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A general equilibrium view of global rebound effects

Taoyuan Wei

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CICERO

Center for International Climate
and Environmental Research
P.O. Box 1129 Blindern
N-0318 Oslo, Norway
Phone: +47 22 85 87 50
Fax: +47 22 85 87 51
E-mail: admin@cicero.uio.no
Web: www.cicero.uio.no

CICERO Senter for klimaforskning

P.B. 1129 Blindern, 0318 Oslo
Telefon: 22 85 87 50
Faks: 22 85 87 51
E-post: admin@cicero.uio.no
Nett: www.cicero.uio.no

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Abstract: How do energy efficiency gains affect energy consumption? The effects are generally called "rebound effects" in the literature. Previous studies have extensively focused on only part of the global economy to study rebound effects, e.g. energy consumption by households, one industry, or one country. However, since the global economy is highly connected among countries, these studies may lead to misleading conclusions if the rebound effects in the rest of the economy are significant. Recently Saunders (2008) analyses the demand side by taking the global economy as a whole. Wei (2007) also provides a general analysis by using Cobb-Douglas production functions for the global economy. The present article expands Wei (2007)'s general analysis to explore the rebound effects from an economist's viewpoint by taking the global economy as a whole and applying general forms of production functions. The analysis provides new insights related to rebound effects. For example, we highlight the role of energy supply as a determinant of rebound and we predict that long term rebound may be lower than short term rebound.

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P.B. 1129 Blindern
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1 Introduction

How do energy efficiency gains affect energy consumption? As put forward very early by Jevons (1865), energy efficiency gains may not reduce energy consumption, but on the contrary, increase energy consumption. The effects are generally called "rebound effects" in the literature. Previous studies have extensively focused on only part of the global economy to study rebound effects, i.e. energy consumption by households, industries, or one country. However, since the global economy is highly connected among countries, these studies may lead to misleading conclusions if the rebound effects are significant in the rest of the global economy, particularly in the long-term.

To provide insights of the rebound effects from a global perspective, this article develops a theoretical framework where the global economy is thought of as one entity. In the framework, we do not specify the forms of production and other functions, which allows us to explore any possible values of the rebound effects. The main insights offered in this article are as follows:

- The supply side of the energy market is of equivalent importance to the demand side in quantitatively determining rebound. Most previous studies do not pay sufficient attention to the supply side.
- The substitution between primary energy and other factors is more relevant to long term rebound, but not very relevant for short term rebound since other inputs are not easy to adjust.
- The theory does not automatically predict higher long-run rebound than short-run rebound. This result can be used to explain the empirical findings by Allan, et al. (2007) and Turner (2009).
- The theory claims super-conservation can happen in both the short term and the long term. In the short term, super-conservation requires introducing externalities, while in the long term, it can happen without externalities, at least from the theoretical perspective.

The present article is organized as follows. The next section explains the basic concepts of rebound effects used in this article. This is followed by Section 3, which motivates the article. Section 4 introduces a general equilibrium model for the global economy. Section 5 contains the insights

from the analysis deriving from the general model. Section 6 provides cautions and limitations related to the analysis. The last section concludes the article. Four appendices give a detailed description of the model and the in-depth analysis of short term and long term rebound effects.

2 Concept of rebound effects

We define the concept of "rebound" as developed in Saunders (2000b, 2005, 2008). Let η denote the elasticity of primary energy use w.r.t energy efficiency gains, which measures the percentage change of primary energy use if energy efficiency gains change by one percent. Then rebound coefficient R is defined by

$$R = 1 + \eta. \tag{1}$$

The definition comes from the meaning of zero rebound. If primary energy use is reduced at exactly the same rate as the energy efficiency gains, this is called zero rebound, i.e. $R = 0$, which corresponds to $\eta = -1$. By this definition, five rebound conditions can be stated as:

$R > 1$ or $\eta > 0$	backfire
$R = 1$ or $\eta = 0$	full rebound
$0 < R < 1$ or $-1 < \eta < 0$	partial rebound
$R = 0$ or $\eta = -1$	zero rebound
$R < 0$ or $\eta < -1$	super-conservation

Backfire means primary energy use is in fact increasing given improved energy efficiency. In this case, energy efficiency gains induce more primary energy use than before. *Full rebound* means no change of primary energy use even though there is technical progress. *Partial rebound* represents the case where primary energy use is decreasing but does not decrease at the same rate as the technology. *Super-conservation* then implies primary energy use is decreasing at a greater rate than the technology gain.

3 Motivation

The modern macroeconomic theory on rebound effects can be traced back to Jevons (1865). Jevons (1865) observed that England's consumption of coal increased considerably after James Watt intro-

duced his improvements to the steam engine. Jevons argued for backfire, i.e. increased efficiency would tend to increase the coal use.

Modern theory has proved that backfire is not the only possible outcome. In the modern theory, the so-called Khazzom-Brookes postulate, named after Khazzoom (1980) and Brookes (1990), states that increased energy efficiency can paradoxically lead to increased energy consumption. The Khazzoom (1980) analysis is by its nature a partial equilibrium analysis since aggregate income and output are taken as given. Brookes (1990) began to develop the argument in a macroeconomic context. Later, Saunders (1992, 2000a,b, 2005, 2008) extends this by applying a neoclassical growth approach. However, Saunders' analyses implicitly ignore the constraints of limited energy supply in the market by assuming constant energy prices. Wei (2007) notices this drawback in the theory and introduces a theoretical general equilibrium model to internalize the energy price, using production functions of the Cobb-Douglas form. However, this specific Cobb-Douglas functional form restricts the interpretive power of the general equilibrium analysis. Hence, the present article extends the general equilibrium analysis by applying a completely general form of the production function. In doing so, we put the details of human activities aside and focus on the key issues related to rebound effects.

There are extensive empirical studies of rebound effects. According to a review provided by Dimitropoulos (2007), the estimates for rebound effects range from 15% to 350% (Dimitropoulos, 2007, Section 7). Hence, empirical evidence is very inconclusive. For example, according to Gardner and Joutz (1996), The short-run rebound effects are negligible and long-run rebound effects are considerable; both, however, are significantly less than the theoretical results in Saunders (2000a,b) and Wei (2007). On the contrary, Allan, et al. (2007) and Turner (2009) show that both short-run and long-run rebound effects are considerable, and surprisingly short-run rebound effects are greater than long-run effects, which seems to contradict previous theoretical studies on rebound effects (e.g. Saunders, 2008; Wei, 2007). New theory is necessary to explain these contradictions.

The methods applied by empirical studies are various. They can be roughly classified into three types: econometric methods, computable general equilibrium (CGE) modeling, and hybrids of the previous two. By their nature, studies using econometric methods always focus on part of the economy, e.g. households and industries. CGE models are more suitable for studying the rebound effects. So far, most studies using CGE models focus on one single country, which may not be suitable

for the study of long-term rebound due to the effects of international trade. Even though there are many empirical studies using CGE models, the general equilibrium theoretical foundation of rebound is still not fully developed. The present article is a response to this theoretical gap.

4 Theoretical framework

An ideal framework to study rebound effects would consider the global economy as one entity within a general equilibrium framework. In the framework, the behavior of all industries, countries and individuals is captured such that we can observe their response to energy efficiency gains. In this way, we can sum up all the energy use by all agents to analyze changes of energy consumption at the global level. This framework is not available in reality due to the complexity of the global economy. However, a good approximation can be made by using a global CGE model to empirically study the rebound effects (e.g. GRACE model developed by Aaheim and Rive, 2005).

In order to interpret the empirical results in the right way, this article develops a highly simplified general equilibrium model to highlight the key elements related to rebound effects. The present article attempts to analyze the rebound effects from a theoretical point of view by concentrating on primary energy at the macro-level. We hope this will add to theoretical literature of this field and help researchers understand their own results more deeply.

We assume a global planner who aims to maximize the global welfare such that the existing resources (or global endowments) are used in an efficient way. The welfare allocation among countries and individuals is assumed to be agreed by everyone and is therefore not at issue. We will assume the global economy always follows an efficient path given the available technology and resources such as labor, capital, and primary energy. In this way, an energy efficiency improvement can shift the economy from one efficient path to another efficient path. Then, we can study the rebound effects by comparing global energy consumption in the two efficient paths.

It is worth noting that we focus only on energy efficiency gains. In other words, we do not consider any measure that can be implemented both before and after an energy efficiency gain since such a measure is not relevant to the energy efficiency gain. For example, we do not consider energy use eliminated by turning off certain processes during times they are not required. Or simply by turning off some lights at night when the process is idle. In fact, we exclude such measures by

the assumption that the global economy always follows an efficient path. If such a measure exists, the global economy is not following an efficient path.

Global welfare is determined by various factors such as produced goods and services, pollutants and waste, and preservation of the natural environment. It is impossible to list all of them. However, in a thought experiment, we can imagine that all of the factors can be observed and determined by the existing resources. For example, the produced goods and services are determined by the production technology and resource inputs such as labor, primary energy, and other productive resources; pollutants and waste are by-products of the production and consumption of households; and the natural environment is considered a natural resource. In essence, global welfare can be thought of as produced by inputs of the existing resources, classified into three types: labor, primary energy, and capital. Primary energy includes all fossil fuel like coal, crude oil, and natural gas. The remaining resources other than labor and primary energy are classified as capital.

The essential feature of primary energy and capital is that they exist at the current time and can directly be used without any additional human activities. Any human activities are assumed to be included in the welfare producing process. For example, underground coal is primary energy, while the coal extracted from the ground is just an "intermediate good" used to produce welfare. The existing equipment produced yesterday is capital while new equipment produced today is an "intermediate good" to produce welfare. Hence, we are able to relate global welfare directly to the existing resources. Hence, the market of existing resources is a pure exchange market.

With the story in mind, we develop a simplified model to simulate the essential idea of the previous description for the global economy. The model is a theoretical general equilibrium model, which is described in Appendix A. Using this simplified general equilibrium model, we calculate the rebound effects in the short term and long term. Several insights are generated; these are described in the next section.

5 The insights

In the model, we distinguish short term and long term. In the short term, we assume all production factors other than primary energy are constant. In the long term, all factors can adapt to energy efficiency gains.

By the definition of rebound (1) and the short term elasticity of energy consumption w.r.t. energy efficiency gains (19) in Appendix B, short term rebound can be expressed as

$$R^s = 1 + \frac{1 + \sigma_e^e}{1/\sigma^s - \sigma_e^e}, \quad (2)$$

where σ_e^e is the own-elasticity of marginal product w.r.t. energy input in the welfare production and σ^s the own-price elasticity of energy supply. Since in the short term, the own price elasticity of energy consumption can be defined as $\sigma^d = 1/\sigma_e^e$, then as shown in (22) in Appendix B, the short term rebound can be estimated by the own price elasticities of energy supply and consumption,

$$R^s = \frac{1 + 1/\sigma^s}{1/\sigma^s - 1/\sigma^d}. \quad (3)$$

If the price of energy is assumed to be constant, which implies the own price elasticity $\sigma^s \rightarrow \infty$, then the short term rebound coincides with the absolute value of the own price elasticity of energy consumption, $-\sigma^d$.

By (40) in Appendix C, long term rebound can be expressed by

$$R^l = 1 + \frac{1 + \theta + \sigma_e^e}{1/\sigma^s - \sigma_e^e - \theta} = \frac{1 + 1/\sigma^s}{1/\sigma^s - \sigma_e^e - \theta}, \quad (4)$$

where θ is a parameter related to own-price elasticities of capital supply and demand as well as cross elasticities of marginal product w.r.t. capital and energy inputs in the production of welfare. In the long term, it is not plausible to interpret own elasticity of marginal product w.r.t. energy σ_e^e as own price elasticity of the primary energy σ^d since primary energy price is also affected by capital demand.

In this section, we assume that all the involved elasticities take "correct" values, i.e. $\sigma_e^e < 0$, $\sigma^d < 0$, $\sigma^s > 0$, and $\theta > 0$. In Appendix B where the short term rebound is analyzed, we also consider some "wrong" values of the elasticities of primary energy, e.g. $\sigma_e^e > 0$, $\sigma^d > 0$, or $\sigma^s < 0$ since these may be found in empirical studies.

By these expressions for short term and long term rebounds, we can derive the conditions for various cases of rebounds. The main results are summarized in the next table (Table 1):

	Super Conservation (R<0)	Zero rebound (R=0)	Partial rebound (0<R<1)	Full rebound (R=1)	Back fire (R>1)
Short term	x	$\sigma_e^e \rightarrow -\infty$ (or $\sigma^d \rightarrow 0$) and $\sigma^s \rightarrow 0$	$\sigma_e^e < -1$ or $-1 < \sigma^d < 0$	$\sigma_e^e = -1$ or $\sigma^d = -1$	$-1 < \sigma_e^e < 0$ or $\sigma^d < -1$
Long term	$1/\sigma^s - \theta < \sigma_e^e < 0$	$\sigma_e^e \rightarrow -\infty$ and/or $\theta \rightarrow \infty$	$\sigma_e^e < -1 - \theta$	$\sigma_e^e = -1 - \theta$	$-1 - \theta < \sigma_e^e < \min\{0, 1/\sigma^s - \theta\}$

The table lists the conditions corresponding to possible rebounds. If only energy efficiency gains disturb the economy, then super-conservation can not happen in the short term. The remaining cases of rebound can happen in certain situations. As shown in the table, short term rebound can be nicely represented as the own elasticity of marginal product w.r.t. energy input σ_e^e or the own price elasticity of primary energy consumption σ^d . However, in the long term, as many researchers have noticed, it is not plausible to consider only the own price elasticity of energy consumption. More elasticities are involved in the long term, including the own price elasticity of energy supply σ^s , the price elasticities of other factors (i.e. capital) and the cross elasticities of marginal products (represented by θ).

5.1 Supply side of energy

The supply side of energy market is of equivalent importance to the demand side. Most previous studies pay insufficient attention to the supply side. Consider an extreme case. If the primary energy supply is fixed, the rebound effects must be zero no matter what the rebound is estimated from micro level.

To show the importance of the supply side, consider a single primary energy market at the macro-level. For simplicity, we assume that the market price is determined by primary energy demand and supply even though there are many other factors affecting the market price in the real world. Figure 1 shows the primary energy demand and supply curves. The upward curve is supply and the downward curves are demand. Suppose initial market equilibrium occurs at Point A. If energy efficiency is improved, the original primary energy demand curve may move to the left or the right side due to increased (backfire) or decreased (e.g. partial rebound) primary energy use. First assume the primary energy use increases and the demand curve moves to the right. Then market price moves to Point B if the primary energy price keeps constant. However, the equilibrium price

in the market should be Point C, which is much closer to point A. This implies "actual" primary energy use might be quite less than the constant price case. On the other hand, if the demand curve moves to the left, constant primary energy price may imply much less primary energy use at Point D than the "actual" primary energy use at Point E. As shown in the figure, the "actual" change of primary energy use in both cases is attenuated greatly compared with the constant price case. In addition, the slope of the supply curve may also shift due to changes of factors other than prices (e.g. income), which may result from energy efficiency gains.

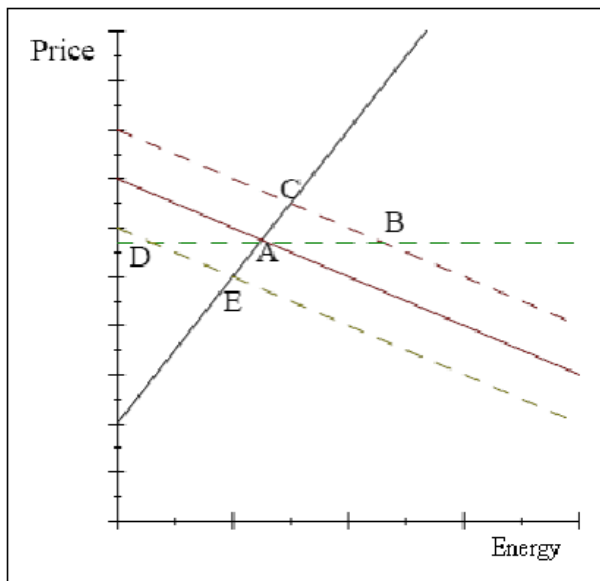


Figure 1. Energy demand and supply

In the quantitative examples of Saunders (2008), rebound is within $(-5, 20)$. It is quite volatile, i.e. the rate of change of primary energy use may be many times the rate of change of energy efficiency gains. However, if the supply of primary energy is assumed to be price inelastic, i.e. $0 < \sigma^s < 1$, then rebound is considerably attenuated. In other words, rebound is always overestimated if the energy price is assumed to be constant, or the supply side of energy is ignored. This holds for both the short term and the long term. For example, if $\sigma^s = 0.5$, $\sigma^d = -0.5$ (or $\sigma_e^e = -2$), then short term rebound is 0.5 if the supply side is ignored and only 0.25 if the supply side is considered. Furthermore, if $\theta = 0.5$, the long term rebound is 0.33 if the supply side is ignored and only 0.14 if the supply side is considered.

When energy efficiency is improved, one direct effect is that less energy is required to produce the same output/welfare and hence demand for energy is reduced. However, when backfire happens,

i.e. the energy efficiency gains lead to more energy demand, then the price of energy can be higher than without the energy efficiency gains. This is an explicit result of the mathematics when the supply side is introduced as shown in (27) of Appendix B and (45) of appendix C.

However, as mentioned in Section 3 of Saunders (2008) , the involvement of the supply side does not essentially change the qualitative findings in both short term and long term. This can be seen in Table 1. the own price elasticity of primary energy σ^s only appears in the condition of super-conservation in the long term and can be dropped for the qualitative findings.

5.2 Substitution between primary energy and other factors

Saunders (1992) first pointed out the importance of substitution between energy and other factors in rebound analysis. Various definitions of substitution elasticities are presented in the literature (e.g. Broadstock, et al, 2007; Thompson, 2006). However, we consider the substitution between energy and other factors by introducing another parameter θ as shown above.

In our model, the short term is defined such that no factors other than primary energy in the production function are allowed to change. Thus, substitution is not an issue in the short term. In the long term, other factors, represented by capital, are allowed to change in the framework and the substitution is relevant.

The substitution effects typically enlarge the value of rebound since by (4), we have the partial derivative

$$\frac{\partial R^l}{\partial \theta} = \frac{1 + 1/\sigma^s}{(1/\sigma^s - \sigma_e^e - \theta)^2} > 0$$

within the interval $\theta > 1/\sigma^s - \sigma_e^e$ or $\theta < 1/\sigma^s - \sigma_e^e$. When $\theta \rightarrow 1/\sigma^s - \sigma_e^e$, rebound goes to an unstable infinity. If in an empirical study, θ happens to be very close to $1/\sigma^s - \sigma_e^e$, then estimated rebound may be quite high and very sensitive to values of these elasticities.

As shown in Table 1, the qualitative results of long term rebound depend on both own elasticity of marginal product of primary energy σ_e^e and the substitution between factors represented by θ . This may lead to super-conservation in certain cases.

5.3 super-conservation

super-conservation says that one percent energy efficiency gains leads to more than one percent reduce in primary energy consumption. This is highly counter-intuitive. As our general equilibrium model shows, super-conservation is possible with very strong substitution between primary energy and other factors. A hypothetical example is the application of solar energy. Assume solar energy is included in the capital since it is non-exclusive and clean to a large extent. If we find methods to replace primary energy with solar energy in most applications, then the demand for primary energy would decline dramatically and super-conservation may happen.

In the short term, we do not expect to see super-conservation since the substitution between primary energy and other factors is extremely limited at the global level¹. However, in the long term, we can expect new technology to develop new energy and super-conservation could happen. This is implied by the expressions (2) for short term rebound and (4) for long term rebound. By (2), short term rebound is always positive. The substitution between primary energy and other factors is suppressed and there is no way to induce super-conservation. By (4), long term rebound becomes more flexible due to the availability of substitution between factors. Then super-conservation may happen when $\theta > 1/\sigma^s - \sigma_e^e$.

The above results hold when an energy efficiency gain τ is the unique external disturbance. If there are other external shocks happening together with energy efficiency gains, then super-conservation may happen even in the short term. Appendix D considers such kinds of cases by introducing accompanying efficiency gains for capital and total factor productivity.

5.4 Higher long term rebound?

Previous theoretical work in this area seems to predict that long term rebound is always higher than short term rebound (e.g. Saunders, 2008; Wei, 2007). This might be true for many common cases. However, as shown by our general equilibrium model in the appendices, long term rebound

¹Harry Saunders acknowledges (personal communication) the correctness of this result that super-conservation is impossible in the short term, and that the mathematical development here is sound. His results showing super-conservation for the Gallant (Fourier) function in the short term (Saunders, 2008) must be incorrect. He has not been able to determine the source of the problem but speculates that precision of the concavity calculation may be to blame and that the short-term function he used may be very slightly convex, leading to a slightly positive implied value for σ_e^e in equation (19) with resulting super-conservation but violation of concavity. In essence, the approach used in Saunders (2008) is consistent with the approach used in the present article, which is shown in Appendix E.

can be lower than short term rebound in certain settings.

By comparing the short term rebound in (2) with the long term rebound (4), we have

$$R^l - R^s = \frac{\theta (1 + 1/\sigma^s)}{(1/\sigma^s - \sigma_e^e) (1/\sigma^s - \sigma_e^e - \theta)} \quad (5)$$

if the values of all the elasticities are assumed to be the same both in the short term and in the long term. Hence, if the substitution effect is so large that $\theta > 1/\sigma^s - \sigma_e^e$, then by (5), long term rebound is less than short term rebound. It is more likely to happen with larger own price elasticity of primary energy supply σ^s . In particular, if the long term price of primary energy is assumed to be constant, i.e. $\sigma^s \rightarrow \infty$, then we only need $\theta > -\sigma_e^e$ to observe lower long term rebound. If in fact σ^s is far from the infinity, e.g. $\sigma^s < 1$, then we are less likely to observe lower long term rebound.

According to (43) in Appendix C, the elasticity of capital K w.r.t. energy efficiency gains τ in the long term can be expressed by

$$\frac{dK}{d\tau} \frac{\tau}{K} = \frac{\sigma_e^k}{1/\sigma_k^s - \sigma_k^k} R^l, \quad (6)$$

which shows that whether disinvestment (i.e. less capital stock) happens or not depends on the rebound R^l and the cross elasticity of the marginal product of capital w.r.t. primary energy σ_e^k . If super-conservation happens, i.e. $R^l < 0$, then disinvestment happens if additional energy inputs raise other marginal products, i.e. $\sigma_e^k > 0$ and investment happens if additional energy inputs reduce other marginal products, i.e. $\sigma_e^k < 0$. On the other hand, if partial rebound, full rebound, or backfire happens, i.e. $R^l > 0$, then disinvestment happens if additional energy inputs reduce other marginal products, i.e. $\sigma_e^k < 0$ and investment happens if additional energy inputs raise other marginal products, i.e. $\sigma_e^k > 0$.

This can be used to explain the empirical findings by Allan, et al. (2007) and Turner (2009). Since the long term rebound of primary energy can be lower than short term rebound, this implies the theory does not exclude the same result for produced energy like gasoline. One special case is that the produced energy is proportional to primary energy input in thermal terms. In addition, at least for this special case, as shown by (6), the supply side of capital represented by σ_k^s is not

crucial for either long term rebound nor investment. That is to say, if we assume a constant capital price in the long term, the same conclusion can be drawn.

6 Cautions and limitations

Our general equilibrium model is based on a number of fairly strict assumptions. We assume that the global economy seeks to maximize global welfare by using existing productive resources such as capital, labor, and primary energy. We assume one production function to connect global welfare with the existing productive resources. Capital and primary energy markets are assumed to be competitive. Labor supply is constant. In the main analysis, we consider only one disturbance in the economy, i.e. energy efficiency gains that are factor-specific to primary energy. In doing so, we restrict the technical effects to energy only, not to other factors. By these assumptions, we are able to focus on the key questions related to global rebound and obtain the above insights.

In our simplified model, the short term assumes constant inputs other than primary energy. In reality, this might not be obtained. If some inputs in the short term are assumed to be variable, then we could observe outcomes that exhibit certain properties of the long term case described in this article. Such outcomes typically appear in some partial analysis where labor are assumed to be flexible. However, in the global context, constant labor supply seems to be more plausible.

For the most part, we assume all involved elasticities take the "correct" values. In some extreme cases, these elasticities may actually take the "wrong" values. For example, if primary energy is close to its exhaustion, then its supply may have to be decreasing even though primary energy price in the market is increasing. This then implies an abnormal supply curve of primary energy, i.e. the own price elasticity of primary energy supply σ^s may be negative. In Appendix B, we consider abnormal values of own price elasticities of primary energy demand and supply. As shown there, the abnormal values may lead to some abnormal results.

Since we are considering the global economy, this means we ignore certain rebound effects identified by studies focusing on one part of the global economy. For example, the "open economy" rebound effect first identified by Allan, et al. (2007) and any rebound effects that cancel out each other in the context of the global economy.

In our model, only two factors (capital and primary energy) are considered for simplicity. In this

way, capital is assumed to include all the factors other than primary energy. Hence, the substitution between capital and primary energy may tend to be more flexible than we have assumed since certain renewable energy is classified as capital. The classification is not precise. However, if we are concerned with other factors that may produce rebound effects, then these factors can be considered as primary energy and still satisfy the intent of our analysis.

In this theoretical model, it seems appropriate to use one production function given our purpose. However, in reality, rebound effects may differ greatly from sector to sector, from country to country, and from one household to another. This indicates it would be helpful for future analyses to decompose the global economy.

We do not say anything about the source of energy efficiency gains. In our model, we only care what effect the technology gains have on energy consumption and the global economy. We just assume the technology becomes available at a given point in time, which may be the result of endogenously determined mechanisms arising from the accumulation of human capital through investments in R&D. But, whatever their source, we only care about the future course of the global economy after the technology gains are available. It is beyond this article's scope to discuss the relations between the technology gains and the accumulation of human capital.

We study the changes between two equilibria before and after the technology gains and ignore the dynamics of possible adjustment processes from one equilibrium to another. The adjustment process may induce huge costs since new technology could call for updating existing machinery and equipment. Such costs may be considerable under certain circumstances.

The general equilibrium model is also instructive for partial equilibrium analysis of the rebound effect. For example, constant factor price implies infinite price elasticity of supply and it can be considered in the model by setting $1/\sigma^s = 0$; Furthermore, if we interpret global welfare as output that demands a specific source of primary energy, the model can be thought of as a partial equilibrium model.

7 Conclusion

By the application of a general equilibrium model to the global economy with a completely general form of the production function, this article has examined rebound effects in the global context and

has drawn several insights related to the role of energy supply, to substitution between energy and other factors, to the comparison of long term rebound with short term rebound, and to the case of super-conservation. These insights may serve to assist the intuitive understanding of results generated from empirical studies. However, this connection between empirical studies and theoretical ones works both ways: several of the findings of this article still lack sufficient empirical support. In particular, at this point it appears super-conservation is mainly a phenomenon in theory only.

Appendix A. Theoretical general equilibrium model

Assume global welfare can be represented by a numerical index Y , which can be aggregate output of global production if global welfare is proportional to output. Global welfare is a complicated function of existing resources: labor, primary energy, and capital. To simplify the notation, this article always assumes constant labor input to the global economy. We assume that primary energy and capital inputs to produce global welfare can be represented by aggregates E^d and K^d respectively. The superscript d indicates these variables are quantities demanded by the production process. Hence, the global welfare function, including all the economic activities in the economy, is assumed to be of the following form and is assumed to be twice continuously differentiable

$$Y = f(K^d, \tau E^d), \tag{7}$$

where labor is ignored in the function since it is assumed to be constant and τ is an energy efficiency parameter. If energy efficiency τ increases by one percent, we can produce the same global welfare level by reducing primary energy input by one percent *ceteris paribus*.

Normally increasing factor inputs increases more welfare, which implies positive derivatives: $f_1 > 0$ and $f_2 > 0$, where f with a number as subscript represents the derivative w.r.t the argument indicated by the number signifying its order in the function f .

If the prices of capital K and primary energy E are given in the global market, the global

planner maximizes the net value of the global welfare:

$$\begin{aligned} \max_{K^d, E^d} \quad & Y - P_k K^d - P_e E^d \\ \text{s.t.} \quad & Y = f(K^d, \tau E^d), \end{aligned}$$

where the price of welfare Y is set as the numeraire (unity), P_k is the rental price of capital, and P_e is the price of primary energy. The two necessary conditions for the maximization problem are,

$$f'_k : f_1(K^d, \tau E^d) = P_k \tag{8}$$

$$f'_e : \tau f_2(K^d, \tau E^d) = P_e, \tag{9}$$

which determine the demand for capital K^d and primary energy E^d respectively if both capital and primary energy can be adjusted by the global planner.

On the other hand, the supplies of capital and primary energy depends on the market prices, which can simply be written as

$$K^s = K^s(P_k) \tag{10}$$

$$E^s = E^s(P_e). \tag{11}$$

Of course there are more drivers than own prices that have effects on these supply curves. Here they are abstracted or assumed to be taken into account in the prices such that we can highlight the key effects of prices on capital and primary energy supplies.

Along the optimal path, the demand and supply of both markets for capital and primary energy are equalized:

$$K^s = K^d = K \tag{12}$$

$$E^s = E^d = E. \tag{13}$$

If the quantities of demand K^d and E^d are known, these four equations from (10) to (13) can be used to determine the market prices P_k and P_e .

Hence, the above 7 equations from (7) to (13) constitute a general equilibrium model, where the variables are global welfare Y , capital demand K^d , primary energy demand E^d , capital supply K^s , primary energy supply E^s , capital price P_k , and primary energy price P_e .

Assume initially the global economy follows an efficient path corresponding to an energy efficiency level $\tau = 1$. An energy efficiency gain, i.e. a change in τ , will disturb the initial global economy and lead the global economy to another optimal path. By comparing these two paths before and after the energy efficiency gain, we can study the rebound effects on primary energy consumption of the energy efficiency gain.

Following the notation used here, the elasticity of primary energy consumption w.r.t the efficiency gains can be expressed as

$$\eta = \frac{d \ln E}{d \ln \tau} = \frac{dE}{d\tau} \frac{\tau}{E}.$$

As explained above, rebound is defined as $R = 1 + \eta$. Zero rebound happens if and only if primary energy E changes proportionally in the opposite direction of the energy efficiency gain. Hence, no rebound in fact assumes only primary energy use is affected by the energy efficiency gains.

Economists argue that after observing energy efficiency gains, the global planner would not keep the same production level since the costs are reduced for the output of global welfare Y . She can benefit more by expanding the production. One way to do so is to use more primary energy in production while keeping other input factors the same, which is called the "short term" response since other factors like capital are not easy to adjust in a short term. If there is enough time for the global planner to adjust other factors, here only the capital, then it is called the "long term" response. Consequently, rebound may happen if the global planner expands her production scale.

Appendix B. Short term effects

In the short term, the global planner is not able to adjust capital level to an energy efficiency gain. On the other hand, she can adjust primary energy input to achieve a higher global welfare level.

By the supply function (11), the reversed demand function can be written as

$$P_e = P_e(E^s), \tag{14}$$

which can be substituted into equation (9) to obtain

$$\tau f_2 \left(K^d, \tau E^d \right) = P_e \left(E^s \right), \quad (15)$$

which holds for any equilibria of the global economy. A change in energy efficiency τ may disturb the global economy. In the short term, capital K^d is assumed given. Then total differentiation on both sides of (15) w.r.t τ , E^d , and E^s yields

$$\frac{dP_e}{dE^s} dE^s - \tau^2 f_{22} dE^d = \left[f_2 + \tau E^d f_{22} \right] d\tau,$$

which can be rewritten as

$$\frac{dP_e}{dE^s} \frac{E^s}{P_e} \frac{P_e}{E^s} dE^s - \frac{\tau E^d f_{22}}{f_2} \frac{\tau f_2}{E^d} dE^d = f_2 \left[1 + \frac{\tau E^d f_{22}}{f_2} \right] d\tau. \quad (16)$$

To simplify the notation, let σ^s denote the *price elasticity of primary energy supply*:

$$\sigma^s = \frac{dE^s}{dP_e} \frac{P_e}{E^s}, \quad (17)$$

which is generally positive since suppliers tend to provide more primary energy with a rise in primary energy price. Since the main source of primary energy today is fossil primary energy, which is an exhaustible resource and non-renewable, primary energy supply must be considered price inelastic. According to Huntington (1991, Table A.3), the price elasticity of U.S. oil supply is between 0.01 and 0.67. Hence, in the present article, the normal price elasticity of primary energy supply is assumed to be between 0 and 1, i.e. $0 < \sigma^s < 1$. In the following analysis, The abnormal value of σ^s is also considered to provide hints that may explain possible empirical results.

Let σ_e^e denote the own elasticity of marginal product w.r.t. primary energy,

$$\sigma_e^e = \frac{\partial f'_e}{\partial E^d} \frac{E^d}{f'_e} = \frac{\tau E^d f_{22}}{f_2}, \quad (18)$$

which is normally negative since more primary energy input lead to lower marginal product.

Substituting (17), (18) and (9) to (16), we have

$$\frac{1}{\sigma^s} \frac{\tau f_2}{E^s} dE^s - \sigma_e^e \frac{\tau f_2}{E^d} dE^d = f_2 [1 + \sigma_e^e] d\tau$$

if $\sigma^s \neq 0^2$. By the market clearing condition (13), the short-term elasticity of primary energy use w.r.t. the energy efficiency gain can be expressed by

$$\eta^s = \frac{dE}{d\tau} \frac{\tau}{E} = \frac{1 + \sigma_e^e}{1/\sigma^s - \sigma_e^e}, \quad (19)$$

which is defined by the price elasticity of primary energy supply and the own elasticity of marginal product w.r.t. primary energy. If $\sigma_e^e = -1$, then primary energy consumption would not change since the effects of the energy efficiency gain are totally offset by the effects on the marginal product of primary energy. The expression (19) also shows that the short term rebound can not result in super-conservation since super-conservation needs $\eta^s < -1$, which implies an elasticity of energy supply $-1 < \sigma^s < 0$.

Furthermore, in the short term, the rebound η^s can be expressed by price elasticity of primary energy demand. Let σ^d denote *the price elasticity of primary energy demand*,

$$\sigma^d = \frac{dE^d}{dP_e} \frac{P_e}{E^d}, \quad (20)$$

which is normally negative, i.e. $\sigma^d < 0$, since less energy demand typically accompanies a high price of energy. In these circumstances, increased primary energy demand is induced by increased production of global welfare. As shown in (9), the marginal product of primary energy use, τf_2 , can be interpreted as the energy price P_e . In the short term, only E^d and P_e are variables in equation (9) given any energy efficiency gain τ . Then the short term price elasticity of primary energy demand can be expressed by

$$\sigma^d = \frac{f_2}{\tau E^d f_{22}} = \frac{1}{\sigma_e^e}, \quad (21)$$

where normally the second order derivative $f_{22} < 0$. In empirical studies, one may encounter the

²That $\sigma^s = 0$ implies supply of primary energy does not affected by market price. It may happen when the primary energy is close to its exhaustion. However, it is not of interest in this article.

case of abnormal value: $f_{22} > 0$ and $\sigma^d > 0$. If so, primary energy demand may increase even though its price is going up for some production combination. Hence, the present article also discusses these abnormal cases.

Applying (21) to (19), short term rebound η^s can be expressed by

$$\eta^s = \frac{dE}{d\tau} \frac{\tau}{E} = -\frac{\sigma^s}{\sigma^s - \sigma^d} (1 + \sigma^d), \quad (22)$$

which shows that short term rebound is determined by price elasticities of both supply and demand of primary energy. If the absolute value of the price elasticity of energy demand is closer to unity, i.e. $\sigma^d \rightarrow -1$, then energy efficiency gains have almost no effect on energy consumption.

If primary energy prices in the market are constant, which implies the price elasticity of primary energy supply $\sigma^s \rightarrow \infty$, then the short-term elasticity of energy consumption w.r.t the energy efficiency gains is

$$\eta_0^s = \frac{dE}{d\tau} \frac{\tau}{E} = -1 - \sigma^d, \quad (23)$$

a relationship which was first put forward by Khazzoom (1980, eq. A7). Normal values of η_0^s satisfy $\eta_0^s > -1$ since $\sigma^d < 0$. In this case, only the price elasticity of energy demand σ^d is relevant to rebound effects. The value of η_0^s can serve as a baseline rebound indicator for the short term analysis. By rearranging the expression, we obtain the price elasticity of energy demand

$$\sigma^d = 1 + \eta_0^s = R, \quad (24)$$

where the second equation is obtained from the definition of rebound (1).

Expression (24) shows an important fact for short term rebound with constant primary energy price: we see that rebound is represented by the price elasticity of primary energy demand alone. If primary energy demand is inelastic with respect to its price, then rebound will not be very volatile by (24). By the definition (20), since the price elasticity of primary energy demand is normally negative, then zero rebound and super-conservation are not expected in the short term. This is also true even if the supply side of primary energy is considered as shown below.

Substituting (23) to (19) yields

$$\eta^s = \frac{\sigma^s}{\sigma^s - \sigma^d} \eta_0^s, \quad (25)$$

which indicates that the short term elasticity of primary energy use w.r.t the energy efficiency gain η^s is proportional to, and always less than, the situation where supply of primary energy is assumed to be always available at a constant market price. A relatively higher absolute value of σ^d compared with σ^s implies smaller rebound effects from the energy efficiency gain. This shows the importance of the supply side in the primary energy market.

If instead, we insert the first equation of (24) to (22) to yield

$$\eta^s = \frac{\sigma^s \eta_0^s}{1 + \eta_0^s + \sigma^s}, \quad (26)$$

this then shows the price elasticity of primary energy supply σ^s plays the same important role as the baseline elasticity of primary energy use w.r.t the energy efficiency gains η_0^s .

The primary energy price change due to τ can be calculated as the elasticity of primary energy price w.r.t τ by using (14) directly,

$$\frac{dP_e}{d\tau} \frac{\tau}{P_e} = \frac{1}{\sigma^s} \frac{dE}{d\tau} \frac{\tau}{E} = \frac{1}{\sigma^s} \eta^s, \quad (27)$$

which shows that the primary energy price change is greater than the change of primary energy use if the price elasticity of primary energy supply is inelastic to the primary energy price, i.e. $\sigma^s < 1$. In addition, equation (27) also shows that primary energy price changes in the same direction as primary energy use, which is consistent with the illustration in Figure 1. Since the primary energy supply curve does not change, then it is natural that the market equilibria go along the primary energy supply curve.

The global welfare change due to τ can be calculated as the elasticity of output w.r.t τ by using (7) directly,

$$\frac{dY}{d\tau} \frac{\tau}{Y} = s_e \left(1 + \frac{dE}{d\tau} \frac{\tau}{E} \right) = s_e (1 + \eta^s), \quad (28)$$

where s_e is the value share of primary energy use in welfare production,

$$s_e = \frac{P_e E}{Y}.$$

Since the rebound coefficient $R = 1 + \eta^s$, the elasticity of output w.r.t τ is determined by the product

of the value share of the primary energy use in output and the rebound coefficient³. However, it is ambiguous whether or not the rate of change of welfare is greater than the rate of change of primary energy use. If $\eta^s < s_e/(1 - s_e)$, then welfare changes at a higher rate than primary energy, which implies that production becomes less primary energy intensive even though more primary energy is used.

Just by comparing (19) with (23), we see the effect on primary energy use of energy efficiency gains is less than the constant energy price case as long as there is some energy price change. In particular, the price elasticity of primary energy supply σ^s is far from infinity. As a result, the change of primary energy use represented by η^s might be considerably less than in the case of constant primary energy price. If the value of the price elasticity of primary energy supply and demand are normal, i.e. $0 < \sigma^s < 1$ and $\sigma^d > 0$, then the short term elasticity of primary energy use η^s satisfies $-1 < \eta^s < 1$, which implies that primary energy use always changes at a rate less than the energy efficiency gains even if it is positive infinity under the constant primary energy price case, $\eta_0^s \rightarrow +\infty$. For example, Even if $\eta_0^s = 19, \sigma^s = 1$, then $\eta^s = 19/21$, which is less than one and only about 5% of η_0^s . The rebound effects is attenuated greatly. Figure 2 shows the normal case for $0 < \sigma^s < 1$ and $-1 < \eta_0^s < 10$.

³Interested readers can check this equation (28) by production functions listed in Saunders (2008).

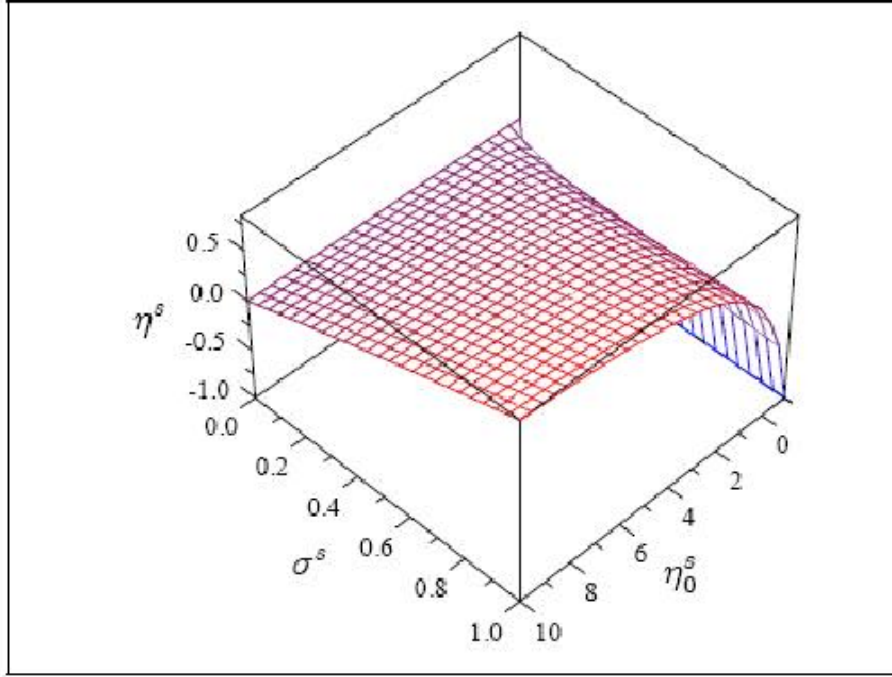


Figure 2. The short term elasticity of primary energy consumption
w.r.t. energy efficiency gains (normal value)

By (19), It is clear there is no room for super-conservation in the normal case. But is it possible if we introduce the abnormal values of price elasticities of primary energy supply or demand?

First consider the abnormal values of price elasticity of primary energy demand. Given normal value of σ^s , as shown in Figure 3 (where the x -axis is η_0^s and the y -axis is η^s . If $\eta_0^s \rightarrow -1 - \sigma$, then $\eta^s \rightarrow \infty$. If $\eta_0^s \rightarrow \infty$, then $\eta \rightarrow \sigma$), the large negative η_0^s always implies higher η^s than the normal values of η_0^s , which shows the reversal of rebound: the *super-conservation case under constant primary energy prices becomes a backfire case after the price change effects are taken into consideration*. This is because the large negative η_0^s implies a large negative price elasticity of primary energy demand σ^d by (24), which means the primary energy demand moves upward instead of downward. The closer to zero the negative values of η_0^s (or σ^d), the closer is the demand curve to the supply curve. Hence, the slight change of the demand curve due to τ would result in a large change of energy use when these two curves become almost the same, which then implies huge η^s .

The possibility of super-conservation happens when $\eta_0^s \rightarrow -1 - \sigma^s$ from the positive direction,

which, by (24), implies the price elasticity of energy demand is almost equated to the price elasticity of primary energy supply, i.e. $\sigma^d \rightarrow -\sigma^s$. This means the primary energy demand curve almost overlaps the supply curve. Since this implies people buy more energy to use when the energy price is higher, such a situation would be very unstable if it should happen in reality. In addition, the result is very sensitive to the change of the initial abnormal value of η_0^s (or σ^d). Hence, this is not a credible explanation for the super-conservation case. Similarly, the backfire case along with $\eta_0^s \rightarrow -1 - \sigma^s$ from the negative direction is also not credible.

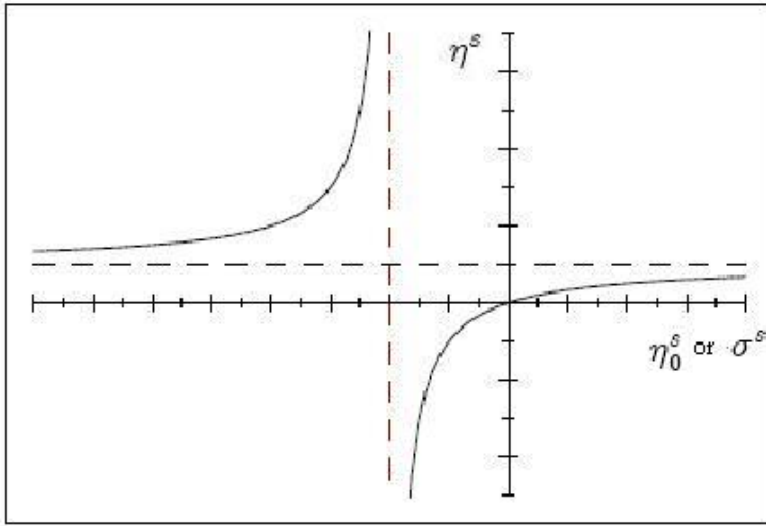


Figure 3. The short term elasticity of primary energy consumption w.r.t. energy efficiency gains. Given one of the two arguments

On the other hand, if $\sigma^d > 0$ is given as a normal value, then by (26), if $\sigma^s \rightarrow -1 - \eta_0^s$, then $\eta^s \rightarrow \infty$. If $\sigma^s \rightarrow \infty$, then $\eta^s \rightarrow \eta_0^s$. Since σ^s and η_0^s are symmetric in the expression of η^s (26), then Figure 3 can also be used for this case. In this case, the x -axis is σ^s , y -axis is η^s . Super-conservation is possible if and only if $\sigma^s \rightarrow -1 - \eta_0^s$ from the positive direction, which is almost the same story as the case with given σ^s and flexible η_0^s . Now the primary energy supply curve is abnormally downward, i.e. more supply with lower prices. This situation with super-conservation is not credible since it happens when the primary energy supply curve is almost the same as the energy demand curve.

Undoubtedly super-conservation may happen if both σ^s and η_0^s have abnormal values. However,

this is not credible. Hence, it is not plausible to explain the super-conservation case by appealing to a non-concave production function or a downward primary energy supply curve.

The above analysis of super-conservation is applicable for any production function or its dual cost function. Hence, if the producer is assumed to pursue profit maximization, the only possible case for super-conservation comes from assuming an abnormal primary energy demand or supply curve, both of which are not credible⁴.

Then does it mean super-conservation is impossible from a theoretical perspective? The answer is both yes and no. If the production function does not change, the answer is yes as shown above. The answer could be no if the external effects on productivity of the energy efficiency gains are considered. We will consider the externalities in Appendix D.

Appendix C. Long term effects

In the long term, capital is allowed to adjust to the energy efficiency gains. Hence, the capital market has to be considered. By the capital supply function (10), the reversed demand function can be written as

$$P_k = P_k(K^s), \quad (29)$$

which can be substituted to equation (8) to obtain

$$f_1(K^d, \tau E^d) = P_k(K^s), \quad (30)$$

which holds for any equilibria of the global economy. At the same time, the equation (15) still holds in the long term. For convenience, we rewrite it here

$$\tau f_2(K^d, \tau E^d) = P_e(E^s). \quad (31)$$

⁴For example, the Gallant (Fourier) function may not be suitable for the rebound study to depict the full range of rebound possibilities as declared in Saunders (2008). Since (28) holds for any cost or production function, then super-conservation always means less output, which implies the initial production is inefficient for a profit-maximizing producer. If super-conservation occurs, it must show that the initial unit cost (or output) can be improved for the producer to obtain more profit even without the energy efficiency gain.

Total differentiation on both sides of (30) and (31) w.r.t τ , K^d , E^d , K^s , and E^s , yields

$$f_{11}dK^d + f_{12} \left(\tau dE^d + E^d d\tau \right) = \frac{dP_k}{dK^s} dK^s \quad (32)$$

$$\tau f_{21}dK^d + f_2 d\tau + \tau f_{22} \left(\tau dE^d + E^d d\tau \right) = \frac{dP_e}{dE^s} dE^s, \quad (33)$$

Both of which can be respectively re-expressed by

$$\frac{f_{11}K^d}{f_1} \frac{f_1}{K^d} dK^d + \frac{\tau E^d f_{12}}{f_1} \frac{f_1}{\tau E^d} \left(\tau dE^d + E^d d\tau \right) = \frac{dP_k}{dK^s} \frac{K^s}{P_k} \frac{P_k}{K^s} dK^s \quad (34)$$

$$\tau \frac{K^d f_{21}}{f_2} \frac{f_2}{K^d} dK^d + f_2 d\tau + \frac{\tau E^d f_{22}}{f_2} \frac{f_2}{E^d} \left(\tau dE^d + E^d d\tau \right) = \frac{dP_e}{dE^s} \frac{E^s}{P_e} \frac{P_e}{E^s} dE^s. \quad (35)$$

We now define

$$\begin{aligned} \sigma_e^k &= \frac{\partial f'_k}{\partial E^d} \frac{E^d}{f'_k} = \frac{\tau E^d f_{12}}{f_1}, & \sigma_k^e &= \frac{\partial f'_e}{\partial K^d} \frac{K^d}{f'_e} = \frac{K^d f_{21}}{f_2} \\ \sigma_k^k &= \frac{\partial f'_k}{\partial K^d} \frac{K^d}{f'_k} = \frac{f_{11}K^d}{f_1}, & \sigma_k^s &= \frac{dK^s}{dP_k} \frac{P_k}{K^s}, \end{aligned}$$

where σ_e^k is the elasticity of the marginal product of capital w.r.t. primary energy input, σ_k^e is the elasticity of marginal product of primary energy w.r.t. capital input, σ_k^k is the elasticity of marginal product w.r.t. capital itself, and σ_k^s is the own-price elasticity of capital supply. The sign of σ_e^e is the same as σ_e^k due to the symmetric cross partials $f_{21} = f_{12}$. Substituting these elasticities together with σ_e^e and σ^s into (34) and (35) yields

$$\begin{aligned} \sigma_k^k \frac{f_1}{K^d} dK^d + \sigma_e^k \frac{f_1}{\tau E^d} \left(\tau dE^d + E^d d\tau \right) &= \frac{1}{\sigma_k^s} \frac{P_k}{K^s} dK^s \\ \tau \sigma_k^e \frac{f_2}{K^d} dK^d + f_2 d\tau + \sigma_e^e \frac{f_2}{E^d} \left(\tau dE^d + E^d d\tau \right) &= \frac{1}{\sigma^s} \frac{P_e}{E^s} dE^s. \end{aligned}$$

Furthermore, by noticing (8), (9), (12), and (13), we have

$$\sigma_k^k \frac{dK}{K} + \sigma_e^k \left(\frac{dE}{E} + \frac{d\tau}{\tau} \right) = \frac{1}{\sigma_k^s} \frac{dK}{K} \quad (36)$$

$$\sigma_k^e \frac{dK}{K} + \frac{d\tau}{\tau} + \sigma_e^e \left(\frac{dE}{E} + \frac{d\tau}{\tau} \right) = \frac{1}{\sigma^s} \frac{dE}{E}. \quad (37)$$

By (36), we know

$$\frac{dK}{K} = \frac{\sigma_e^k}{1/\sigma_k^s - \sigma_k^k} \left(\frac{dE}{E} + \frac{d\tau}{\tau} \right), \quad (38)$$

which is then inserted to (37) to obtain

$$\left[1 + \frac{\sigma_k^e \sigma_e^k}{1/\sigma_k^s - \sigma_k^k} + \sigma_e^e \right] \frac{d\tau}{\tau} = \left[\frac{1}{\sigma^s} - \frac{\sigma_k^e \sigma_e^k}{1/\sigma_k^s - \sigma_k^k} - \sigma_e^e \right] \frac{dE}{E}. \quad (39)$$

If we let

$$\theta = \frac{\sigma_k^e \sigma_e^k}{1/\sigma_k^s - \sigma_k^k},$$

which should be positive for a normal capital market since σ_e^e has the same sign as σ_e^k . Then by (39), we have the long term elasticity of primary energy use w.r.t energy efficiency gains

$$\eta^l = \frac{dE}{d\tau} \frac{\tau}{E} = \frac{1 + \theta + \sigma_e^e}{1/\sigma^s - \sigma_e^e - \theta} \quad (40)$$

if $1/\sigma^s - \sigma_e^e \neq \theta^5$. Long term rebound effects depend on three elements: price elasticity of primary energy supply, price elasticity of primary energy demand, and another element related to the cross elasticities of marginal product w.r.t. factors and price elasticities of capital supply and demand. By (40), super-conservation can happen in the long term if $1/\sigma^s - \sigma_e^e < \theta$.

By using (19), the long term rebound can be expressed by short term rebound plus one additional term. We have

$$\eta^l = \frac{dE}{d\tau} \frac{\tau}{E} = \eta^s + \frac{\theta (1 + 1/\sigma^s)}{(1/\sigma^s - \sigma_e^e) (1/\sigma^s - \sigma_e^e - \theta)}. \quad (41)$$

Since the denominator of the second term on the r.h.s of (41) is indeterministic, long term rebound may be greater or less than the short term rebound.

One extreme is that if θ goes to zero, then the long term elasticity of primary energy consumption w.r.t τ coincides with the short term one. By the definition of θ , this may happen if both σ_k^e and σ_e^k are very close to zero, which always implies one factor has almost no effect on the marginal product of another factor⁶. Another extreme is that if θ goes to infinity, then the long term elasticity of primary energy consumption w.r.t τ goes to negative unity by (40), which corresponds to zero

⁵If $1/\sigma^s - \sigma_e^e = \theta$, the l.h.s. of (39) equals zero and $\theta = -(1 + \sigma_e^e)$ must hold since $d\tau/\tau$ is not zero in the current context. Then we have $1/\sigma^s - \sigma_e^e = \theta = -(1 + \sigma_e^e)$, which holds only if $\sigma^s = -1$, which implies an abnormal supply curve of primary energy and is therefore not of interest.

⁶Zero σ_k^s implies a constant capital supply, which is exactly the short term case.

rebound. Moreover, by (41), the long term rebound is always less than the short term rebound since the second term on the r.h.s. of (41) goes to a negative number. By the definition of θ , this extreme can happen in three cases. One case is that at least one of σ_k^e and σ_e^k goes to infinity, i.e. one factor has huge effects on the marginal product of the other factor. Another case is that σ_k^s goes to infinity in the long term, which implies a constant capital price. The third case is that σ_k^k goes to zero, which implies an almost linear welfare function w.r.t. capital.

Otherwise, if $\theta < 1/\sigma^s - \sigma_e^e$, then long term rebound is always greater than short term rebound by (41) for the normal supply and demand curves of primary energy and capital. If super-conservation is impossible for the short term, then it become more impossible for the long term in this case. On the contrary, if $\theta > 1/\sigma^s - \sigma_e^e$, then long term rebound is always less than the short term one for the normal supply and demand curves of primary energy and capital. In this case, super-conservation becomes possible in the long term.

If the long term capital price P_k is assumed to be constant, which implies the price elasticity of capital supply goes to infinity, then θ goes to and is always less than a positive number $-(\sigma_k^e \sigma_e^k) / \sigma_k^k$, which implies that θ is greater than other cases with variable capital price and long term rebound is more likely to be less than short term rebound. Hence, the assumption of constant capital price in the long term is not crucial to explaining the case that long term rebound is less than short term rebound.

Furthermore, if the long term primary energy price P_e is assumed to be constant, which implies the price elasticity of primary energy supply goes to infinity, then by (41), the long term rebound can be expressed by

$$\eta^l = \frac{dE}{d\tau} \frac{\tau}{E} = \eta^s + \frac{\theta}{(-\sigma_e^e)(-\sigma_e^e - \theta)}, \quad (42)$$

which still does not indicate whether long term rebound is greater or not. However, if both primary energy and capital prices in the long term are assumed to be constant, then long term rebound must be greater than short term rebound if $-\sigma_e^e - \theta = -\sigma_e^e + (\sigma_k^e \sigma_e^k) / \sigma_k^k > 0$, i.e. $\sigma_k^e \sigma_e^k > \sigma_e^e \sigma_k^k$. For example, the Cobb-Douglas function with the property of constant returns to scale always satisfies this condition. Hence, it is a natural result that long term rebound is always greater than short term rebound in the model described by Wei (2007).

Next we calculate the effects on capital, global welfare, and prices of the energy efficiency gains.

By (38), the elasticity of capital w.r.t τ is

$$\frac{dK}{d\tau} \frac{\tau}{K} = \frac{\sigma_e^k}{1/\sigma_k^s - \sigma_k^k} (1 + \eta^l), \quad (43)$$

which shows whether or not capital increases along with the long term rebound depends on the sign of σ_e^k . If $\sigma_e^k > 0$, then only super-conservation ($\eta^l < -1$) implies disinvestment. If $\sigma_e^k < 0$, then disinvestment can happen for any rebound other than super-conservation ($\eta^l < -1$).

The elasticity of capital price w.r.t τ can be calculated by the inverted capital supply function (29),

$$\frac{dP_k}{d\tau} \frac{\tau}{P_k} = \frac{1}{\sigma_k^s} \frac{dK}{d\tau} \frac{\tau}{K} = \frac{1}{\sigma_k^s} \frac{dK}{d\tau} \frac{\tau}{K}, \quad (44)$$

which goes in the same direction as the equilibrium quantity of capital in the market.

The elasticity of primary energy price w.r.t τ can be calculated by the inverted primary energy supply function (14),

$$\frac{dP_e}{d\tau} \frac{\tau}{P_e} = \frac{1}{\sigma^s} \frac{dE}{d\tau} \frac{\tau}{E} = \frac{1}{\sigma^s} \eta^l, \quad (45)$$

which goes in the same direction as primary energy consumption.

However, the elasticity of global welfare w.r.t τ differs a little since capital is now changing. By the welfare production function (7),

$$\begin{aligned} \frac{dY}{d\tau} \frac{\tau}{Y} &= s_k \frac{\tau dK}{K d\tau} + s_e (1 + \eta^l) \\ &= \left(\frac{s_k \sigma_e^k}{1/\sigma_k^s - \sigma_e^e} + s_e \right) (1 + \eta^l), \end{aligned} \quad (46)$$

which shows that global welfare goes in the same direction as rebound if $\sigma_e^k > 0$ and the opposite if $\sigma_e^k < 0$.

Appendix D. Externalities and super-conservation in short term

Is it possible that the primary energy is used much less than in the zero rebound case, i.e. $\eta < -1$, in the short term? The case is called super-conservation. Economically the answer is no if the global planner is assumed to pursue welfare maximization using the same production function.

One obvious fact for super-conservation is that primary energy is reduced below its initial state, which, in the short term, leads to less welfare (or output) if the production function stays the same. On the other hand, if less welfare is preferred by the welfare-maximizing planner, then she must have chosen this before the energy efficiency gains. Hence, if super-conservation happens, the original production must be inefficient given other things being equal.

One possibility for super-conservation to happen in short term is due to the introduction of externalities caused by the energy efficiency gains. For example, suppose that during the energy efficiency gain, the global planner discovers some hidden causes that make production inefficient. She eliminates these causes and production becomes more efficient than before. Such externalities are not considered in typical methods for introducing technology to improve energy efficiency. However, externalities may be considered that induce capital efficiency gains $\varphi(\tau)$ or gains of total factor productivity (TFP) $\phi(\tau)$. To consider these externalities in the production function, we can rewrite the production function as

$$Y = \phi(\tau) f\left(\varphi(\tau) K^d, \tau E^d\right), \quad (47)$$

where $\varphi(\tau)$ and $\phi(\tau)$ are positive functions of τ and may be greater or less than one. As long as the effects of these externalities on welfare are less than the improved energy efficiency itself, the new production function is more preferable for the global planner.

We can replace the production function (7) in Appendix A with (47) and re-solve the short term problem associated with the general equilibrium framework. In this way, we are able to look at the effects of externalities on short term rebound.

Now the equivalence to equation (15) is

$$\tau \phi(\tau) f_2\left(\varphi(\tau) K^d, \tau E^d\right) = P_e(E^s), \quad (48)$$

which holds for any equilibria of the global economy. Total differentiation on both sides of (48) w.r.t τ , E^d , and E^s yields

$$\frac{dP_e}{dE^s} dE^s - \tau^2 \phi f_{22} dE^d = \left[\phi f_2 + \tau \frac{d\phi}{d\tau} f_2 + \tau \phi f_{21} \frac{d\varphi}{d\tau} K^d + \tau \phi E^d f_{22} \right] d\tau,$$

which can be rewritten as

$$\frac{dP_e}{dE^s} \frac{E^s}{P_e} \frac{P_e}{E^s} dE^s - \frac{\tau E^d f_{22}}{f_2} \frac{\tau \phi f_2}{E^d} dE^d = \phi f_2 \left[1 + \frac{d\phi}{d\tau} \frac{\tau}{\phi} + \frac{f_{21} \psi K^d}{f_2} \frac{\tau d\psi}{\psi d\tau} + \frac{\tau E^d f_{22}}{f_2} \right] d\tau.$$

Then by using (17), (18) and (48), we have

$$\frac{1}{\sigma^s} \frac{\tau \phi f_2}{E^s} dE^s - \sigma_e^e \frac{\tau \phi f_2}{E^d} dE^d = \phi f_2 \left[1 + \frac{d\phi}{d\tau} \frac{\tau}{\phi} + \frac{f_{21} \psi K^d}{f_2} \frac{\tau d\psi}{\psi d\tau} + \sigma_e^e \right] d\tau \quad (49)$$

if $\sigma^s \neq 0$. Define

$$\epsilon_\phi = \frac{d\phi}{d\tau} \frac{\tau}{\phi} \text{ and } \epsilon_\varphi = \frac{\tau d\psi}{\psi d\tau},$$

and now we also have

$$\sigma_k^e = \frac{f_{21} \psi K^d}{f_2}.$$

Application of these terms ϵ_ϕ , ϵ_φ , and σ_k^e to (49) yields

$$\frac{1}{\sigma^s} \frac{\tau}{E^s} dE^s - \sigma_e^e \frac{\tau}{E^d} dE^d = [1 + \epsilon_\phi + \epsilon_\varphi \sigma_k^e + \sigma_e^e] d\tau. \quad (50)$$

By the market clearing condition (13), the short-term elasticity of primary energy use w.r.t the energy efficiency gain can be expressed by

$$\begin{aligned} \eta_{sc}^s &= \frac{dE}{d\tau} \frac{\tau}{E} = \frac{1 + \epsilon_\phi + \epsilon_\varphi \sigma_k^e + \sigma_e^e}{1/\sigma^s - \sigma_e^e} \\ &= \frac{1 + \sigma_e^e}{1/\sigma^s - \sigma_e^e} + \frac{\epsilon_\phi + \epsilon_\varphi \sigma_k^e}{1/\sigma^s - \sigma_e^e}. \end{aligned} \quad (51)$$

When compared with (19), we see the second term on the right hand side of (51) is an additional term, which is related to the externalities on the TFP and capital. If the second term is small enough, then the super-conservation may be possible.

Super-conservation implies $\eta_{sc}^s < -1$, which gives us by (51)

$$\epsilon_\phi + \epsilon_\varphi \sigma_k^e < -\frac{1}{\sigma^s} - 1. \quad (52)$$

Even though we have no idea about the signs of ϵ_ϕ , σ_k^e , and ϵ_φ , we are able to find a case satisfying the condition. For example, if $\epsilon_\phi = 0$ is assumed, i.e. no externality on TFP, and $\sigma_k^e < 0$, less capital is used with an increase in primary energy price, and then the condition becomes

$$\epsilon_\varphi > \frac{1}{-\sigma_k^e} \left(\frac{1}{\sigma^s} + 1 \right),$$

which is possible for certain kinds of production functions. If let $\sigma^s = 0.5$, and $\sigma_k^e = -10$, then we need $\epsilon_\varphi > 0.3$, which implies capital productivity increases by 0.3 percent together with a one percent energy efficiency gain. If the primary energy price is assumed to be constant, i.e. $\sigma^s \rightarrow \infty$, and we assume capital has huge effects on the marginal product of primary energy, i.e. $\sigma_k^e \rightarrow -\infty$ from the negative side, then we only require $\epsilon_\varphi > 0$, which is much easier to be satisfied since a small positive effect on capital productivity due to energy efficiency gains can lead to super-conservation.

Another possibility is that $\epsilon_\phi < 0$ and $\epsilon_\varphi = 0$. Then if the primary energy price is assumed to be constant, i.e. $\sigma^s \rightarrow \infty$, we require $\epsilon_\phi < -1$ by (52). This implies that an energy efficiency gain leads to a large decrease in TFP. In this case, global welfare would be reduced significantly by (7). This is not good for the economy and probably not happen in reality without other external impacts.

Hence, we conclude that, in the short term, super-conservation is possible theoretically if one assumes the existence of large externalities.

Appendix E. Approach for short term rebound in Saunders (2008)

Using the notion in the present article, the approach for short term rebound in the cost functions by Saunders (2008) can be described as follows.

Assume the dual unit cost function corresponding to production function (7) is

$$c = c \left(P_k, \frac{P_e}{\tau} \right). \quad (53)$$

By Shephard's Lemma, we have

$$c_2 \left(P_k, \frac{P_e}{\tau} \right) Y = \tau E^d. \quad (54)$$

Substituting production function (7) into (54) yields

$$c_2 \left(P_k, \frac{P_e}{\tau} \right) f \left(K^d, \tau E^d \right) = \tau E^d. \quad (55)$$

In the short term, as assumed in Saunders (2008), we assume fixed capital $K^d (= K)$ and constant primary energy price P_e . Constant primary energy price implies all the primary energy demand can be satisfied by the market, i.e. we always have market equilibrium quantity of primary energy $E = E^d$. Hence, in the following, we ignore the superscript d of K and E . Total differentiation on both sides of (54) yields

$$c_{21} f dP_k - c_{22} \frac{P_e}{\tau} f \frac{d\tau}{\tau} + c_2 f_2 d(\tau E) = d(\tau E),$$

which can be rewritten as

$$c_{21} f dP_k - c_{22} \frac{P_e}{\tau} f \frac{d\tau}{\tau} = (1 - c_2 f_2) \tau E \left(\frac{dE}{E} + \frac{d\tau}{\tau} \right) \quad (56)$$

since $d(\tau E) = \tau E \left(\frac{dE}{E} + \frac{d\tau}{\tau} \right)$. By (56), we can express the short term rebound as

$$R^s = 1 + \frac{dE}{E} \frac{\tau}{d\tau} = \frac{c_{21} f dP_k \frac{\tau}{d\tau} - c_{22} \frac{P_e}{\tau} f}{(1 - c_2 f_2) \tau E}, \quad (57)$$

which is essentially the expression applied by Saunders (2008) to derive short term rebound for cost functions like Gallant (Fourier) function (Gallant, 1981). Next I will show this expression is consistent with the expression (2) if Shephard's Lemma for capital price also holds, i.e. $c_1(P_k, P_e/\tau) Y = K^d$, which corresponds to the first order condition (8) of production function.

Total differentiation of both sides of the first order condition (8) yields

$$f_{12} \tau E \left(\frac{dE}{E} + \frac{d\tau}{\tau} \right) = dP_k,$$

which gives us

$$dP_k \frac{\tau}{d\tau} = f_{12} \tau E \left(\frac{dE}{E} \frac{\tau}{d\tau} + 1 \right) = f_{12} \tau E R^s. \quad (58)$$

Substituting (58) into (57) and rearranging, we obtain

$$R^s = \frac{-c_{22} \frac{P_e}{\tau} f}{(1 - c_2 f_2 - c_{21} f_{12} f) \tau E}. \quad (59)$$

On the other hand, substituting first order conditions (8) and (9) into (55) yields

$$c_2 (f_1 (K, \tau E), f_2 (K, \tau E)) f (K, \tau E) = \tau E. \quad (60)$$

Total differentiation on both sides of (60) yields

$$c_{21} f_{12} f d(\tau E) + c_{22} f_{22} f d(\tau E) + c_2 f_2 d(\tau E) = d(\tau E),$$

which implies

$$c_{22} f_{22} f = 1 - c_2 f_2 - c_{21} f_{12} f \quad (61)$$

as long as zero rebound ($d(\tau E) = 0$) is not the case. Substituting (9) and (61) into (59), we obtain

$$R^s = -\frac{f_2}{f_{22} \tau E} = -\frac{1}{\sigma_e^e}, \quad (62)$$

where the second equation is obtained by directly using the definition of σ_e^e . The expression of short term rebound (62) is the same as the expression (2) with infinite price elasticity of primary energy supply $\sigma^s \rightarrow \infty$, which implies constant primary energy price P_e .

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