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# EFFICIENT CONTRACTS IN A GAME OF NATIONS PURSUING GREENHOUSE GAS EMISSIONS ABATEMENT<sup>1</sup>

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## ABSTRACT

A group of nations contract to limit greenhouse gas emissions. They are facing incentive problems in terms of moral hazard since individual emission abatement efforts cannot be observed by other nations. I discuss contracts that correct the incentive problem by specifying a liability to an international policing institution if some chosen emission abatement target is not met. A nation with a low risk aversion and high marginal effort disutility must face a relatively high liability. All deviations by one or more nations are prevented if each nation's liability is high enough to discourage a unilateral deviation. The necessary liability is lower for a non-budget-balancing contract as compared to a budget-balancing contract. In a repeated game of risk-neutral nations where observation of total efforts is uncertain, a trigger mechanism can be constructed to induce nations to exert the higher cooperative effort level. If the chosen target is not met, a fixed-length non-cooperative punishment period is triggered. Cooperative periods will be intervened by punishment periods of fixed length. The nation that gains most from a unilateral deviation to a lower effort level and has the highest expected benefit in a non-cooperative period sets the lower bound on the punishment period length and the upper bound on the emission abatement target.

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## 1. INTRODUCTION

One of the main environmental issues the last decade has been the greenhouse problem, or global climate change caused by release of various gases to the atmosphere. The most important of these gases is carbon dioxide, released through burning of fossil fuels and reduced biomass stocks, mainly from clearing of forest areas. Other important greenhouse gases are methane, nitrous oxide and chlorofluorocarbons. The available climate models predict an increase in global average temperature of 2.2 to 4.8 °C from pre-industrial time (taken to be 1765) to year 2070, with a best estimate of 3.3 °C<sup>2</sup>. The increase in temperature will be greater at higher latitudes than at the equator. Even if the uncertainty of these predictions is large, a climate change of this magnitude could have important consequences for human activities like agriculture and forestry, for human settlements and for the biosphere.

Due to the possibility of large and harmful consequences of global climate change from carbon dioxide and other greenhouse gases, there has been an international debate on policy measures to reduce such emissions. To have any effect, such policy measures would have to cover most or all nations of the world, especially the larger economies, and nations where the growth potential is large.

Participating nations need to establish an agreement or contract, committing each nation to some greenhouse gas emission limitation. The global climate change reduction depends on the total emission abatement effort, where a benefit compared to a *laissez-faire* policy will accrue to each nation. The climate state can be considered as a public good, which is suboptimally supplied in a free market. Both the cost of reducing emissions and the benefits of reduced climate change vary from nation to nation.

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<sup>2</sup> A good overview of the greenhouse problem is given in Houghton et al. (eds.) (1990) and (1992).

A recent study of a related problem is Mäler (1989). He studies a game of sulfur emissions among European nations, and compares the non-cooperative equilibria with the fully cooperative solution. The EMEP meteorological model is used to describe a country times country transfer matrix relating sulfur emissions to sulfur deposits for each country. The transfer matrix, the net control cost function, and the damage function define a non-cooperative game. Each country is assumed to have full information, and the implications of incomplete information are not discussed.

One problem of an agreement is the difficulty of observing emissions abatement efforts undertaken by a nation. The effort is known by the nation itself, but is arduous and costly to observe by other nations. Consequently there is a free rider problem. An incentive for shirking is found since a shirking nation will receive the full benefit of the others' effort but does not bear the full cost of one's own lack of effort.

A similar information problem is analyzed in the literature on nonpoint pollution control. Segerson (1988) discusses a general incentive scheme for controlling nonpoint water pollution in a situation where the effects from abatement on environmental quality are uncertain, and where only the combined effects from the actions of several polluters are observable. The general mechanism combines penalties for substandard water with rewards for water quality above some given standard. A dynamic analogue of Segerson's mechanism is developed by Xepapadeas (1992). The effects from the dynamic process of the pollutant accumulation on individual behavior are now accounted for. These effects arise through the incentive scheme as a restriction on the polluters' profit-maximization problem.

I will assume that the aggregate effort level is observable by all nations through monitoring of the atmospheric concentration of greenhouse gases. If one (large) nation or a group of nations shirk, the atmospheric concentration of the greenhouse gases will increase faster than expected. Thus I assume that the relationship

between emissions and atmospheric concentration is well known.

For some greenhouse gases the asymmetric information on national efforts is less pronounced than for others. Since a major part of carbon dioxide emissions is closely related to fossil fuel use, where reasonably reliable data series on energy production, export, import and consumption exist, shirking is difficult to hide for this greenhouse gas. However, for domestically produced fossil fuels there could be an incentive to report too low figures. For methane, nitrous oxide and chlorofluorocarbons shirking is less likely to be observed by other nations.

In this paper I will study the incentive problems in a group of nations committed to greenhouse gas emissions abatement through a contract. Some nations may choose to refrain from such a contract, since there is a cost to committing to it. These nations may find the climate change problem less threatening than other nations, or find their effort costs larger, or benefits from reduced climate change smaller than the other nations. I will not discuss how nations could be persuaded to participate, but make the simplifying assumption that all nations of the world are sufficiently concerned about the possible costs from climate change to be willing to participate. The least willing nations may be given incentives to participate through side payments from the initial group of contracting nations. Side payments are not within the scope of the models presented in this paper, however.

The incentive problem is a type of *moral hazard* with hidden actions (Rasmusen (1989)). To avoid shirking the contract must be made conditional on the level of emissions abatement reaching some optimal level as a Nash equilibrium. If the nations fail to reach this emissions abatement level, the contract must specify some punishment or liability (in monetary terms) to be paid by the nations. Since the shirker(s) can not be identified, the contract must specify a punishment for all nations, or a randomized scheme for punishing some of the nations.

To establish and enforce the contract an international policing institution is needed.

The ideal policing institution should be independent of all nations. Especially in the case of punishment of all contracting nations, the contract must be enforced by this institution. Since the contracting nations *ex post* will be better off canceling the punishment when a low aggregate effort level is observed, the international institution must be able to prevent this type of renegotiation. Such an international policing institution does not exist today, and will probably not be established in the near future. Nations are not likely willing to transfer sovereignty to some international institution in the present regime of sovereign national states. Nevertheless, the approach taken here makes it possible to discuss some important mechanisms and problems of international climate policy in a setting of asymmetric information.

Surveys of principal-agent models are found in Hart and Holmström (1987), Rasmusen (1989), and Fudenberg and Tirole (1991). Important contributions to the field are found in Holmström (1979) and Grossman and Hart (1983). A branch of principal-agent models focus on *team production*. Holmström (1982) defines a team loosely as "a group of individuals who are organized so that their productive inputs are related". He assumes that there is an incentive problem due to asymmetrical information, caused by actions that are hidden to other agents and cannot be contracted for directly. The main role of the principal is to administer incentive schemes (or contracts). These may not be budget-balancing, i.e. they may require penalties that waste output or bonuses that exceed output. The principal enforces the penalties or finances the bonuses. Holmström shows that efficient incentive schemes exist for risk neutral agents as long as budget balance is not required. Rasmusen (1987) discusses moral hazard in risk-averse teams, and his article is the main inspiration for the present paper. He shows that efficient budget-balancing contracts exist for risk-averse teams.

So far I have assumed that the efforts are not observable by the other agents and the international institution (the principal). A more realistic approach is to assume that a nation has some information about the effort level of other nations. Neighboring nations, nations with similar economies, important trading partners,

and nations with similar cultures, might have some information on the efforts exerted by the other nations based on own efforts. Correlated information is discussed by Demski and Sappington (1984) and Crémer and McLean (1988).

Demski and Sappington (1984) assume that only two states of nature are possible, interpreted as states of low and high productivity. For both risk-neutral and risk-averse agents the slightest correlation between their private information is - under some circumstances - used by the principal to induce the *ex post* Pareto efficient (i.e. full information efficient) effort levels. Crémer and McLean (1988) assume that information among bidders in an auction for a single indivisible object is correlated. The efficient solution is to sell the object to the bidder with the highest willingness to pay. They show that the seller can construct an auction that enables him to capture for himself the total increase in social welfare induced by transferring the object to the bidder with the highest willingness to pay.

These studies show that correlated information among agents - under some circumstances - implies that it is possible to find a contract that induces a first best Pareto optimal solution. Correlated private information is more to the principal's benefit, since it may make him able to withhold all rents from the agents.

The role of the international institution in the climate change problem addressed here is not to withhold rent, but to ensure an efficient solution and efforts from all participating nations. The more information (efforts) between two nations is correlated, the more difficult for one of the nations to hide its shirking for the other of these two nations. One of these nations could then inform upon the other nation's shirking to the international institution. Even if shirking might be difficult to verify, it would become more risky. Both nations would have something to gain from collusion, on the other hand, since shirking in that case could be hidden to other nations (with smaller correlation coefficients) and to the international institution. Thus the information and effort correlation structure of nations could be one of the factors explaining collusion between groups of nations. In this paper I will not discuss correlated information further, but use the simplifying



assumption of efforts that are unobservable to other nations and the international institution.

In the next section of the paper I develop a model of team production for a group of nations that commit themselves to greenhouse gas abatement, inspired by Rasmusen (1987). I discuss some contracts that can induce efficient effort levels. Efficient budget-balancing contracts exist for risk-averse nations (agents). Efficient non-budget-balancing contracts exist for both risk-neutral and risk-averse nations, but risk aversion makes such contracts possible for smaller liabilities. Some numerical illustrations are given for the case of identical nations, and for the case of one small and two large nations.

In the final section of the paper I develop a repeated game version of a team production model, which is inspired by the trigger mechanism found in Green and Porter (1984). This is a more realistic assumption since a contract is likely to be renewed after some period of time. In the one-period model the aggregate emission abatement efforts could be inferred from the atmospheric concentration of greenhouse gases with certainty. I now include uncertainty in the model. This uncertainty can be caused by errors in the measurement of atmospheric concentration of greenhouse gases, or by some stochastic element in the relation between emissions and atmospheric concentration. The latter may be a feature of the complex dynamic system involving the atmosphere, the lithosphere (terrestrial areas of the earth), the biosphere and the hydrosphere (oceans and lakes) (Houghton et al. (1990) and (1992)). The discussion is limited to the case of risk-neutral nations. The group of nations coordinate their efforts by agreeing on some aggregate emission abatement level. If the observed aggregate effort is lower than the level agreed upon, there is a fixed-length punishment period where all nations reduce their efforts. This means a faster climate change and consequently reduced benefits for all nations. Due to the possibility of measuring a higher atmospheric concentration than the trigger level - even if all nations exert the necessary efforts - there will be periods of fixed-length punishment and normal periods of variable length.

## 2. A ONE-PERIOD GAME

### 2.1 A game with hidden actions

A group of nations are concerned about climate change caused by release of greenhouse gases to the atmosphere. Due to the free rider nature of the problem the incentives to commit to emissions reduction is smaller the smaller share of global emissions the participating nations have. In addition the efficiency of the climate policy is reduced if nations with a large share of global greenhouse gas emissions do not participate. This would mean larger global emissions and thereby a larger climate externality. In this paper I will assume that all nations of the world participate. The nations recognize that collaboration is needed to carry out an efficient climate policy and agree on a contract. The free rider problem remains, however. It is impossible to force a nation to participate and commit itself to emissions reductions in the present global regime of sovereign nations. The contracts analyzed in this paper only focus on how to correct the incentive problems caused by unobservable individual emission abatement efforts. For each nation the contract specifies a punishment or liability in money terms. If aggregate emission abatement is too small, a punishment is carried out by an international contract policing institution (the principal).

There are  $n$  nations or agents. Each nation makes an effort  $e_i \geq 0$ , where  $i=1, \dots, n$ , to reduce national emissions. The effort is observable neither to the other agents, nor the principal. It is worthwhile to define the two effort vectors:

$$(1) \ e_{-i} = (e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_n)$$

$$(2) \ e = (e_i, e_{-i})$$

The effort vector (1) consists of all efforts except nation  $i$ 's effort, whereas the

complete effort vector can be expressed like (2).

All nations and the international institution can observe the atmospheric concentration of greenhouse gases. In the present single-period model the atmospheric concentration of greenhouse gases is equal to  $Y_0$  in the absence of any contracts. Under an emissions abatement contract the atmospheric concentration is reduced to  $Y$ . The abatement level,  $y$ , is then defined as the reduction in atmospheric concentration of greenhouse gases,  $y(e) = Y_0 - Y(e)$ , which is a function of the emissions abatement effort level  $e$  under the contract. Thus I assume that the global effort and abatement level can be observed indirectly through the atmospheric concentration of greenhouse gases.  $y$  is specified as:

$$(3) \quad y = y(e) \quad ; \quad y'_i > 0, \quad y''_{ii} < 0$$

and is thus assumed to be strictly increasing and concave in each nation  $i$ 's effort, and twice continuously differentiable. Increased efforts result in increased abatement, equivalent to reduced atmospheric concentration. The value of reduced atmospheric concentration of greenhouse gases pertaining to nation  $i$ ,  $x_i$ , is given by the strictly increasing and concave function:

$$(4) \quad x_i = X_i[y(e)] \equiv x_i(e) \quad ; \quad x'_i > 0, \quad x''_{ii} < 0$$

where the benefit function  $x_i(e)$  is assumed to be twice continuously differentiable and measured in money terms, for instance USD. Emission reductions can be evaluated on the basis of a damage cost function, relating emissions to costs due to various kind of damages from climate change.

If the aggregate emission abatement target stated in the contract is not met, each nation will face a punishment or liability in money terms. The contracting nations have to agree on the liability for each nation. The international institution will

measure the atmospheric concentration of greenhouse gases after some pre-specified period of time and state if the target is met or not. If a punishment is due, the liabilities are paid to the institution. The money is redistributed between nations according to some randomized scheme, or spent by the international institution. In the latter case an idea is to invest the money in a global research and development fund on renewable energy technologies, e.g. solar thermal power, solar photovoltaics, wind power, wave power, geothermal energy, and biomass fuels. The point is to redistribute or employ the money in a way that minimizes the influence on the incentive mechanism of the contract.

Thus the net compensation to nation  $i$ ,  $z_i(e)$ , depends on the efforts and the abatement target being met or not:

$$(5) \quad z_i(e) \equiv \begin{cases} x_i(e) & \text{if target is met} \\ x_i(e) - \mu_i & \text{if target is not met} \end{cases}$$

where  $\mu_i$  is the liability of nation  $i$ . For simplicity agent  $i$ 's utility function is assumed to be separable in the value of net compensation  $z_i(e)$  and the effort disutility  $v_i(e_i)$ , and given by:

$$(6) \quad u_i(z_i, e_i) = m_i[z_i(e)] - v_i(e_i)$$

Exerting effort  $e_i$  gives a disutility  $v_i(e_i)$ , which is assumed strictly convex, increasing and twice continuously differentiable. The first part of the utility function,  $m_i(z_i)$ , is specified as a concave function with constant absolute risk aversion equal to  $\beta_i > 0$ :

$$(7) m_i(z_i) = - e^{-\beta_i z_i}$$

$$(8) \beta_i = \frac{-m_i''}{m_i'} > 0$$

The specification in (7) is chosen since it is simple, differentiable and implies a convenient risk aversion parameter.

A reason why nations (governments) may be risk averse in this context is that climate change due to greenhouse gas emissions is likely to significantly affect total activity in the economy, and thereby total consumption. Given that national welfare depends on total consumption through a concave utility function, risk aversion arises also in this case and can not be diversified away.

Assuming that a global utility function exists the first-order condition for the social optimum can be expressed as:

$$(9) \frac{\delta v(e)}{\delta e} = \beta e^{-\beta x(e)} \frac{\delta x(e)}{\delta e}$$

where  $v(e)$  is the global effort disutility function and  $x(e)$  is the global benefit function. The first-order condition for nation  $i$ , given the effort level  $e_i$  by other nations and *no contracts* (i.e.  $z_i(\cdot) = x_i(\cdot)$ ), can be found from eq. (9) by substituting  $v_i(e_i)$  for  $v(e)$  and  $x_i(e_i, e_i)$  for  $x(e)$ . For a large nation there can be an interior solution described by eq. (9) in this case. For a small nation, however,  $\delta x_i(\cdot)/\delta e_i$  is close to zero and the convexity of the effort disutility function is likely to be stronger. Consequently a corner solution  $e_i = 0$  is likely to be the best choice. If the other nations exert zero effort, nation  $i$  will choose the effort defined by the national optimum in this case, which may be positive for a large nation (with a relatively low convexity of the effort disutility function). A Nash equilibrium where every nation exerts zero effort exists if a unilateral deviation to some positive effort level does not pay for the nation with the strongest marginal increase in benefit

from a marginal effort increase and/or the lowest marginal increase in effort disutility.

We want to find a contract of compensation rules such that the noncooperative game with payoffs:

$$(10) \quad m_i \{z_i[y(e)]\} - v_i(e_i) \quad ; \quad i = 1, \dots, n$$

has a Nash equilibrium in terms of expected utility. From this perspective I will analyze two types of contracts, the massacre contract and the general punishment contract.

## 2.2 The massacre contract

In the massacre contract there is a randomized redistribution scheme for the liabilities. All but one agent are punished when output (emissions abatement) is low. The lucky agent is randomly chosen, and receives the entire output (liabilities of the other nations). Consequently this contract balances the budget, and there are no net payments to the principal. The massacre contract is defined as:

$$(11) \quad z_i(e) \equiv \begin{pmatrix} x_i & \text{if } y \geq y(e^*) \\ x_i - \mu_i & \text{w. prob. } \frac{n-1}{n} \text{ if } y < y(e^*) \\ x_i + \sum_{j=1; j \neq i}^n \mu_j & \text{w. prob. } \frac{1}{n} \text{ if } y < y(e^*) \end{pmatrix}$$

where  $y(e^*)$  is the aggregate emission abatement target given by the Nash equilibrium effort vector  $e^*$ . The contracting nations must determine an optimal emission abatement level  $y(e^*)$  based on the best available information on cost and benefits of emission abatement. From the optimal emission abatement level the efficient effort vector  $e^*$  can be found. This Nash equilibrium is implemented through a contract specifying a liability for each nation. As we shall see later only

some values of the parameters  $\beta_i$  and  $\mu_i$  will generate a Nash equilibrium. In the Nash equilibrium there will be no shirking, and efficient effort levels will be realized.

The effort vector  $e^*$  can be a Nash equilibrium if and only if any agent's utility of choosing an effort  $e_i^*$  is higher than from cheating and choosing any other (lower)  $e_i$ . Thus a Nash equilibrium requires the following condition to be satisfied for all efforts  $e_i < e_i^*$  and all nations  $i=1, \dots, n$ :

$$(12) \quad Y_{im} \equiv \{m_i [x_i(e^*)] - v_i(e_i^*)\} - \left\{ \frac{(n-1)}{n} m_i [x(e_{-i}^*, e_i) - \mu_i] + \frac{1}{n} m_i [x(e_{-i}^*, e_i) + \sum_{j=1, j \neq i}^n \mu_j] - v_i(e_i) \right\} > 0$$

After some rearrangement inequality (12) can be expressed as:

$$(13) \quad \mu_i > M_{im} \equiv \frac{1}{\beta_i} \log \frac{n}{n-1} + x_i(e_{-i}^*, e_i) + \frac{1}{\beta_i} \log \left[ v_i(e_i^*) - v_i(e_i) + e^{-\beta_i x_i(e^*)} - \frac{1}{n} e^{-\beta_i [x_i(e_{-i}^*, e_i) + \sum_{j=1, j \neq i}^n \mu_j]} \right]$$

The liability of each nation  $i$  must be larger than the right-hand side of the inequality, which is the lower liability bound represented by  $M_{im}$ .  $M_{im}$  depends on the benefit function  $x_i(\cdot)$ , the effort disutility function  $v_i(\cdot)$ , the Nash equilibrium  $e^*$ , the deviation  $e_i$ , the sum of the liabilities of other nations, the number of nations  $n$ , and the coefficient of absolute risk aversion  $\beta_i$ .

From inequality (13) the lower liability bound  $M_{im}$  increases with the marginal effort disutility and convexity of  $v_i(e_i^*)$ . Furthermore,  $M_{im}$  is independent of the risk aversion of other nations, but must increase with the sum of the liabilities of other nations, which can be shown by taking the derivative of  $M_{im}$  with respect to  $\sum_{j \neq i} \mu_j$ .

An increased liability is needed to balance the increased expected utility from an effort deviation caused by an increased sum of liabilities of other nations. The relation between  $M_{im}$  and nation size depends on the closer specification of the benefit and disutility functions and on how these depend on nation size.

The derivative of  $M_{im}$  with respect to  $\beta_i$  is negative if:

$$(14) \quad \beta_i \left( x_i(e^*) e^{-\beta_i x_i(e^*)} - \frac{1}{n} [x_i(e_{-i}^*, e_i) + \sum_{j \neq i} \mu_j] e^{-\beta_i [x_i(e_{-i}^*, e_i) + \sum_{j \neq i} \mu_j]} \right) + B \log \frac{Bn}{n-1} > 0$$

where B is the expression within the log-parenthesis of inequality (13). B is positive if i)  $u_i[x_i(e^*), e_i^*] < 1/n m_i[x_i(e_{-i}^*, e_i) + \sum_{j \neq i} \mu_j] - v_i(e_i)$ , and ii)  $v_i(e_i) < v_i(e_i^*)$ . Condition i) is equivalent to the Nash equilibrium utility being smaller than the utility in the case where nation i deviates and earns the liabilities of all the other nations, adjusted for the probability of this case, which is  $1/n$ . This condition is satisfied since the absolute value of the gross utility function  $m_i(z_i)$  decreases in  $z_i$ , with zero as an upper bound, and we assume that  $x_i(e^*) < x_i(e_{-i}^*, e_i) + \sum_{j \neq i} \mu_j$ . In addition the utility level approaches zero when divided by n. Consequently the parenthesis of the left-hand side of (14) is most likely positive. Condition ii) is satisfied since  $v_i(e_i)$  is increasing in the effort level, and since the deviation is to an effort level  $e_i$  which is lower than  $e_i^*$ . The log-term may be negative if B is less than one. This could be the case for a sufficiently small marginal effort disutility and convexity of the effort disutility function, and/or a small effort deviation, and/or a large optimal Nash equilibrium benefit level, and/or a small sum of the other nations' liabilities. Given the benefit and effort disutility function assumptions and a massacre contract implemented by sufficiently large liabilities, inequality (14) is most likely satisfied. Consequently  $M_{im}$  most likely decreases in the absolute risk aversion coefficient  $\beta_i$ .

Let us assume that nation i deviates from  $e_i^*$  to  $e_i$ , where  $0 \leq e_i < e_i^*$ . Nation i's



optimal deviation is now to  $\hat{e}_i$ , where  $0 \leq \hat{e}_i < e_i^*$ . From eq. (9), expressed in terms of nation  $i$ , we find that the optimal effort of nation  $i$ ,  $\hat{e}_i$ , may increase if the efforts of the other nations are reduced. If nation  $i$  is small the optimal deviation can still be the corner solution of zero effort. If nation  $i$  is large and there is an interior solution, however,  $\hat{e}_i$  is higher than the effort in the unilateral deviation case. Consequently we have that  $e_i \leq \hat{e}_i$ , where  $e_i$  is the optimal deviation effort in the unilateral deviation case from inequality (13).

The difference between the lower liability bound in the unilateral deviation case,  $M_{im}$ , and the present case,  $N_{im}$  is equal to:

$$\begin{aligned}
 M_{im} - N_{im} = & \\
 & x_i(e_{-i}^*, e_i) - x_i(e_{-1,i}^*, e_1, \hat{e}_i) \\
 (15) \quad & + \frac{1}{\beta_i} \left\{ \log \left[ \frac{1}{n} m_i [x_i(e_{-i}^*, e_i) + \sum_{j \neq i} \mu_j] - v_i(e_i) - m_i[x_i(e^*)] + v_i(e_i^*) \right] \right. \\
 & \left. - \log \left[ \frac{1}{n} m_i [x_i(e_{-1,i}^*, e_1, \hat{e}_i) + \sum_{j \neq i} \mu_j] - v_i(\hat{e}_i) - m_i[x_i(e^*)] + v_i(e_i^*) \right] \right\}
 \end{aligned}$$

Assuming that nation  $i$  in its deviation does not fully compensate for the reduced effort by nation  $1$  due to the externality, the benefit level is smaller when both nation  $1$  and  $i$  deviate, i.e.  $x_i(e_{-1,i}^*, e_1, \hat{e}_i) < x_i(e_{-i}^*, e_i)$ . The first log-parenthesis is larger than the second log-parenthesis since  $m_i(\cdot)$  is negative and upper bounded at zero when  $x_i(\cdot)$  increases, and since  $v_i(\cdot)$  is increasing in  $e_i$ . Consequently expression (15) is positive. This means that the highest lower liability bound  $M_{im}$  is needed to prevent a unilateral deviation by nation  $i$ . Thus if  $\mu_i$  prevents a unilateral deviation by nation  $i$ , deviation by nation  $i$  is also prevented for all other cases where one or more nations deviate, including the case where all other nations deviate to zero effort. It is less tempting for two or more nations to deviate since the resulting benefit is reduced compared to a unilateral deviation. Even if a Nash equilibrium

where every agent exerts zero effort exists, the nations are induced to stay in the Nash equilibrium of positive efforts as long as the liabilities are large enough to prevent unilateral deviations, and thereby all other deviations.

Summing up, to induce each nation to choose the Nash equilibrium a nation with a low absolute risk aversion and high effort disutility must face a relatively high liability. If the liability prevents a unilateral deviation by a nation all possible deviations by the nation and one or more of the other nations are also prevented. If the nations only differed in terms of risk aversion (i.e. had identical utility functions except for the risk aversion coefficient), the nation with the lowest risk aversion would have to face the highest liability to induce all nations to choose the Nash equilibrium supported by the contract.

A possible problem of the massacre contract is the *ex ante* incentive for nations to establish mutual insurance schemes. Even if the expected utility is not changed, there is a utility gain to sharing the sum of liabilities received by the lucky nation since the nations are risk averse. If all nations should contract to redistribute the output according to their liabilities, there would be no risk of punishment left. Thus the massacre contract would have no incentive correcting effect on emission abatement efforts. This is also - to a smaller extent - the case for a contract of this type where fewer nations participate - the incentive correction effect is reduced. On the other hand insurance contracts must be secret and hidden from the policing institution. Consequently it will be difficult to make these contracts binding, and thus, if the situation occurs, force the lucky nation to share the money with the other contracting nations. As long as there is some incentive for mutual insurance contracts, the policing institution must be able to deter nations from participating in these. Massacre type contracts would otherwise be less attractive. This is a common problem for incentive correcting contracts.

### **2.3 The general punishment contract**

In this contract all nations are punished with certainty if emissions abatement is

too low. The liabilities are paid to the international institution, and the contract is consequently not budget-balancing. The international institution must employ the money in a way that minimizes the effect on the incentive mechanism of the contract. Holmström (1982) shows that efficient contracts exist for risk-neutral agents as long as a budget balance is not required. As above the liabilities must be large enough to induce the agents to choose the efficient effort level as a Nash equilibrium. In the present model some degree of risk aversion is assumed, and we thus discuss the general punishment contract in the case of risk averse nations. The general punishment contract is defined as:

$$(16) \quad z_i(e) \equiv \begin{pmatrix} x_i & \text{if } y \geq y(e^*) \\ x_i - \mu_i & \text{if } y < y(e^*) \end{pmatrix}$$

To induce the optimal Nash equilibrium, the effort level  $e_i^*$  for all nations  $i=1, \dots, n$  must give higher utility than any single deviation  $0 \leq e_i < e_i^*$ :

$$(17) \quad Y_{ig} \equiv \{m_i [x_i(e^*)] - v_i(e_i^*)\} - \{m_i [x_i(e_{-i}^*, e_i) - \mu_i] - v_i(e_i)\} > 0$$

$$\forall 0 \leq e_i < e_i^*$$

Every agent  $i$  must make the effort  $e_i^*$  to reach the optimal output  $y(e^*)$ , and thus the Nash equilibrium. Given that the other agents have chosen the efficient effort, cheating by one or more agents would mean a punishment of  $\mu_i$  for every nation  $i$  ( $i=1, \dots, n$ ). In this case the liability of each nation would be paid to the international policing organization. A separate deviation to increase the effort level from the Nash equilibrium level would earn the deviating agent only a marginal increase in output  $x_i$ , whereas the full disutility of the extra effort would be met. Thus single deviations from this Nash equilibrium in either direction would not pay for the agent. Condition (17) assures that the efficient effort level is exerted and that the optimal Nash equilibrium is reached.

Inequality (17) can be rearranged to yield:

$$(18) \mu_i > M_{ig} \equiv x_i(e_i^*, e_i) + \frac{1}{\beta_i} \log [v_i(e_i^*) - v_i(e_i) + e^{-\beta_i x_i(e^*)}]$$

The lower liability bound  $M_{ig}$  depends on the benefit function  $x_i(\cdot)$ , the effort disutility function  $v_i(\cdot)$ , the Nash equilibrium efforts  $e^*$ , the deviating effort  $e_i$ , and the coefficient of absolute risk aversion  $\beta_i$ . From inequality (18)  $M_{ig}$  increases in the marginal effort disutility and convexity of  $v_i(e_i^*)$ .

The derivative of  $M_{ig}$  with respect to  $\beta_i$  is negative if:

$$(19) \quad \beta_i x_i(e^*) e^{-\beta_i x_i(e^*)} + [v_i(e_i^*) - v_i(e_i) + e^{-\beta_i x_i(e^*)}] \log [v_i(e_i^*) - v_i(e_i) + e^{-\beta_i x_i(e^*)}] > 0$$

The expression within the parentheses at the left-hand side of inequality (19) is positive since  $e_i < e_i^*$  and  $v_i(\cdot)$  is strictly increasing in the effort level by assumption. The left-hand side of (19) may be negative if the expression within the log-parenthesis is sufficiently smaller than one. This could be the case for a sufficiently low marginal effort disutility and convexity of the effort disutility function, and/or a small effort deviation, and/or a large optimal Nash equilibrium benefit level. The first left-hand term is positive, and the most likely case given the benefit and disutility function assumptions and an implemented general punishment contract is for inequality (19) to be satisfied. Thus a relatively high liability must be faced by a nation with a low risk aversion to induce each nation to choose the Nash equilibrium, which is similar to the situation for the massacre contract.

If nation 1 deviates from  $e_1^*$  to  $e_1$  and nation  $i$ 's optimal deviation in this case is  $\hat{e}_i$ , confer the discussion for the massacre contract, the change in  $M_{ig}$  from the unilateral deviation case can be expressed as in eq. (15) without the first term within the log-parentheses. Similarly to the situation for the massacre contract this expression is positive, meaning that the highest lower liability bound  $M_{ig}$  is needed to prevent a unilateral deviation by nation  $i$ . Thus if  $\mu_i$  prevents a unilateral

deviation by nation  $i$ , a deviation by nation  $i$  is also prevented for cases where one or more nations deviate, including the case where all other nations deviate to zero effort. It is less tempting for two or more nations to deviate since the resulting benefit is reduced compared to a unilateral deviation. Even if a Nash equilibrium where every nation exerts zero effort exists, the nations are induced to stay in the Nash equilibrium of positive efforts supported by the contract as long as the liabilities are large enough to prevent unilateral deviations, and thereby all other deviations.

Assuming that a massacre contract and a general punishment contract support the same Nash equilibrium for the same vector of efforts  $e^*$ , we find that the difference between the lower liability bound for the massacre contract  $M_{im}$  and the general punishment contract  $M_{ig}$  is equal to:

$$(20) \quad M_{im} - M_{ig} = \frac{1}{\beta_i} \log \left[ \frac{n}{n-1} - \frac{e^{-\beta_i [x_i(e_i^*, e_i) + \sum_{j \neq i} \mu_j]}}{(n-1) [v_i(e_i^*) - v_i(e_i) + e^{-\beta_i x_i(e_i^*)}]} \right]$$

which is positive if  $[\exp\{-\beta_i [x_i(e_i^*, e_i) + \sum_{j \neq i} \mu_j]\}] / [v_i(e_i^*) - v_i(e_i) + \exp\{-\beta_i x_i(e_i^*)\}] < 1$ . This condition is satisfied since  $x_i(e_i^*) < x_i(e_i^*, e_i) + \sum_{j \neq i} \mu_j$  and  $0 < v_i(e_i^*) - v_i(e_i)$ , noticing the negative argument in the exp-functions. The lower liability bound is consequently larger for the massacre contract than for the general punishment contract. This is due to the budget-balancing feature of the massacre contract as opposed to the general punishment contract, and the property that one nation must be given a large reward (i.e. the sum of liabilities of the other nations) in the case of punishment.

To sum up, the general punishment contract can be efficient for both risk neutral and risk averse nations, but the higher the level of risk aversion and the lower the marginal effort disutility, the smaller the level of liabilities required to make the contract efficient. The lower liability bound compared to the benefit level is likely to decrease with the size of a nation. If the liability level is high enough to prevent

a unilateral deviation by a nation all deviations by the nation and one or more additional nations are also prevented. It is less tempting for two or more nations to deviate since the resulting benefit is reduced compared to a unilateral deviation. The liabilities must be higher in the massacre contract than in the general punishment contract due to the non-budget-balancing feature of only the latter contract type. Moreover, the massacre contract never works under risk neutrality, while the general punishment contract generally does.

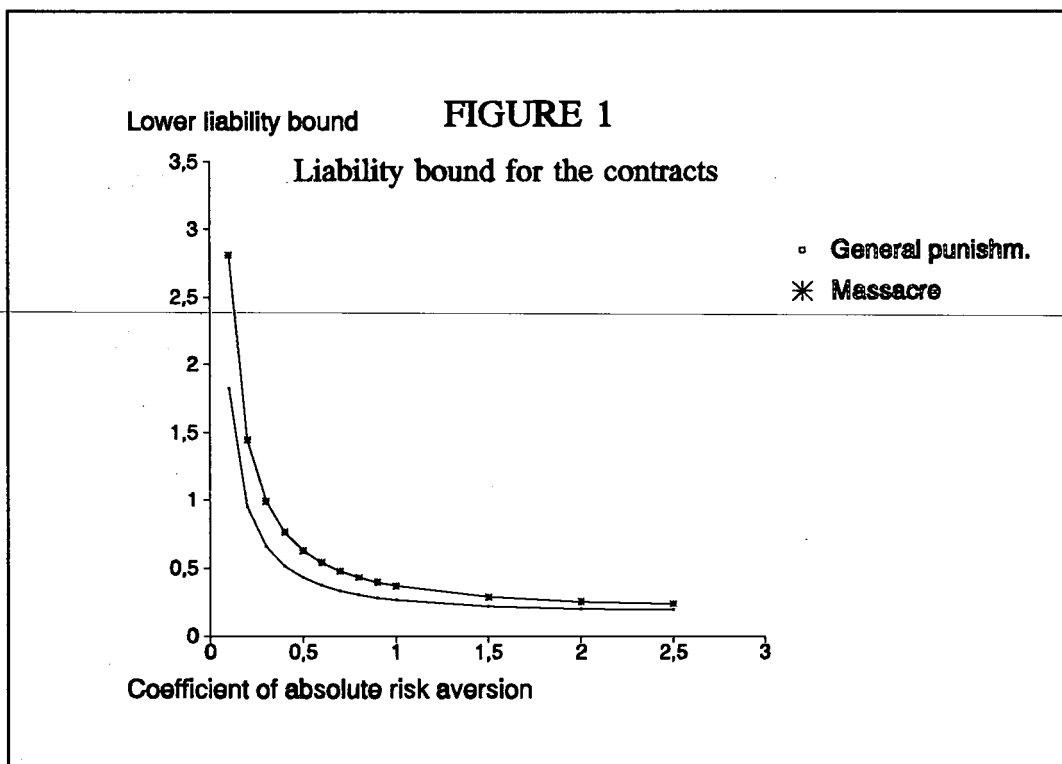
## 2.4 A numerical example featuring identical nations

As an illustration of the two contract types and the relation between the level of risk aversion and the liability level needed to induce the nations to choose the efficient effort level supported by a contract, some calculations are given for the special case of identical nations. Both the utility functions and the parameter values, including the coefficient of absolute risk aversion and the liability level, are consequently identical for all nations in this example. The calculations are made for the case of 10 nations. Since this is a global model, 10 must be interpreted as the number of regions and/or nations the world is divided into. Due to these simplifying assumptions we only need to specify one benefit function  $x(e)$  and one effort disutility function  $v(e)$ . For simplicity these functions are given a quadratic specification, keeping in mind the requirements of a strictly increasing and concave benefit function, and a strictly convex effort disutility function. The utility function is thus specified as:

$$(21) \quad u(e_i, e_j) = -e^{-\beta\{(n-1)e + e_i - b[(n-1)e + e_i]^2\}} - [ae_i + ge_i^2]$$

where  $e_i$  is the effort of nation  $i$  and  $e$  is the effort of each of the other nations. Furthermore  $a > 0$ ,  $g > 0$ ,  $b > 0$ , and for all  $n$  nations, indexed by  $j$ ,  $0 \leq e_j$ ,  $e_j \leq e_j^* \leq 1/2bn$  to satisfy the restrictions on the functions.  $e_j^*$  is the efficient effort level supported as a Nash equilibrium by a contract, which is the same for all nations/regions. The model is solved in three steps. First the utility is maximized

for a nation when the emissions abatement level is constrained to a higher level  $y^*$  than the freely chosen level (which is suboptimal due to the climate externalities). This implies that the effort level  $e^*$  and the benefit level  $x^*$  also are constrained to a higher level than the freely chosen level, which defines the Nash equilibrium to be supported by a contract. Then the optimal deviation by nation  $i$  is found when the rest of the nations stick to this higher effort level  $e^*$ . Finally the lower bound on the liability is calculated for these effort levels. Due to the specification constraints  $b$  is fixed at 0.5,  $a$  fixed at 2 and  $g$  fixed at 0.02. The liability bound is calculated for different levels of absolute risk aversion from 0.1 to 2.5 for the same efforts  $e^*$ , and thus for the same Nash equilibrium. Figure 1 presents the resulting curves for the massacre contract and the general punishment contract.



The general result of a negative relation between the liability level and the coefficient of absolute risk aversion is demonstrated in Figure 1. The more risk averse a nation is, the more worried it is for punishment, and the lower the necessary liability. Especially for small levels of risk aversion an increase in risk

aversion means a large reduction in the necessary level of liability. The lower liability bound at the same risk aversion level is reduced for the general punishment contract compared to the massacre contract. This is caused by the non-budget-balancing feature of the general punishment contract. In the massacre contract a deviating nation participates in a lottery where the prize is the sum of liabilities of all other nations. Since there is a possible gain from this lottery the nations' liability must be higher than in the general punishment contract, where every nation has to pay the liability and no one gains. Consequently the non-budget-balancing general punishment contract can be implemented for a lower liability and/or a lower risk aversion than the massacre contract.

## 2.5 A numerical example featuring one small and two large nations

In this example we will study an example illustrating the role of the relative size of nations in the two contract types considered. The world is now divided into  $2m+1$  identical regions. These regions are divided into two large blocks of  $m$  regions, referred to as two large nations, and a single region, referred to as a small nation. Thus  $m$  may be interpreted as nation size parameter. In this example  $m$  is fixed at 10, which means that the large nation is ten times larger than the small nation. The emissions abatement efforts and policy are perfectly coordinated within each nation. The utility function is (21), but the parameter values are now fixed at  $b = 0.5$ ,  $a = 0$ ,  $g = 1$ , and  $\beta = 1$ . Thus the restriction on the effort levels to satisfy the restrictions on the functions are, for each of the three nations, which are indexed by  $j$ ,  $0 \leq e_j$ ,  $e_j \leq e_j^*$ , and  $\sum_j e_j \leq 1/2b = 1$ .  $e_j^*$  is the efficient effort level supported as a Nash equilibrium by a contract. An illustration of a global optimum and various Nash equilibrium efforts are presented in Table 1. The global optimum is found by maximizing the global utility function, defined as a region's individual utility function multiplied with  $2m+1$ , where the efforts are coordinated over all regions. Then the effort levels of two different Nash equilibria are presented. In the first Nash equilibrium the large nations coordinate their efforts, whereas the efforts are not coordinated in the second Nash equilibrium.



Table 1 show that global (aggregate) efforts are reduced in the two Nash equilibria compared to the global optimum, with the lowest level in the non-coordinated Nash equilibrium, which should be expected. The small nation's effort is reduced in the Nash equilibria compared to the global optimal effort, especially when the large nations coordinate their efforts. In the latter case the small nation's effort, adjusted for the size parameter (dividing the large nation's liability bound by 10), is ten times smaller than the large nations' effort. In the Nash equilibria the large nations partly compensate the reduced effort by the small nation, increasing their effort compared to the global optimum.

**TABLE 1 Nash equilibrium effort levels for one small and two large nations**

Nash equilibrium	Effort of large nation	Effort of small nation	Global effort
Global optimum	0.472656	0.0472656	0.992578
Nash eq. where large nations coordinate	0.494685	0.0024734	0.991843
Nash eq. where large nations does not coordinate	0.489483	0.0048948	0.983861

Table 2 shows the lower liability bounds of the small and large nations for the general punishment contract and massacre contract which support the global optimum mentioned above. The results demonstrate the difficulty of giving a small nation incentives to exert the contracted effort level and not deviate to a lower effort level. This is emphasized by the results for the general punishment contract. By adjusting the liability bounds under this contract for the size difference of the nations (equal to 10), we find that the liability bound of the small nation must be 80 times higher than for the large nations. As expected the liability bounds are

higher for the massacre contract than the general punishment contract. There is an extra incentive for a nation to deviate to a lower effort under the massacre contract and trigger a lottery where it may win the liabilities of the other nations. Under the same contract the liability bound adjusted for nation size is still larger for the small nation, indicating that the liability must be relatively higher to restrain the small nation from exerting a lower effort. On the other hand the non-adjusted liability bound for a large nation (presented in Table 2) is larger than for the small nation. The massacre contract appears not to be very effective for inducing cooperation since much higher liabilities are required than for the general punishment contract.

**TABLE 2 Liability bounds for one small and two large nations**

Contract type	Large nation	Small nation
Lower liability bound General punishm. contract	0.0003224	0.0025575
Lower liability bound Massacre contract	0.288725	0.0313978

## 2.6 Summing up

Comparing the contract types in terms of which is the most realistic as part of an international treaty, the massacre contract would be more difficult to realize than the general punishment contract. In the massacre contract one nation would gain all the liabilities if the emission abatement target is not met, which probably will be considered unfair, whereas the money would be collected by the international institution for the other contract type. Another disadvantage of the massacre contract is the need for higher liabilities at the same level of risk aversion as compared to the non-budget-balancing general punishment contract. A higher

liability is necessary to discourage a nation from participating in the lottery for the other nations' liabilities under the massacre contract.

Under the general punishment contract it may seem unfair that all nations are punished if one (or a few) nations cheat. This may also be the case for the massacre contract, where all but one nation are punished if one nation cheat. The idea is to give participating nations strong incentives to be honest, however, given that the identity of the cheating nation(s) is unknown.

Even if efficient contracts exist in theory, the levels of risk aversion may be so low that the necessary liabilities need to be very large to secure the existence of an optimal Nash equilibrium. Consequently it may be difficult to commit nations to the necessary set of liabilities. To implement contracts like these the participating nations would have to negotiate a liability of each nation that supports some Nash equilibrium. Obviously there are many difficulties involved in such negotiations. Nations would have different opinions and interests as to the optimal aggregate level of emissions, and thereby to the optimal aggregate effort level, and the Nash equilibrium to aim for. Furthermore the nations would have different interests as to the distribution of efforts and liabilities. To support this Nash equilibrium some level of liability is needed for each nation, and this level can not be known with certainty. Thus each nation would seek to lower its liability and increase the other nations' liabilities. Finally there is a problem of mutual insurance schemes between nations since these distort the incentive-correcting feature of the contracts.

### **3. A REPEATED GAME**

So far the discussion has been restricted to a single-period game. A repeated game featuring many time periods is more realistic, however. If some type of contract between nations is established, it is likely to be renewed after some period of time. This will enable the nations to base their strategies on observations from earlier periods. Furthermore I started out by assuming that the relation between emissions

and atmospheric concentration of greenhouse gases is well known. According to Houghton et al. (1990) and (1992) it is not possible to make exact predictions on the future atmospheric concentrations of greenhouse gases even if human-made emission paths are given. Future concentrations will depend upon the biospheric processes that control the greenhouse gas exchange between the atmosphere, oceans and the terrestrial biosphere. These greenhouse gas feedbacks are again influenced by changes in the climate and other environmental conditions. We are still not able to fully understand these complicated dynamic processes. In the repeated game model presented below, the uncertainty is represented by a stochastic variable in the emission abatement function. The model is inspired by Green and Porter (1984). This is a model of collusion under uncertainty as a Nash equilibrium in contingent strategies.

There is now an infinite time horizon, where the time index is  $t=1,..,\infty$ . Let us assume that the emission abatement  $y(e)$  from eq. (3) is uncertain due to measurement problems or incomplete knowledge of the relation between efforts, emissions and atmospheric concentration of greenhouse gases. Let us furthermore assume that the uncertainty can be represented by a stochastic variable  $\theta_t$  in the emission abatement function (3):

$$(22) Y_t = \theta_t y_t(e_t)$$

where  $\theta_t$  for all  $t$  are identically and independently distributed, and where  $E \theta_t = 1$ . All nations can observe the realization of the stochastic variable  $Y_t$ , whereas  $y_t(e_t)$  is latent. The emission abatement net benefit  $X_{it}$  for nation  $i$  is then defined as:

$$(23) X_{it} = X_{it}[e_{it}, Y_t] = X_{it}[e_{it}, \theta_t y_t(e_{-it}, e_{it})]$$

$$\frac{\delta E X_{it}}{\delta e_{it}} < 0, \quad \frac{\delta^2 E X_{it}}{\delta e_{it}^2} < 0$$

where the expected marginal benefit from unilaterally increasing efforts is assumed to be negative and concave for all nations  $i$ . We limit the discussion to the case of risk-neutral nations. Each nation  $i$  maximizes the expected present value:

$$(24) \quad E \left\{ \sum_{t=1}^{\infty} \rho^{t-1} X_{it} [e_{it}, \theta_t y_t(e_{-it}, e_{it})] \right\}$$

where  $\rho$  is the discount factor. Each nation  $i$  develops a contingent effort strategy  $s_i = (s_{i1}, s_{i2}, \dots)$ , where  $s_{i1} = e_{i1}$  and  $e_{it} = s_{it}(Y_1, \dots, Y_{t-1})$ . Thus the contingent strategy depends on the earlier observations of emissions abatement (the history). The emissions abatement observations depend on earlier efforts by all nations, but are uncertain due to the error term.

A Nash equilibrium in contingent strategies (denoted by  $*$ ) is defined by:

$$(25) \quad E_{s_1^* \dots s_i^* \dots s_n^*} \left\{ \sum_{t=\tau}^{\infty} \rho^{t-1} X_{it} [e_{it}, \theta_t y_t(e_{-it}^*, e_{it}^*)] \right\} \leq \\ E_{s_1^* \dots s_i^* \dots s_n^*} \left\{ \sum_{t=\tau}^{\infty} \rho^{t-1} X_{it} [s_{it}^*(Y_1, \dots, Y_{t-1}), \theta_t y_t(e_{-it}^*, e_{it}^*)] \right\} \\ \forall e_{it}, i$$

where  $i=1, \dots, n$ , and  $E_{s_1^* \dots s_n^*}$  represents expectation with respect to the probability distribution on the space of sequences of emission abatement levels  $Y_t$ . A stochastic process of emission abatement levels is determined recursively by a strategy profile  $(s_1, \dots, s_n)$ . The Nash equilibrium strategy for nation  $i$  is represented by  $s_{it}^*(Y_1, \dots, Y_{t-1})$ . If inequality (25) is satisfied for all  $\tau=1, \dots, \infty$ , this is a perfect equilibrium.

In this game the participating nations must coordinate their strategies in terms of the emission abatement target  $\Psi$  and the length of the non-cooperative punishment period  $T$ . The nations must agree on these parameters. If a lower value than  $\Psi$  is

Let the optimal efforts be  $e$  in non-cooperative periods and  $q$  be the effort level chosen by nation  $i$  in normal periods, where nation  $i$  expects the other nations to make the efforts  $e_i$  and  $\hat{e}_i$ . The expected benefit for nation  $i$  in a normal period is  $N_i$ , whereas the expected benefit in a non-cooperative period is  $R_i$ :

Period  $t$  is normal if either  $t=1$ , or if  $(t-1)$  was normal and  $\Psi \leq \gamma_{t-1}$ , or if  $(t-T-1)$  was normal and  $\gamma_{t-1} > \Psi$ . Otherwise period  $t$  is not normal. Consequently period  $t$  is normal if it is the first period, if the target was met in the preceding and normal period, or if the nations just finished a non-cooperative punishment period. A suitable  $\Psi$  and  $T$  choice will generate a suitable risk for punishment. There is a positive correlation between  $\Psi$  and  $T$  since a more ambitious emission abatement target requires a longer punishment period to induce the cooperative Nash equilibrium. Different pairs of these parameters will support different Nash equilibria. The interests of nations in this respect differ since costs and benefits of an emission abatement policy differ. However, an equilibrium requires the nations to negotiate and agree on one set of  $\Psi$  and  $T$  before the game is opened. Since the optimal strategy only depends on history through the present state (which is either normal or not), each nation must solve a  $(T+1)$ -step stationary Markov dynamic programming problem.

$$(26) \quad s_{it} = \begin{cases} e_{it} & \text{if } t \text{ is normal} \\ \hat{e}_{it} & \text{if } t \text{ is not normal} \end{cases}$$

observed all nations ( $i=1, \dots, n$ ) will choose a non-cooperative emission reduction effort  $\hat{e}_i$ , which by assumption is lower than the cooperative effort  $e_i$ . The non-cooperative emission reduction effort  $\hat{e}_i$  is an alternative Nash equilibrium, namely a Nash equilibrium in noncontingent strategies. No matter what happens in the non-cooperative period the nations will choose the lower effort level. After  $T$  periods they will increase their efforts to the higher level  $e_i$  again. A period is either normal, where all nations choose the cooperative effort level, or not normal, where the nations choose the lower non-cooperative effort level. Thus a *trigger strategy* is defined by:

$$(27) N_i(e_{-i}, q) = E_{\theta} \{ X_i [q, \theta y(e_{-i}, q)] \}$$

$$(28) R_i(\hat{e}) = E_{\theta} \{ X_i [\hat{e}_i, \theta y(\hat{e}_{-i}, \hat{e}_i)] \}$$

where I assume that  $R_i < N_i$  for all nations  $i$ . The expected present value of nation  $i$ ,  $V_i(q)$ , is equal in all normal periods and can be expressed as:

$$(29) \quad V_i(q) = N_i(e_{-i}, q) + \rho \Pr [\Psi \leq \theta y(e_{-i}, q)] V_i(q) \\ + \Pr [\Psi > \theta y(e_{-i}, q)] \left[ \sum_{t=2}^{T+1} \rho^{t-1} R_i(\hat{e}) + \rho^{T+1} V_i(q) \right]$$

where  $\Pr[\Psi \leq \theta y(e_{-i}, q)]$  is the probability of next period being normal (cooperative), and the probability of the next period being non-cooperative is  $\Pr[\Psi > \theta y(e_{-i}, q)]$ . Eq. (29) can be solved with respect to  $V_i(q)$  and rewritten as:

$$(30) \quad V_i(q) = \frac{R_i(\hat{e})}{1 - \rho} + \frac{N_i(e_{-i}, q) - R_i(\hat{e})}{1 - \rho + [\rho - \rho^{T+1}] \Pr [\Psi > \theta y(e_{-i}, q)]}$$

According to eq. (30) the expected present value for nation  $i$  should equal the present value in the non-cooperative phase (the first right-hand term) plus the appropriately discounted expected increase in benefit from cooperation .

The Nash equilibrium condition (25) can be expressed as:

$$(31) \quad V_i(q) \leq V_i(e_i) \quad \forall q \text{ and } i$$

which defines a perfect equilibrium when the inequality is satisfied for all possible choices of  $q$  and for all nations. The first order condition with respect to  $e_i$  is:

$$(32) \quad V_i'(e_i) = 0 \quad \forall i$$

Employing eq. (30) the first order condition is satisfied if:

$$(33) \quad 0 = \{1 - \rho + [\rho - \rho^{T+1}] Pr[\Psi > \theta y(e_{-i}, e_i)]\} \frac{\delta N_i(e_{-i}, e_i)}{\delta e_i} \\ - \{[\rho - \rho^{T+1}] \frac{\delta Pr[\Psi > \theta y(e_{-i}, e_i)]}{\delta e_i}\} \{N_i(e_{-i}, e_i) - R_i(\hat{e})\}$$

where the signs of  $\delta N_i/\delta e_i$  and  $\delta Pr[.]/\delta e_i$  are negative. Thus a nation's marginal benefit from decreasing its efforts in normal periods ( $-N'_i(e)$ ) must be balanced by the marginal increase in risk from facing reduced benefits ( $-[N_i(e)-R_i(\hat{e})]$ ) through triggering a non-cooperative period of length T. This condition must be satisfied for all nations, and will place n constraints on the n-dimensional effort vector e. Owing to this, additional constraints may not be supportable, e.g. maximizing the sum of benefits over all nations.

The Nash equilibrium condition (31) can - with the help of (30) - after some rearrangement be expressed as:

$$(34) \quad \frac{N_i(e_{-i}, e_i) - N_i(e_{-i}, q)}{\rho - \rho^{T+1}} \\ + \frac{N_i(e_{-i}, e_i) Pr[\Psi > \theta y(e_{-i}, q)] - N_i(e_{-i}, q) Pr[\Psi > \theta y(e_{-i}, e_i)]}{1 - \rho} \\ + \frac{R_i(\hat{e}) \{Pr[\Psi > \theta y(e_{-i}, e_i)] - Pr[\Psi > \theta y(e_{-i}, q)]\}}{1 - \rho} \geq 0$$

which must be satisfied for all effort levels q and for all nations i. Since  $E X_i$  is concave in  $e_i$  (confer eq. (23)) I focus on deviations from the Nash equilibrium effort level  $e_i$  to some lower level q, which is lower bounded at the non-cooperative effort level  $\hat{e}_i$ . Thus  $\hat{e}_i \leq q < e_i$ .



Taking the derivative of the left-hand side of (34) with respect to  $T$ , we find that the sign is positive since  $N_i(e_{-i}, e_i) < N_i(e_{-i}, q)$  due to  $\delta(E X_i) / \delta e_i < 0$ . This means that the Nash equilibrium condition is more likely to be satisfied for a given nation, the longer the punishment period lasts. The longer the non-cooperative punishment period, the larger the expected reduction in benefit compared to the marginal short-term benefit from a nation's unilateral deviation to a lower effort. The derivative of the left-hand side of (34) with respect to the abatement target  $\Psi$  is negative if:

$$(35) \quad \frac{\delta \Pr[\Psi > \theta y(e_{-i}, q)] / \delta \Psi}{[N_i(e_{-i}, q) - R_i(\hat{e})]} < \frac{\delta \Pr[\Psi > \theta y(e_{-i}, e_i)] / \delta \Psi}{[N_i(e_{-i}, e_i) - R_i(\hat{e})]}$$

Since  $R_i(\cdot) < N_i(\cdot)$  and  $N_i(e_{-i}, e_i) < N_i(e_{-i}, q)$  the left-hand side denominator is larger than the right-hand side denominator. Inequality (35) is thus satisfied unless the left-hand side numerator is much larger than the right-hand side numerator, which is equivalent to the marginal increase in the probability distribution function  $\Pr[\Psi > \theta y(e_{-i}, e_i)]$  from a marginal increase in emission abatement target  $\Psi$  being much smaller at the Nash equilibrium effort level  $e_i$  than at the lower effort level  $q$ . These derivatives depend on the probability density function of  $\theta$ . Furthermore the difference between the derivatives depends on the difference in the emission abatement level  $y(e_{-i}, e_i) - y(e_{-i}, q)$ , which is small due to the large number of participating nations. For a larger nation this difference is larger, but still the marginal change of the probability distribution function has to dominate the difference of the denominators to change the inequality sign in (35). Taken together this means that the difference of the derivatives is most likely dominated by the difference of the denominators, which supports the inequality sign in (35). Consequently a lower emission abatement target  $\Psi$  means that the Nash equilibrium condition is more likely to be satisfied for nation  $i$ , and then also for all nations.

Analyzing (34) we find that it is most difficult to induce the Nash equilibrium

cooperative effort level for the nation with the largest  $[N_i(e_{-i}, q) - N_i(e_{-i}, e_i)]$  and  $R_i(\hat{e})$ . This follows from  $0 < \delta y(\cdot) / \delta e_i$  and  $q < e_i$ , which lead to  $\Pr[\Psi > \theta y(e_{-i}, e_i)] < \Pr[\Psi > \theta y(e_{-i}, q)]$ . In other words the abatement target must be low enough and the punishment period long enough to induce the cooperative effort level for the nation that will gain most in terms of expected benefit from a unilateral deviation to a lower effort level, and has the highest expected benefit in a non-cooperative period. Since the nations are different they have different interests in  $T$  and  $\Psi$ . The more different the interests of the nations are, the more difficult it is to agree on the abatement target and the length of the punishment period. However, to induce all nations to choose the cooperative effort level, the nation which requires the longest punishment period and the lowest emission abatement target sets the lower bound on the punishment period and the upper bound on the emission abatement target. If one or more countries are outliers in terms of a high  $T$  and/or low  $\Psi$  required to induce the Nash equilibrium, the other nations may be better off without these nations participating. The expected benefit from the trigger contract may be higher without the outlier nations participating since this means a shorter punishment period and a more ambitious emission abatement target.

The frequency of non-cooperative periods is given by  $\Pr[\Psi > \theta y(e)]$  in equilibrium. Even if nations realize that a non-cooperative episode is triggered by the emission abatement (observation or limited knowledge) error term, they will stick to the non-cooperative punishment for the fixed number of periods. The reason is the incentive feature of the equilibrium. Should the nations refrain from punishment if the emission abatement is below the target, cooperation would no longer be optimal for the individual nation. Either the trigger mechanism contract would no longer be valid, and the nations would revert to the low non-cooperative effort level. Or, alternatively, the nations could decide to skip the punishment period this time, but stick to the contract thereafter. Since the contracting nations *ex post* will be better off canceling the punishment when a low aggregate effort level is observed, this is a dominant Nash equilibrium, and the contract is not renegotiation-proof. For the same reason the solution (i.e. the punishment period) is not Pareto optimal. If the punishment period is canceled, however, the trigger

feature of the contract would not be very convincing, and the nations could be left in a situation where the cooperative high effort level is no longer individually optimal for a nation.

#### 4. CONCLUSIONS

We have seen that optimal incentive contracts exist for nations pursuing greenhouse gas emissions abatement, given large enough liabilities and degree of risk aversion for the participating nations. The contracts may or may not be budget-balancing. An international policing institution is needed to prevent renegotiation of the contract. To induce all nations to choose the efficient Nash equilibrium effort level, the liability must depend on the risk aversion of each nation. A nation with a low risk aversion and high marginal effort disutility must face a relatively high liability. All deviations by one or more nations are prevented if each nation's liability is high enough to discourage a unilateral deviation. A lower liability is needed to implement a non-budget-balancing contract as compared to a budget-balancing contract at the same level of risk aversion.

In a repeated game of hidden emission abatement efforts a trigger mechanism can be constructed to give risk neutral nations individual incentives for cooperation, increasing their emissions abatement effort level. Total emission abatement efforts cannot be observed with certainty, and there is a probability of triggering a punishment period even if all nations cooperate. If the emission abatement target is not met, there is a fixed-length non-cooperative punishment period of lower efforts. There will be cooperative periods interrupted by fixed-length punishment periods. The interests of the nations may differ, but they must coordinate their strategies in terms of agreeing on an emission abatement target and the length of the punishment period. The nation that gains most from a unilateral deviation to a lower effort level and has the highest expected benefit in a non-cooperative period will be the most difficult to induce to choose the cooperative Nash equilibrium effort level. These relations suggest that the marginal nation is small

(since aggregate efforts are only slightly reduced from a small nation's unilateral deviation to a lower effort), with high abatement costs and low damage costs from climate change. To give all nations incentives to choose the cooperative effort level, the marginal nation sets the lower bound on the punishment period length and the upper bound on the emission abatement target. Consequently the rest of the nations may be better off without the marginal nation (or nations) participating since the expected benefit from the trigger contract may increase for a shorter punishment period and a higher emission abatement target. If side payments could be administered from the participating nations to the marginal nation(s) a Pareto improvement could be possible. The expected benefit could increase for both the marginal nation and the participating nations. A higher emission abatement target could be accomplished, and a shorter punishment period would be needed to support the Nash equilibrium.

## REFERENCES

- Crémer, Jacques and Richard P. McLean (1988), "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions", *Econometrica*, Vol. 56, No. 6, pp. 1247-57.
- Demski, Joel S. and David Sappington (1984), "Optimal Incentive Contracts with Multiple Agents", *Journal of Economic Theory*, Vol. 33, pp. 152-171.
- Fudenberg, Drew and Jean Tirole (1991), *Game Theory*, The MIT Press, Cambridge and London.
- Green, Edward J. and Robert H. Porter (1984), "Noncooperative Collusion Under Imperfect Price Information", *Econometrica*, Vol. 52, No. 1, pp. 87-100.
- Grossman, Sanford J. and Oliver D. Hart (1983), "An Analysis of the Principal-Agent Problem", *Econometrica*, Vol. 51, No. 1, pp. 7-45.
- Hart, Oliver and Bengt Holmström (1987), The theory of contracts, in Truman F. Bewley (ed.), *Advances in Economic Theory - Fifth World Congress*, Cambridge University Press, Cambridge and New York.
- Holmström, Bengt (1979), "Moral Hazard and Observability", *Bell Journal of Economics*, Vol. 10, pp. 74-91.

- Holmström, Bengt (1982), "Moral hazard in teams", *The Bell Journal of Economics*, Vol. 13, No. 2, pp. 324-40.
- Houghton, J. T., B. A. Callander and S. K. Varney (eds.) (1992), *Climate Change 1992 - The Supplementary Report to the IPCC Scientific Assessment*, World Meteorological Organization and United Nations Environmental Program, Intergovernmental Panel on Climate Change, Cambridge University Press, Cambridge and New York.
- Houghton, J. T., G. J. Jenkins and J. J. Ephraums (eds.) (1990), *Climate Change - The IPCC Scientific Assessment*, World Meteorological Organization and United Nations Environmental Program, Intergovernmental Panel on Climate Change, Cambridge University Press, Cambridge and New York.
- Mäler, Karl-Göran (1989), The Acid Rain Game, in H. Folmer and E. van Ierland (eds.), *Valuation Methods and Policy Making in Environmental Economics*, Elsevier, Amsterdam.
- Rasmusen, Eric (1987), "Moral Hazard in Risk-Averse Teams", *RAND Journal of Economics*, Vol. 18, No. 3, pp. 428-35.
- Rasmusen, Eric (1989), *Games and Information - An Introduction to Game Theory*, Basil Blackwell, Oxford and Cambridge.
- Segerson, Kathleen (1988), "Uncertainty and Incentives for Nonpoint Pollution Control", *Journal of Environmental Economics and Management*, Vol. 15, pp. 87-98.
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- Xepapadeas, A. P. (1992), "Environmental Policy Design and Dynamic Nonpoint-Source Pollution", *Journal of Environmental Economics and Management*, Vol. 23, pp. 22-39.