index may lead to inefficiencies (Fuglestvedt et al., 2003), a much less studied topic is how costly the GWP index potentially may turn out to be. The extra costs that may be incurred by relying on the GWP index is our main concern here.

Few studies have addressed this issue. Godal and Fuglestvedt (2002) examined the effects of using different indices on the composition of least cost abatement of various gases in Norway. The effects were found to be relatively small. However, since the impact on climate beyond emissions in terms of  $CO_2$  equivalents was not modeled, the actual costs of using one particular index as compared to another were not investigated for a more comprehensive climate measure (say radiative forcing or temperature change). Also, due to its limited regional relevance, the study begged for a broader analysis where the costs of using the GWP index could be more properly assessed in a global long-term framework.

This broader analysis was accomplished by O'Neill (2003), who provided a first order estimate of the issue addressed here. The study used a cost-effectiveness framework and included two gases ( $CO_2$  and  $CH_4$ ), and it was found that the costs of climate policy over the next 100 years was reduced by 2% if the flexible approach was used rather than an approach with fixed GWPs. From this it was concluded that the flexible framework would not substantially decrease the total costs of climate policy for a given constraint on the level of radiative forcing, as compared to a case when the GWPs were used, although it could have a substantial effect on the least cost mix of reductions of the various gases.

We follow O'Neill (2003) in the sense that we apply a cost-effectiveness dynamic framework. This choice is not obvious, but keeping in mind the level of understanding of the nature of monetary impacts that may result from climate change, we believe it is as good as any other particular choice. Our analysis differs from O'Neill (2003) in several ways. For instance, we do not rely on abatement cost functions for  $CO_2$  that are derived from a computable general equilibrium model, but rather explicitly model the intertemporal production, consumption and investment decisions. We also include additional gases. Together with O'Neill (2003), our analysis may give some guidance to policy makers as to whether or not they would want to implement a different index than GWPs for future climate agreements.

We organize the paper as follows: In section 2 we outline the main characteristics of the model; the details are relegated to an Appendix. We first spell out the problem of the free, flexible case, and then turn to the additional constraint for the cases when using GWPs and when only  $CO_2$  emissions are targeted. Section 3 provides a brief summary of the choice of functions and parameters and draws up the various scenarios. The numerical results appear in section 4, while section 5 discusses some limitations and concludes.

<sup>&</sup>lt;sup>2</sup> According to Article 3.3. "... policies and measures should ... cover all relevant sources, sinks and reservoirs of greenhouse gases..."

# 2 Model

### 2.1 A flexible multi-gas climate policy

The model is highly stylized, just detailed enough to capture some important features of the issues addressed here. It should be clear from the outset that it is deterministic. There is one global region where households at any time  $t \in [0,T]$  derive utility from consuming energy  $x_u(t)$  and "other goods"  $x_c(t)$ . The objective is to maximize the total welfare over the given time horizon, that is,

$$W = Max \int_{t=0}^{T} V(x_c(t), x_u(t)) e^{-\delta t} dt , \qquad (1)$$

where  $V(x_c(t), x_u(t))$  measures the instantaneous welfare at time t, and  $\delta$  is the consumers' pure rate of time preference. The decision variables are specified in the Appendix.

There are two production sectors, i = 1,2. Sector 2 produces energy – sector 1, other goods (and services). Both sectors demand energy and capital, such that the production function in sector *i* is given by  $f^i(u_i(t), k_i(t))$ , where  $u_i(t)$  is the use of energy and  $k_i(t)$  the use of capital in sector *i* at time *t*.

The total capital stock of the economy, k(t), is divided between the two production sectors

$$k(t) = k_1(t) + k_2(t).$$
(2)

Capital goods are produced in sector 1, "other goods", only. In addition to providing capital goods, sector 1 also provides consumption goods,  $x_c$ , and input to the abatement of GHG emissions. Denote by  $a_j$  the use of products from sector 1 to the abatement of the gas j = 1, ..., n. The growth in the total stock of capital of the economy is then given by

$$\dot{k}(t) = f^{1}(u_{1}(t), k_{1}(t)) - x_{c}(t) - \sum_{j=1}^{n} a_{j}(t).$$
(3)

Note that the depreciation term for sectors 1 and 2 is omitted to facilitate the presentation. The differential equation (3) clears the market for "other goods": The output from this sector is partly spent on consumption and abatement. The remaining is saved and invested to increase the total stock of capital.

Moreover, equilibrium in the energy market requires that at any time t it has to hold that

$$f^{2}(u_{2}(t),k_{2}(t)) = x_{u}(t) + u_{1}(t) + u_{2}(t).$$
(4)

Emissions stem from the use of energy.<sup>3</sup> Thus, there is an emission coefficient attached to energy use for each greenhouse gas, j = 1, ..., n which we denote by  $b_{i,j} \ge 0$ , to account for the fact that the emissions coefficient varies across sectors, *i*, including households that we denote as sector i = 3. The emissions can be reduced by reducing energy use, or by spending amount

<sup>&</sup>lt;sup>3</sup> The numerical calculations include also exogenous emissions, which we omit in this description.

 $a_j(t)$  on direct abatement of gas *j*. The abatement  $a_j(t)$  reduces emissions of gas *j* by  $G^j(a_j(t))$ , where  $\partial G^j/\partial a_j > 0$  and  $\partial^2 G^j/\partial a_j^2 < 0$ .

Emissions increase the atmospheric concentrations by the constant  $B_j$ . We can then redefine the emission coefficients and the reductions from direct abatement in order to express them in terms of concentrations units to  $\beta_{i,j} = B_j b_{i,j}$  and  $g^j(a_j) = B_j G^j(a_j)$ , respectively.

For a given flow of emissions of the various gases, the concentration of each gas evolves according to a specific process. For CO<sub>2</sub> (*j*=1) this is a rather complicated process, which we shall simplify considerably for programming reasons. Hasselmann et al. (1997) provides a simplified representation of results from the more advanced general circulations models, consisting of a weighted sum of five exponential terms with different but constant decay rates. We simplify Hasseslmann et al's relation further by dividing the CO<sub>2</sub> concentration into two components only. One part,  $\psi \in (0,1)$ , accumulates totally in the atmosphere, while the remaining part, (1- $\psi$ ), declines naturally at the constant rate  $1/\tau_j$ , *j* = 1, where  $\tau_j$  is what is usually referred to as the lifetime of gas *j*.<sup>4</sup>

Thus, the degrading part of the CO<sub>2</sub> concentrations,  $dS_1^D(t)$ , evolves according to

$$\dot{S}_{1}^{D}(t) = -\frac{1}{\tau_{1}}S_{1}^{D}(t) + (1-\psi)\left(\beta_{3,1}x_{u}(t) + \sum_{i=1,2}\beta_{i,1}u_{i}(t) - g^{T}(a_{1}(t))\right), \quad (5)$$

whereas, the perfectly accumulating part develop according to

$$\dot{S}_{1}^{A}(t) = \psi \left( \beta_{3,1} x_{u}(t) + \sum_{i=1,2} \beta_{i,1} u_{i}(t) - g^{T}(a_{1}(t)) \right).$$
(6)

The total amount of CO<sub>2</sub> concentration at any time is therefore given by

$$S_1(t) = S_1^D(t) + S_1^A(t) .$$
<sup>(7)</sup>

The concentrations of all the other gases  $S_i(t)$ , j = 2,...,n, evolves according to

$$\dot{S}_{j}(t) = -\frac{1}{\tau_{j}}S_{j}(t) + \beta_{3,j}x_{u}(t) + \sum_{i=1,2}\beta_{i,j}u_{i}(t) - g^{j}(a_{j}(t)), \quad j = 2,...,n, \quad (8)$$

which has the same form as the variable part of the  $CO_2$  response (5).

By definition, changed concentrations of GHGs contribute to radiative forcing, denoted  $\Phi(S_1(t),...,S_n(t))$ , although in quite different manners. It is therefore convenient to make further specification of the index *j*. Denote CH<sub>4</sub> by *j* = 2, nitrous oxide (N<sub>2</sub>O) by *j* = 3 and sulfur hexaflouride (SF<sub>6</sub>) by *j* = 4. We shall restrict our analysis to these four gases.<sup>5</sup> The

<sup>&</sup>lt;sup>4</sup> The lifetime of a gas is defined as the time it takes for some amount of concentration to decrease to 1/e of its initial level, where e = 2.718...

 $<sup>^{5}</sup>$  CO<sub>2</sub>, CH<sub>4</sub>, and N<sub>2</sub>O are currently the main contributors to radiative forcing among the gases targeted in the Kyoto Protocol, and are likely to remain so in the future. We have included the industrial gas SF<sub>6</sub> mainly because of its long life-time, to see how the control of this gas changes when GWPs are replaced by a long-term target on forcing. The contributions to radiative forcing from the remaining

relationship between the concentrations of the various gases and the total radiative forcing function is based on IPCC, (2001), that is,

$$\Phi(t) = \alpha_1 \ln(S_1(t)/S_1^0) + (1+\mu)\alpha_2(\sqrt{S_2(t)} - \sqrt{S_2^0}) + \alpha_3(\sqrt{S_3(t)} - \sqrt{S_3^0}) + \alpha_4(S_4(t) - S_4^0)$$
(9)

where  $a_j$ , j = 1...4, are constant scaling parameters that transform the concentration function of gas *j* into radiative forcing.  $S_j^0 = S_j(0)$  is the initial concentration of gas *j*, which is year 2000 in the numerical model. The parameter  $\mu$ , encapsulates what is known as the indirect effects of methane, and is set to 0.3 in accordance with IPCC (2001). The so-called "overlap term" which accounts for the interactions between N<sub>2</sub>O and CH<sub>4</sub> on radiative forcing, has thus been omitted here.

Finally, our approach is such that radiative forcing in the last period cannot exceed a specific level  $\overline{\Phi}$ , that is,

$$\Phi(T) \le \overline{\Phi} \ . \tag{10}$$

A more common formulation is to assume that the target could not to be exceeded at any instant  $t \in [0, T]$ . However, to simplify computations, we apply (10). It is worth stating that the only motivation for positive abatement in this optimal control problem is due to the constraint on the level of radiative forcing in year *T* as given in (10), a so-called transversality condition.

What remains for the specification of the model is a transversality condition for real capital to avoid the stock of capital to be consumed towards the last period. We require that the present value of real capital not be lower in the terminal year t = T, than at t = 0, that is,

$$r_1(T)K(T) = r_1(0)K(0) \tag{11}$$

where  $r_1(T)$  is the user price of capital in the terminal year. The user price is endogenous, and (11) can be interpreted as a conditional requirement for sustainability, related to the present productivity of the capital stock. The initial quantities of the stock variables, K(0) and  $S_j(0)$ , including the share of constant and variable concentrations of CO<sub>2</sub>,  $\psi$ , are known at t = 0. The description of the objective and the constraints is now complete. The solution is presented in the Appendix. What follows below is a brief economic interpretation of the optimum conditions.

Household demand is determined by the condition that the price equals the marginal utility of the two goods. The subjective utility of energy use must, however, be adjusted for the shadow cost of emissions, which illustrates the optimal emissions charge. In both production sectors, the product prices equal the marginal productivity of the two input factors, energy and capital. In the case of energy use, the shadow cost of associated emissions is subtracted from marginal productivity. The capital stock is to be allocated to assure that the value of the marginal productivity of capital is equal in the two sectors.

gases in the Kyoto Protocol, PFCs and HFCs, are similar to that of  $SF_6$  – i.e. very small. They may, of course, be important for the implementation of climate policy in specific sectors of the economy, and even to a country in some cases. With the world perspective, they may be left out without loss of principal insight.

A shadow cost is attached to the emissions of each gas. In optimum, the marginal cost of abatement must be equal to the marginal implicit value of reducing concentrations, which is related to the target for radiative forcing. This is the same as requiring that an additional  $\in$  spent on abatement of a gas is to reduce the cost of emissions of this gas implicitly imposed by the target by one  $\in$ .

The intertemporal optimum conditions describe the growth process of the economy. The consumption-saving problem corresponds to the so-called Ramsey rule (see e.g. Blanchard and Fisher, 1989). This rule says that, at each point in time, the temporary marginal welfare loss by abstaining from consumption "now" equals the marginal welfare gains from investing "now" in future consumption.

The growth in abatement depends on two gas-specific characteristics. The first is the abatement cost functions, or more precisely, the rate of change in the "productivity" of abatement expenditures,  $g^{i}(a_{j})$ , which must compare with the productivity of investments expressed by the endogenous discount rate. Since the abatement cost functions are specified for each gas, the optimal composition of abatement will depend on the shape of these functions. As will soon be explained, it may be noted that this requirement is not met when using GWPs. The second characteristic determining the growth in abatement is the lifetime of each gas: the shorter the lifetime, the higher the abatement growth rate. A high growth rate normally indicates a 'low' initial abatement level. The intuition behind this result is that early abatement is inefficient for gases with short life-times because it will not have any significant effect when the target is approached by the end of the period.

The final composition of abatement of the different gases is determined in the year when the target is approached, which is the terminal year in the calculations in this study. Then in optimum, the marginal cost of abating one 'unit' of radiative forcing is equal for all the gases.

It might be added that the negative impacts from climate change could be related also to the rate of change, for example in radiative forcing or temperature change, and indeed several analysts have calculated optimal climate policy including such effects. The typical approach taken is to simply include a constraint  $\partial \Phi(t) / \partial t \leq \overline{\Phi}$  in the calculations (see e.g Michaelis, 1999). It is, however, unclear whether possible rate dependent impacts are well represented by such a function, especially within an optimization model. The optimization itself will usually contribute to stabilize the rate of increase in the chosen climate parameter, here being radiative forcing.

The model gives rise to an independent path for the abatement of each gas, linked only by the transversality constraint. From each path we can derive the composition of abated quantities of the gases. We will refer to these as "flexible weights", as opposed to the fixed weights when GWPs are used. Note that the marginal abatement costs may be interpreted as the efficient emissions charge on the various gases under a tax regime.

### 2.2 The fixed GWP and the CO<sub>2</sub> control only cases

To compare the use of "flexible weights" with the use of GWPs, it is necessary to formalize an additional constraint that encapsulates such a framework. We do this by replacing the optimum condition for optimal composition of abatement with the constraint that relative marginal abatement cost between the various gases should equal the values of the GWP index at all instances. This is appropriate since cost-minimizing behavior in such a regime implies this condition. Hence, this constraint amounts to<sup>6</sup>

<sup>&</sup>lt;sup>6</sup> It is more common to think of abatement cost functions as the cost required to abate a specific amount of emissions, in which case relative marginal abatement costs should equal the relative GWPs.

$$\frac{\partial g^{\prime}(a_1(t))}{\partial g^{\prime}(a_j(t))} / \frac{\partial a_1(t)}{\partial a_j(t)} = GWP_j \quad \text{for } j = 2, \dots, n.$$

$$(12)$$

Thus, only the path of one gas, notably CO<sub>2</sub>, is independently determined in this alternative.

In the  $CO_2$  control only case, we simply set the abatement of the other gases exogenously to zero. Recall, however, that the emissions of these gases may still be reduced under emissions targets as a result of reductions in energy use.

# 3 Parameters and Scenarios

### 3.1 Parameters

The model was calibrated with data for economic activity, energy use, emissions and abatement costs applied on the world total. Data were provided by the Energy Modeling Forum (EMF), and abatement cost data for non-CO<sub>2</sub> gases were provided by the United States Environmental Protection Agency (EPA) through EMF. Direct abatement costs for CO<sub>2</sub> are very difficult to obtain. The figures used in our numerical analysis must be considered only as qualified guesses. Household consumption was calibrated to a linear expenditure system, and both production sectors are represented by a CES production technology.

In calibrating the model, initial consumption, production and emissions would have to correspond to the optimal solution. The simple structure of the model, plus the constraints on the terminal capital stock and concentrations applied to the world total suggest that the initial level of the control variables may deviate substantially from the observed values. In the numerical model, consumption of non-energy goods exceeded the observed by 8 percent, total energy use by 10 percent and emissions by  $\pm 10$  percent for the different gases. Generally, energy use is somewhat overestimated. In order to attain a reasonable level and growth rate for consumption, the pure rate of time preference,  $\delta$ , was set as low as  $\frac{1}{2}$  percent.

The data for the abatement cost functions were adapted by log-linear functions,  $g^{j}(a_{j}) = A_{j}a_{j}^{\theta_{j}}$ , where  $A_{j}$  and  $\theta_{j}$  are constants. This functional form did not fit the estimated costs very well for all the gases. For N<sub>2</sub>O, in particular, the potential for direct emissions reductions is much larger in the model than indicated by the abatement data, which suggest a very low cost up to 30% reductions, but infinitely high above 40%. The EPA emphasizes, however, the uncertainties in these estimates, especially for non-developed countries. Table 1 provides some of the gas-specific parameters used in the numerical analysis.<sup>7</sup>

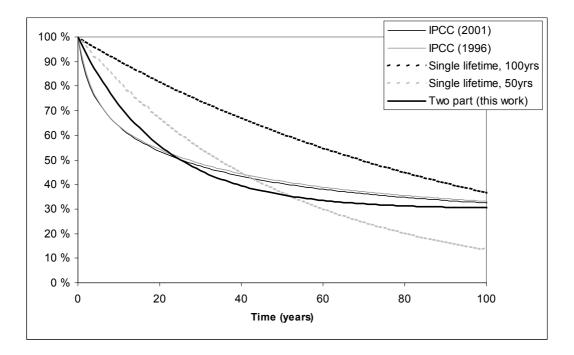
However, in our model formulation, cost functions are measured in terms of the abatement achieved by spending a specific amount of dollars, which is thus the inverse of the more commonly used form.

<sup>&</sup>lt;sup>7</sup> Mt are million metric tons; Gt is giga metric ton; C is carbon; N is nitrogen; ppmv, ppbv and pptv are parts per million, billion and trillion volume respectively; yr is year; and US\$ are United States (1995) dollars.

**Table 1.** Some parameter values. Units are:  $A_j$ ,  $\theta_j$  are such that productivity of abatement is measured in terms of GtC, MtCH<sub>4</sub>, MtN and MtSF<sub>6</sub> all per trillion US\$; GWPs in tonne CH<sub>4</sub>, N<sub>2</sub>O and SF<sub>6</sub> per tonne CO<sub>2</sub> respectively; lifetimes,  $\tau_j$ , in years;  $\alpha_j$ , are such that radiative forcing is given by W/m<sup>2</sup>; initial concentrations in ppmv (CO<sub>2</sub>), ppbv (CH<sub>4</sub> and N<sub>2</sub>O) and pptv (SF<sub>6</sub>). The lifetime of CO<sub>2</sub> is discussed below Figure 2.

Parameter	Symbol	CO <sub>2</sub>	CH <sub>4</sub>	N <sub>2</sub> O	$SF_6$
Abatement cost function parameters	$A_{j}$	0.60	61.18	0.46	0.0038
	$\theta_{j}$	0.35	0.23	0.12	0.17
Global Warming Potential	GWP	1	23	296	22 200
Lifetime	$ au_j$		12	120	3 200
Radiative forcing	$\alpha_j$	5.35	1.30	0.12	0.52
Initial concentration	$S_j^0$	368	1 759	316	4.2

Regarding the parameterization of the climate module, we have relied on formulation and values provided by the IPCC (2001), with the exception of the response function for  $CO_2$ . Figure 1 shows how our  $CO_2$  model compares to those given in IPCC (2001 and 1996) and to a single lifetime formulation.



**Figure 1.** The response function for  $CO_2$  applied here as compared to IPCC (1996), IPCC (2001), fixed single lifetime of 100 years and 50 years, and the one applied here. Percent remaining after emitting a unit pulse at time zero.

We found that setting  $\psi$  to 0.3 (such that 30% of the CO<sub>2</sub> emissions accumulates totally) and the remaining degrades at an annual rate of 5% ( $\tau_1$  is 20 years) fitted reasonably well. It is

clear from Figure 1 that the simplified  $CO_2$  model applied here is an improvement over the single lifetime approach often applied in similar optimal control problems (e.g. by Michaelis (1992;1999)). However, as compared to the IPCC carbon cycles, our formulation yields somewhat higher concentrations the first two decades, after which it is somewhat below the better approach.

It should be pointed out that we have chosen to apply the GWPs reported in IPCC (2001) when these are forced onto the optimization problem. This choice is not obvious since the GWPs that are to be used in the Kyoto Protocol are actually the ones given in IPCC (1996). However, since those values are based on parameters that are not consistent with the more recently published parameters that we choose to apply in our climate module, they were not chosen.

### 3.2 Scenarios

An immediate question in our analysis is: What could constitute an appropriate radiative forcing constraint? What is "*dangerous anthropogenic interference with the climate system*" as formulated by the climate convention? In our reference scenario, without emissions abatement, radiative forcing from the four gases considered here approaches about 5.7 W/m<sup>2</sup> towards the end of the century, as compared to the year 2000, which is within the range given by the so-called SRES-scenarios provided by the IPCC (2001).<sup>8</sup> We have chosen two cases, setting this constraint to a level of 4.0 W/m<sup>2</sup> and 2.58 W/m<sup>2</sup> respectively. The latter corresponds to a CO<sub>2</sub> concentration of 550 ppmv in the flexible alternative, which is often used in policy analysis.

To sum up, our numerical analysis considers six scenarios. Three different policies (flexible multi-gas, fixed GWPs multi-gas, and  $CO_2$  only) each studied under the constraint of limiting radiative forcing in 2100 to 4.0 and 2.58 W/m<sup>2</sup>, respectively. The corresponding naming is FLEX-4.0, FGWP-4.0, CO2-4.0, FLEX-2.58, FGWP-2.58 and CO2-2.58. In addition, the reference scenario without any emissions abatement is named REF.

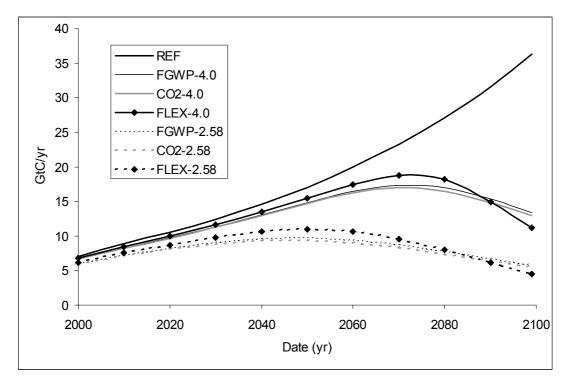
# 4 Results

### 4.1 Emissions and radiative forcing

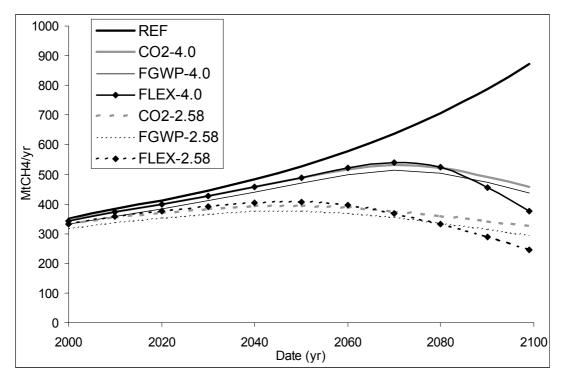
The optimal emissions trajectories for the two most important gases,  $CO_2$  and  $CH_4$  for the various cases are given below.

Figure 2 yields some intuitive and some surprising results. Fist of all, and as commonly known, stabilizing the level of radiative forcing to  $4 \text{ W/m}^2$  or below requires some tough abatement policies towards the last half of the century. As is intuitive, the GWP case allows for more emissions of CO<sub>2</sub> throughout the period as compared to the CO<sub>2</sub>-only cases. However, in the flexible multi-gas cases, the path yields more emissions until around 2090, after which emissions are reduced even below the CO<sub>2</sub> control only case. This is mainly because cutting energy use is also a means reducing emissions of methane, which turns out to be of great value towards the end of the time horizon. Reductions in the emissions of CO<sub>2</sub> then follow as a consequence.

<sup>&</sup>lt;sup>8</sup> For those gases included here, the development of emissions and concentrations in our reference case closely fits the so-called A2 marker scenario given by the IPCC. Our reference radiative forcing trajectory is somewhat lower.



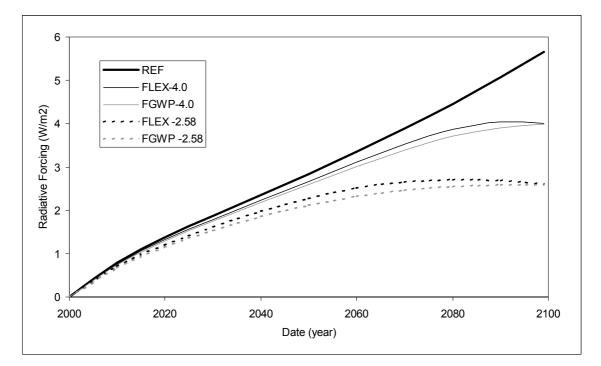
**Figure 2.** Optimal emissions profiles of  $CO_2$  in the six policy scenarios and in the reference case.



**Figure 3.** Optimal emissions profiles of  $CH_4$  in the six policy scenarios and in the reference case.

Figure 3 also contains some surprises and some trivialities. First of all, we see that in the flexible cases, the emissions of  $CH_4$  are lower than in the other cases towards the end of the time horizon. This is quite intuitive since the remaining time until the cap is to be met is short, which encourages abatement of short-lived gases such as  $CH_4$ . We also observe that the methane emissions are always lower when GWPs are used as compared to a  $CO_2$  control only policy. However, it is interesting to note that methane emissions are reduced quite substantially even in the  $CO_2$  only control scenario when no direct abatement of methane is carried out. The reason for this is that a large source of methane emissions is related to the use of energy, which is reduced from a  $CO_2$  perspective.

Together with the other gases, these optimal emissions profiles give rise to concentrations, which again contribute to the more relevant and common measure, radiative forcing.



**Figure 4.** Development of radiative forcing as compared to the year 2000 in the various cases. The  $CO_2$  only control strategy is almost an exact superimposition of the GWP cases and is left out to improve presentation.

Since the reference case yields a higher level of radiative forcing than the imposed constraint, emissions must be reduced in the various scenarios to satisfy the constraint. The first observation from Figure 4 is that, regardless of the cap on radiative forcing, in the GWP cases (and also in the  $CO_2$  only case) the radiative forcing trajectory is below the corresponding flexible one. Basically this means that the flexibility in the tradeoffs between the various gases allows for this higher path.

It is also clear (from the underlying figures) that in both of the flexible cases, radiative forcing slightly exceeds the cap before the terminal year, hence declining in the last decade to meet the target. Had the constraint been applied over the whole time horizon, that would obviously not have been feasible.

### 4.2 Prices and flexible weights

When comparing various climate policies, a common measure used in the literature is the price of carbon. This measure is of course interesting because of the role that activities leading to such emissions play in the production and consumption of key economic goods.

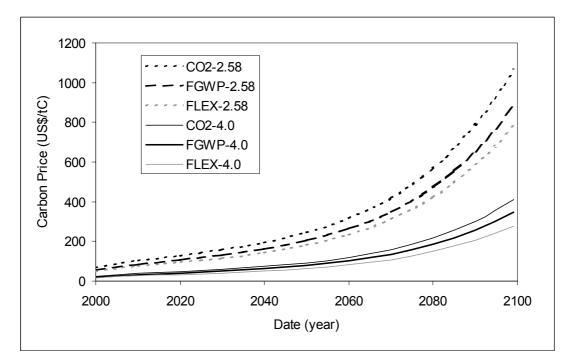


Figure 5. Development of carbon prices in the six cases.

When comparing the development of the carbon prices, we see from Figure 5 that reducing the cap on radiative forcing from 4.0 to 2.58 W/m<sup>2</sup> increases the marginal value of carbon emissions by a factor of almost three throughout the period. From the underlying data it is also clear that in the case when radiative forcing is constrained at 4 W/m<sup>2</sup>, prices are about 25% higher in the GWP case and close to 50% higher in the CO<sub>2</sub> control only case, both as compared to the flexible case. When the cap is 2.58 W/m<sup>2</sup>, carbon prices are 13 and 35% higher in the GWP and CO<sub>2</sub> only cases respectively, as compared to the flexible case.

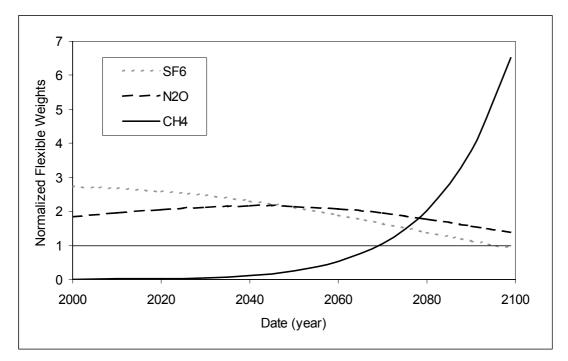
Thus, using different approaches towards the multi-gas issue yields quite different results in terms of carbon prices. But how do the prices of the other gases compare to that of  $CO_2$  and to that of the GWPs?

The trajectories depicted in Figure 6 are the graphs of the functions  $h^{j}(t)$  given by

$$h^{j}(t) = \frac{\partial g^{+}(a_{1}(t)) / \partial a_{1}(t)}{\partial g^{+}(a_{j}(t)) / \partial a_{j}(t)} \frac{1}{GWP_{j}} \quad \text{for } j = 2, 3, 4.$$
(13)

evaluated at the optimal path. In words, and recalling the definition of the functions  $g^{j}(a_{j}(t))$ , the first part of the right hand side of (13) gives the price of a non-CO<sub>2</sub> gas at each instant in the flexible case, divided by the price of CO<sub>2</sub> in the same flexible case for the same instant. We call these optimal weights that could then be used as an index to guide cost-

minimizing agents in a quota regime in an efficient manner. In order to compare with GWPs, the weights were then divided by the GWP value for the gas at hand (recall also that the GWP value for  $CO_2$  by definition equals 1). Hence, Figure 6 gives valuable information in terms of how the efficient prices of the non-CO<sub>2</sub> gases compare both to that of  $CO_2$  and to the GWP value for each of the gases.



**Figure 6**. Optimal flexible weights normalized to the GWPs in the case when the radiative forcing cap is  $4 \text{ W/m}^2$ .

We see that optimal weights would guide the emitting agents towards focusing more on methane later, relative to  $CO_2$ , as compared to the (constant) GWP index. The opposite is true for SF<sub>6</sub>. Our results confirm the findings of Michaelis (1992) and Manne and Richels (2001) in the case when there is no constraint on the growth rate of radiative forcing.

We also observe that the final optimal weight for methane (compared to  $CO_2$ ) is more than six times higher than the GWP value for methane, the value of each being 150 and 23 respectively. At a first glance the value 150 seems unreasonably high keeping in mind that the instantaneous effect on radiative forcing by methane emissions is well below 100 times that of  $CO_2$  with today's chemical composition of the atmosphere. However, in the terminal year, when we only care for the instantaneous effect on radiative forcing, this composition is significantly different from what it is today. The non-linear relations between concentrations and forcing for these gases, then drives this particular result.

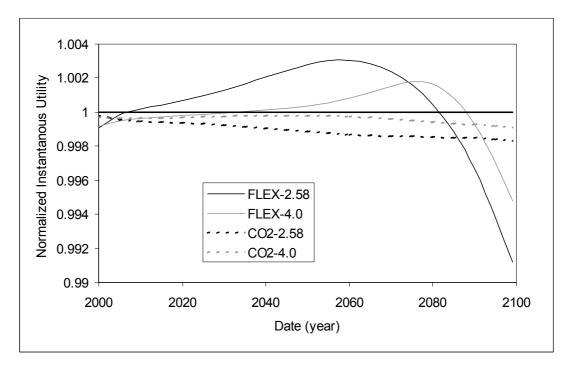
From the underlying figures, it is also interesting to note that the optimal weights when the radiative forcing cap is set at 2.58  $W/m^2$  are almost identical to the ones displayed in Figure 6 for methane, but slightly closer to one (i.e closer to the GWPs for the other gases).

### 4.3 Instantaneous utility and welfare

The results presented thus far are well in line with the existing numerical literature, in particular Manne and Richels (2001) and Michaelis (1992), and adds nothing particular new.

So, we turn to the issues that motivated this study, namely the welfare effects of using a flexible approach. Or in other words, what are the costs of using the GWP weights compared to a flexible approach in the implementation of a multi-gas policy?

However, before turning to that, we shall discuss how instantaneous utility changes across the scenarios. This gives an indication of "who" (in an intergenerational perspective) may benefit or lose from a flexible approach. When using the GWP case as a normalization, the results are as presented in Figure 7.



**Figure 7.** Changes in instantaneous utility normalized to the case when using GWPs (FGWP).

Regardless of the level of the cap on radiative forcing, we see that the cases where we control only  $CO_2$  emissions are below the GWP case. Hence, from an intergenerational point of view, using a multi-gas approach with GWPs is an improvement for everyone as compared to a  $CO_2$  control only strategy. The flexible cases, however, benefit most strongly those living during the first three quarters of the century, at the expense of those living the last couple of decades. This is because the optimal path implies quite intense abatement in that period. With the intergenerational perspective, however, it is worth mentioning that the *level* of utility monotonically increases over the whole time horizon in all cases, hence those living towards the end are better off than the previous generations, although they would have preferred the use of GWPs since that would have yielded more early action.

However, in order to compare the results in a more meaningful way, we also have to compare gains and losses at different points in time. Although this is taken care of by the welfare function *per se*, a comparison of the numerical value of W in the different alternatives does not give a good conception as to whether the cost of using GWP is large or small. To provide a more intuitive measure, we compare the alternatives by means of what we call a "constant annual consumption equivalent", or  $\bar{x}_c$  for short, defined such that the following holds

$$W^* = \int_{t=2000}^{2100} V(\bar{x}_c, x_u^*(t)) e^{-\delta t} dt .$$
 (14)

In (14)  $W^*$  is the welfare attained by consuming the optimal  $x_u^*(t)$  and  $x_c^*(t)$ . The measure  $\overline{x}_c$  is therefore the constant level of consumption over the 100-year period that would give the same total welfare  $W^*$  as the total welfare attained by the consumption path in the alternative. Hence,  $\overline{x}_c$  varies for the alternative scenarios and is highest in the flexible case.

**Table 2.** The costs (i.e. difference in constant annual consumption equivalent) of climate policy in the various scenarios as compared the value of the constant consumption program with no action (the reference case).

	Flexible	GWP	CO2-only
Radiative Forcing Cap, 4.0 W/m2			
Billion US\$/yr	1 006	1 022	1 128
Percent	1.70 %	1.73 %	1.91 %
Radiative Forcing Cap, 2.58 W/m2			
Billion US\$/yr	3 633	3 739	4 000
Percent	6.15 %	6.33 %	6.77 %

From Table 2 we see that when the radiative forcing cap is  $4.0 \text{ W/m}^2$ , climate policy costs are about one trillion US\$ per year, or about 1.7–1.9% of the value of the constant annual consumption equivalent in the reference case. These costs are increased about three-fold if radiative forcing is constrained to 2.58 W/m<sup>2</sup>. Most of these costs occur towards the last half of the century, since that is when the major cuts in emissions take place.

If we use the flexible case as a basis for comparison, the added cost of the two alternative policies are as shown in Table 3.

Table 3. The additional costs of climate policy as compared to the flexible case.

	GWP	CO <sub>2</sub> -only
Radiative Forcing Cap, 4.0 W/m2		
Billion US\$/yr	16	121
percent	1.6 %	12.1 %
Radiative Forcing Cap, 2.58 W/m2		
Billion US\$/yr	106	367
percent	2.9 %	10.1 %

When radiative forcing is limited to  $4 \text{ W/m}^2$ , we see from Table 3 that the additional costs of using GWPs is about 16 billion US\$ per year as compared to the flexible case which is about 1.6% of the costs of flexible climate policy itself. A one-gas (CO<sub>2</sub>) approach increases costs

by about 12% as compared to the flexible case. When the cap is reduced to 2.58  $W/m^2$ , the additional costs of using GWPs or a CO<sub>2</sub>-only control strategy as compared to the flexible case increase substantially in absolute terms. This is quite intuitive: The more ambitious the climate policy chosen, the more important becomes the choice of method to compare GHGs.

The approach in this study was to limit the level of radiative forcing only in the terminal year. In the flexible cases, and only there, the level was somewhat exceeded in the decades before the year 2100. Hence, as compared to a model where the level of radiative forcing is constrained over the entire period, we granted ourselves a little more flexibility, and thus overestimate the benefits of a flexible approach as compared to the case with a continuous, constant cap.

# 5 Some Limitations and Concluding Remarks

We have compared a climate policy that is flexible in terms of the timing and abatement of the various GHGs with two alternatives: climate policies that use fixed GWPs, and targeting  $CO_2$  emissions only. It is clear that the study has evident general shortcomings, as any deterministic model forecasting climate and economic activities a hundred years into the future would have. In particular, the motivation for mitigating emissions given by constraining radiative forcing in the year 2100 cannot be claimed to be a very realistic representation of the climate problem. It is also clear that more detailed numerical models exist, models that include more gases, sources and sinks, have a better description of the economic and climate system, and so on. However, with some exceptions these models usually rely on the use of GWPs from the outset, and can therefore not solve for a flexible gas policy.

Some more specific areas of improvement to the current analysis deserve attention. First, in the cause-effect chain, we stopped at radiative forcing. Moving on to temperature would make sense, but adds to programming, particularly since this relationship is non-linear. Second, we assumed that the GWPs used in the non-flexible case remained fixed throughout the period. Since GWPs depend on the composition of the atmosphere, these values are likely to change for chemical and radiative (not economic) reasons that will be reflected in updates in the literature and by the IPCC. Such scenario-dependent changes were ignored. The important choice of the time horizon, where we followed customary practice, was somewhat arbitrary and may be seen as a compromise between the sluggishness of the climate system and the lack methods for reliable long-term economic forecasting. Lastly, no constraint on the rate of change of radiative forcing was applied. Other studies show that if the objective is to slow the rate of change in the near term, short-lived gases like methane could play an important role at the beginning of the period.

Given our approach, two conclusions from the numerical simulations deserve particular mention. First,  $CO_2$  is the main problem. By including non- $CO_2$  gases and using GWPs, the costs of climate policy decrease only by around 10%. This is about twice as much as Michaelis (1999) found using a similar but simpler model. Second, the additional cost savings from the flexible refinement as compared to using fixed GWPs is about 2%, which is very similar to what reported by O'Neill (2003). In monetary terms, this amounts to 16-106 Billion US \$ per year, depending on the level of the climatic constraint. Hence, most cost savings that stem from including non- $CO_2$  greenhouse gases in climate policy are realized through the use of GWPs, even though GWPs are quite different from the efficient gas tradeoffs.

# Appendix

A thorough presentation of a similar but somewhat more simplified model is given in Aaheim (1999). The Hamiltonian of the problem analyzed here is:

$$H = \max_{x_{c}(t),x_{u}(t),u_{1}(t),u_{2}(t),a_{i}(t),...,a_{4}(t),k_{2}(t)} V(x_{c}(t),x_{u}(t))e^{-\delta t}$$

$$+ r_{1}(t) \left[ f^{1}(u_{1}(t),k(t)-k_{2}(t)) - x_{c}(t) - \sum_{j=1}^{4} a_{j}(t) \right]$$

$$+ r_{2}(t) \left[ f^{2}(u_{2}(t),k_{2}(t)) - u_{1}(t) - u_{2}(t) - x_{u}(t) \right]$$

$$+ q_{1}^{D}(t) \left[ (1-\Psi) \left( -\frac{1}{\tau_{1}}S_{1} + \beta_{3,1}x_{u}(t) + \sum_{i=1}^{2}\beta_{i,1}u_{i}(t) - g^{1}(a_{1}(t)) \right) \right]$$

$$+ q_{1}^{A}(t) \left[ \Psi \left( \beta_{3,1}x_{u}(t) + \sum_{i=1}^{2}\beta_{i,1}u_{i}(t) - g^{1}(a_{1}(t)) \right) \right]$$

$$+ \sum_{j=2}^{4} q_{j}(t) \left[ -\frac{1}{\tau_{j}}S_{j}(t) + \left( \beta_{3,j}x_{u}(t) + \sum_{i=1}^{2}\beta_{i,j}u_{i}(t) - g^{j}(a_{j}(t)) \right) \right]$$

$$+ \lambda(t) \left[ \Phi(S_{1}(t),...,S_{4}(t)) - \overline{\Phi} \right]$$
(A1)

where  $r_1(t)$  is the price of real capital,  $r_2(t)$  is the energy price,  $q_1^A(t)$ ,  $q_1^D(t)$ ,  $q_2(t)$ , ...,  $q_4(t)$  are the associated shadow prices of concentrations, and  $\lambda(t)$  is the shadow price of the radiative forcing target. Note that  $k_1$  is left out of the problem above, and replaced by  $(k-k_2)$  from equation (2), where k is a state variable and  $k_2$  is a control variable.

The instantaneous equilibrium conditions for the energy market can be written as:

$$r_1 = \frac{\partial V}{\partial x_c} e^{-\delta t} \tag{A2}$$

$$r_{2} = \beta_{3,1}(\Psi q_{1}^{A} + (1 - \Psi)q_{1}^{D}) + \sum_{j=2}^{4} \beta_{3,j}q_{j} + \frac{\partial V}{\partial x_{u}}e^{-\delta t}$$
(A3)

$$r_{2} = \beta_{1,1}(\Psi q_{1}^{A} + (1 - \Psi)q_{1}^{D}) + \sum_{j=2}^{4} \beta_{1,j}q_{j} + \frac{\partial f^{1}}{\partial u_{1}}$$
(A4)

$$r_{2} = \beta_{2,1}(\Psi q_{1}^{A} + (1 - \Psi)q_{1}^{D}) + \sum_{j=2}^{4} \beta_{2j}q_{j} + \frac{\partial f^{2}}{\partial u_{2}}$$
(A5)

$$-r_{1} = \left[\Psi q_{1}^{A} + (1 - \Psi) q_{1}^{D}\right] \frac{\partial g^{1}}{\partial a_{1}}$$
(A6)

$$-r_1 = q_2 \frac{\partial g^2}{\partial a_2} \tag{A7}$$

$$-r_1 = q_3 \frac{\partial g^3}{\partial a_3} \tag{A8}$$

$$-r_1 = q_4 \frac{\partial g^4}{\partial a_4} \tag{A9}$$

$$\frac{r_1}{r_2} = \frac{\frac{\partial f^{1}}{\partial k_1}}{\frac{\partial f^{2}}{\partial k_2}}$$
(A10)

Equation (A2) defines the user price of capital as the marginal discounted utility of consuming 'other goods'. Equations (A3) - (A5) define, accordingly, the user price of capital in the energy sector, and require that this user price equal the net value of energy use in each sector. The net value of energy is defined as the marginal utility of energy use in households or the productivity of energy in the two sectors plus the cost of emissions, expressed by the sum of (negative) user prices of all greenhouse gases multiplied by their corresponding emission coefficient.

Equations (A6) - (A9) state that the marginal value of direct abatement is the same for all gases and equal to the user price of capital. Thus, by (A2) it is assured that the value of spending money on consumption or abatement of any greenhouse gas is the same.

(A10) implies that the relative price of capital in the two sectors reflect the differences in the marginal productivity of capital. One may ask why the marginal productivity of capital should not be equal in the two sectors. Recall, then, that capital in the energy sector aims at controlling also the concentrations of greenhouse gases. It thus becomes subject to the long-term target on radiative forcing, which affects the timing of investments in energy production, and thereby the value of energy capital.

We can write the evolution of the shadow prices as

$$-\dot{r}_1 = r_1 \frac{\partial f^1}{\partial k_1} \tag{A11}$$

$$-\dot{q}_{1}^{A} = \Psi \lambda \frac{\partial \Phi}{\partial S_{1}} \tag{A12}$$

$$-\dot{q}_{1}^{D} = (1 - \Psi) \left[ -\frac{q_{1}^{D}}{\tau_{1\nu}} + \lambda \frac{\partial \Phi}{\partial S_{1}} \right]$$
(A13)

$$-\dot{q}_2 = -\frac{q_2}{\tau_2} + \lambda \frac{\partial \Phi}{\partial S_2}$$
(A14)

$$-\dot{q}_3 = -\frac{q_3}{\tau_3} + \lambda \frac{\partial \Phi}{\partial S_3}$$
(A15)

$$-\dot{q}_4 = -\frac{q_4}{\tau_4} + \lambda \frac{\partial \Phi}{\partial S_4}$$
(A16)

which give rise to the intertemporal equilibrium conditions, expressed by the optimal growth in consumption of ordinary goods,  $x_c$ , and abatement costs for each of the greenhouse gases. Consumption growth is found by equating the time derivative of equation (A2) with (A11), which gives the familiar Ramsey rule,

$$\frac{\dot{x}_{c}}{x_{c}} = -\frac{\frac{\partial V}{\partial x_{c}}}{x_{c}\frac{\partial^{2} V}{\partial x_{c}^{2}}} \left[\frac{\partial f^{1}}{\partial k_{1}} - \delta\right].$$
(A17)

The growth rates for abatement over time can be found by the time derivatives of (A6) – A(9), inserting for the time derivatives of the shadow prices in (A12) – (A16) and replacing  $\dot{r}_1/r_1$  from (A11). Then, for abatement of CO<sub>2</sub> we find

$$\frac{\dot{a}_{1}}{a_{1}} = -\frac{\frac{\partial g^{1}}{\partial a_{1}}}{a_{1}\frac{\partial^{2} g^{1}}{\partial a_{1}^{2}}} \left[ \frac{\partial f^{1}}{\partial k_{1}} + \frac{(1-\Psi)^{2} q_{1}^{D}}{\Psi q_{1}^{A} + (1-\Psi) q_{1}^{D}} \frac{1}{\tau_{1}^{D}} - \frac{\lambda}{q_{1}^{D}} \frac{(1-2\Psi)}{\Psi q_{1}^{A} + (1-\Psi) q_{1}^{D}} \frac{\partial \Phi}{\partial S_{1}} \right].$$
(A18)

The expression outside the brackets characterizes the curvature of the abatement productivity function  $g^{l}(a_{l})$ , and is positive since g'' is negative. For the three other greenhouse gases, the growth in abatement over time is

$$\frac{\dot{a}_{j}}{a_{j}} = -\frac{\frac{\partial g^{j}}{\partial a_{j}}}{a_{j}\frac{\partial^{2} g^{j}}{\partial a_{j}^{2}}} \left[ \frac{\partial f^{1}}{\partial k_{1}} + \frac{1}{\tau_{j}} - \frac{\lambda}{q_{j}}\frac{\partial \Phi}{\partial S_{j}} \right], (j = 2, ..., 4).$$
(A19)

The expressions within the brackets of (A18) and (A19) are rates of change in the 'tradeoff' between the productivity of real capital and the concentrations of each gas. What determines the weighting of gases over time is determined by the relative productivity of abatement of the different gases, expressed by the elasticity of intertemporal substitution in abatement defined outside the brackets on the right-hand side, and the lifetime of each gas. The last term within the brackets is zero if the target is not binding, that is,  $\lambda$  is then zero. In the calculations,  $\lambda > 0$  only in the terminal year. Then (A18) and (A19) give the conditions for the optimal composition of abatement under a binding constraint.

The conditions (A2) – (A19) determine the instantaneous and intertemporal allocation of energy use, consumption and abatement over time, whereas the levels are given by the initial quantities of capital  $k_0$  and the concentrations  $S_j$ , j = 1, ..., 4 and terminal conditions for capital and radiative forcing, which are given in equations (10) and (11). Note that, in most cases, the

optimal path implies that radiative forcing increases over the entire period because of the long lifetimes of the greenhouse gases. In these cases, (10) reduces to an ordinary transversality condition. Then the composition of the abatement of different gases will be determined in this terminal year, which is the only for which  $\lambda > 0$ . In that case, we have,

$$\frac{\partial g^{j}}{\partial a_{j}}\frac{\partial \Phi}{\partial S_{j}} = \frac{\partial g^{j}}{\partial a_{j}}\frac{\partial \Phi}{\partial S_{i}}, \forall i, j \text{ if } \lambda > 0.$$
(A20)

In some of the cases with flexible weights, the maximum radiative forcing is met before the terminal year. In the numerical calculations, we compare the different alternatives by setting the target in the terminal year for alternatives with monotonically increasing radiative forcing equal to the maximum forcing in the flexible cases.

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