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**An economic approach  
to monitoring  
pollution accidents**

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## 1 INTRODUCTION

The primary aim of this paper is to present an economic framework broad enough to allow monitoring industrial pollution accidents (such as oil spill phenomena or chemical spills) without imposing differences in the approach between types of spills "a priori". We want a model in which all types of spills are probabilistically subject to reduction through changes in legislation and enforcement, that is we focus on preventive measures rather than spill response (Anderson et al., 1993). The underlying economic process is the maximization of expected profits by the facility (e.g. vessel owner). We assume that none of the choice variables in the profit maximization problem have deterministic relationships either with the amount of oil spilled or with the frequency with which oil spills occur. The oil spill phenomena is fundamentally stochastic in nature. More specifically, oil spills depend on a large number of factors, the year in which the ship was built, skills of the crew, level of maintenance of the equipment, weather conditions, many of which are under the control of the ship owner, but no one of them has deterministic relationship with the quantity spilled. These factors give rise to a systematic pattern in the series of oil spills, and what policy should do is to find out which policy instruments are susceptible of modifying the distribution of this series of events in the desired way.

Over the past few years there has been a series of models to explain the economic costs of marine pollution, e.g. Charnes et al. (1979), Cohen (1986, 1987), Epple and Visscher (1984), Smets (1982) and Burrows et al. (1974).

Compared to these attempts the present model has two main features to explain oil spilling phenomena, first, it is based on viewing pollution accidents as Poisson type processes, and second, it considers such modelling form as the basis of exploring regulatory intervention via principal-agent relationships (Gottinger, 1994).

The situation modelled is for the principal (the regulatory agency) to write a 'regulatory' contract for each agent (ship owner) specifying an output schedule and the agents choosing their activity levels (and levels of care). In such context, 'truth-telling' must be a dominant strategy among all agents or must be enforced by the principal (Ching-to-Ma and J. Moore, 1986). We further assume that the principal observes the outcomes and is risk neutral, minimizing expected social damage. This is in line with considerations on general deterrence and accident cost reduction (Calabresi, 1970, Chapters 7-9; Burrows, 1980, Chapter 4).

As comprehensively covered in the Appendix, we assume a stochastic process (Poisson Process) generating oil spills where both the actions that might be taken by ship owners to reduce the frequency and size of oil spills and the policies of the pollution control agency that might affect these actions are ignored. Suppose the goal of the agency is to minimize the social damage caused by oil spills during a period of time  $(0, \tau]$ . The social damage function is, of course, related to the damage function that assigns a cost level to each spill. For simplicity we take the social damage from all spills to be also the sum of the cost associated with each individual spill. Define  $D(V(\tau))$  as the expected social damage caused by the volume of oil spilled during a period  $(0, \tau]$  and  $d(x_{ik})$  as the social cost of a spill  $x_{ik}$  (i.e. the  $k$ -th spill from a ship type  $i$ ). Thus assuming that  $z_i$  spills have taken place during period  $(0, \tau]$  from ships type  $i$ , then  $D(V(\tau)) = \sum_{i=1}^M \sum_{k=1}^{z_i} d(x_{ik})$ .<sup>1</sup> If  $d(x_{ik}) = aX_{ik}^2$  for some constant  $a$  then the damage caused at the end of a period of time would not depend on the size of each of the spills but only on the final amount of oil spilled during the period  $(0, \tau]$ . If  $d(x_{ik})$  is not linear the temporal damage function can depend not only on the number of oil spilled but also on the number of spills and their respective size. At the moment we assume that  $d(x_{ik})$  is a linear function of the volume of oil spilled. Thus minimizing the expected volume of oil spilled is equivalent to minimizing the social damage  $D(V(\tau))$ .

The agency can use several types of policy measure. At present two main types of pollution control policies are used: technological standards and economic incentives. With the use of technological standards, the government typically tells firms what type of equipment they must install and how the equipment must be operated. It could involve an annual check for compliance with pollution prevention and navigation safety. Economic incentives are incorporated into the expected penalty function facing ship owners for pollution. The expected penalty conditioned on a spill being made is the product of the fine levied for pollution and the probability of a polluter being detected. Naval police monitoring uses two methods for detecting oil spills: first, they randomly monitor transfer operations and, second they patrol harbours and other areas looking for oil spills (Tebeau and Lissauer, 1993). The probability of detection and the response of ship owners to these measures depends on the hours devoted to them. In this paper we also consider how the economic incentives affect the probability of spilling and the spill size. These measures (i.e. monitoring transfer operations and harbour patrols) have a long run and short run effect on the ship behaviour. The agency reallocates effort periodically around some long average. Ship

owners do not know "ex ante" the pollution control enforcement effort level for the period of their arrival in the harbour, but they are assumed to know the long run average level of total enforcement effort. They react to the overall long run expected enforcement level.

Once a ship arrives in harbour, the ship operator learns the number of ships in harbour and the enforcement effort level during this period. We expect ship owners to increase the level of care if the frequency of harbour patrols and the probability of monitoring during their stay in the harbour are larger than the average. Also we expect that if the ship is chosen to be monitored the ship owner will further increase the measures against pollution. Those are the short run or immediate effects of the pollution control policies. The optimization problem is described in terms of the volume of oil spilled and spill size. The pollution control policy instruments give incentives to decrease the expected volume of oil spilled. We assume that ship owners act so as to decrease the expected volume of oil spilled with increase in the level of enforcement effort. However, the volume spilled is equal to the number of spills times the average spill size. The optimization problem is solved only for two types of ships: tankers and barges. There is no modelling loss of generality, the conclusions can be generalized to the case of  $m$  types of ships. This model does not imply that the agency randomly chooses the ships to be monitored. The agency policy takes into account the differences between types of ships, the model permits classification of ships in different categories depending on their history of pollution prevention and safety violations.

First we look at the long run effects of the enforcement effort. Second at the effect of policy instruments in the short run, on an individual ship. Third we solve the short run agency problem of choosing the optimal number of ships monitored and the optimal frequency of harbour patrols. And fourth we derive the comparative static results and policy implications when some parameters of the problem change.

## 2 LONG RUN EFFECTS OF ENFORCEMENT EFFORT

We assume that ship owners in deciding the long run level of investment in pollution control and prevention equipment only take into consideration the long run expected cost in the harbours of interest. Some pollution prevention equipment represents a large investment for ship owners. To build a tanker with double hull, slop or segregated ballast tanks or to install discharge containers is too costly to respond only to short run changes in pollution prevention measures. Pollution control measures include, in addition to monitoring of transfer operations and harbour patrols, the level of fines, frequency of examinations, clean up costs and level of equipment standards required by legislation. Also, since ships operate in different harbours, ship owners do not know the specific harbour they will visit when they make the long run investment decisions. Therefore, they will have to consider the average enforcement level across harbours. The agency optimizes the allocation of effort each period of time but ship owners are assumed to only take into account the steady state (i.e. long run) expected enforcement effort level for investment in pollution prevention equipment.

If there are any costs to be saved by reducing the frequency and size of oil spills profit maximizing ship owners will reduce them. We assume that in the long run, given the cost in pollution prevention equipment, ship owners take into account the overall level of enforcement effort, but in the short run when they actually arrive in a harbour they also react to that day's enforcement effort. Short run policy measures are important because most oil spills done during routine operations are caused by improper operations of equipment and human error. These accidents can be eliminated or reduced with policy measures that give incentives to perform appropriately each operation in the moment that the operation is taking place. The only way to lower expected pollution cost is having the ships perform the operations carefully. We expect the probability and size of spills to decrease with increases in the probability of monitoring and frequency of transfer operations. Next we discuss in detail these short run effects.

### 3 SHORT RUN EFFECTS OF ENFORCEMENT EFFORT

We begin by considering the short run responses of individual ships to pollution prevention measures. There are three types of short-run responses: (i) the response of a specific ship whose oil transfer operations is monitored, (ii) the responses of all ships of a specific type to the probability of monitoring that type of ship, and (iii) the responses of all ships of all types to the amount of harbour patrols.

The goal of the agency in the short run is to minimize the volume of oil spilled during a period of time  $(0, \tau]$ . During this period the agency decides how to allocate optimally the enforcement effort hours available among the different type of policy measures. We assume the regulatory agency chooses the number of each type of ship to be monitored and the harbour patrol frequency to minimize the volume of oil spilled.

In each period of time  $(0, \tau]$  the agency faces a constraint. It has to allocate the total man-hours available  $e(t)$ , to monitoring transfer operations and to harbour patrols. An optimal allocation of enforcement effort by the agency, in general, will depend on the response of the ship owners to the policy measures. The ship owner's short run behaviour is not explicitly modelled in this paper. Assuming that the owners choose the pollution prevention measures that maximize their profit, the frequency and spill size distributions will be influenced by these short run ship owners profit maximization decisions. We expect the volume of oil spilled to decrease when either the probability of inspection or the frequency of harbour patrols increase.

When a ship enters in a harbour, there is a level of investment in pollution control equipment that has been chosen by ship owners depending on the long run steady state level of enforcement effort. The long run characteristics of the spill size distribution function cannot be changed, but there are actions that ship owners can take in the short run to lower the expected pollution. For example, increasing the number of crew members in performing transfer operations, increasing the maintenance level of the pollution prevention equipment, and following more closely safety procedures. These measures will reduce the probability of a spill occurring and the spill size.

Monitoring transfer operations has two types of effects in the short run: a public and a private good effect. The probability of being monitored affects all ships. Since ships "ex-ante" do not know if they will be monitored, they can be expected to increase the pollution prevention measures and level of care with increases in the probability of being monitored -



this is the public deterrence effect. A proportion of ships are actually monitored, and the reduction on the expected volume of oil spilled on these ships is the private good effect. Define  $\alpha_T$  as the proportion by which a tanker's expected volume of oil spilled is reduced if the tankers is monitored.<sup>3</sup> That is,  $\alpha_T$  represents the private good effect of monitoring a transfer operation.

The volume of oil spilled by a tanker depends on the probability of monitoring a tanker  $P_T$ , and on the man-hours devoted to harbour patrols  $f_H$  during the period of time  $(0, \tau]$  that the tanker is in the harbour. The probability,  $P_T = \frac{m_T}{n_T}$  where  $m_T$  is the number of tankers monitored in the harbour and  $n_T$  is the number of tankers that will transfer oil in the harbour during the period  $(0, \tau]$ . We assume that when the agency chooses the enforcement effort level for that period of time the number of ships of each type that visit the harbour is known. Similarly, the volume of oil spilled by barges depends on the probability of monitoring a barge,  $P_B$ , and on the man-hours devoted to harbour patrols  $f_H$ .

Define  $V^T(P_T f_H | \mathcal{E})$  and  $V^B(P_B f_H | \mathcal{E})$  as the random volume of oil spilled by a tanker and barge, respectively, before being monitored, where  $\mathcal{E}$  is the long run enforcement effort level. Let  $S_T$  be the random number of spill that occur during a transfer operation from a ship type T. Thus, the volume of oil spilled by a tanker during its stay in the harbour  $(0, \tau]$  if it has not been monitored is:

$$V^T(P_T f_H | \mathcal{E}) = \sum_{Z_i=0}^{S_T} X_{Z_i}^T(P_T f_H | \mathcal{E}). \quad (3.1)$$

and for a barge,

$$V^B(P_B f_H | \mathcal{E}) = \sum_{Z_i=0}^{S_B} X_{Z_i}^B(P_B f_H | \mathcal{E}). \quad (3.2)$$

The volume of oil spilled by ships that have been actually monitored is  $\alpha_T V_T$  and  $\alpha_B V_B$  where  $0 \leq \alpha_T \leq \alpha_B \leq 1$ .

Define  $EV^T(P_T f_H | \mathcal{E})$  and  $EV^B(P_B f_H | \mathcal{E})$  to be the expected volume of oil spilled by tankers and barges, respectively, before being monitored. Then recalling that  $S_T(t)$  is the random variable that represents the number of oil spills that a ship type T made by time  $t$  after its arrival in the harbour, the expected volume of oil spilled by tanker is,

$$\begin{aligned}
EV^T(P_T f_H | \mathcal{E}) &= E \left( \sum_{Z=0}^{S_T} X_{Z_i}^T(P_T f_H | \mathcal{E}) \right) \\
&= E(S_T)E(X^T). \\
&= \lambda_T E(X^T).
\end{aligned} \tag{3.3}$$

and for the case of barges,

$$\begin{aligned}
EV^B(P_B f_H | E) &= E \left( \sum_{Z=0}^{S_B} X_{Z_i}^B(P_B f_H | \mathcal{E}) \right) \\
&= E(S_B(t))E(X^B). \\
&= \lambda_B E(X^B).
\end{aligned} \tag{3.4}$$

(with  $\lambda$  as appropriately defined in Appendix 1).

Given that there are  $n_T$  tankers in the harbour of which  $m_T$  have been monitored, the expected volume of oil spilled by tankers in the harbour is:

$$(n_T - m_T)EV^T(P_T f_H) + m_T \alpha_T EV^T(P_T f_H).$$

Monitoring a transfer operation includes checking that the pollution prevention and safety requirements are satisfied and witnessing that all steps of the transfer operation are done properly. For ship owners it should be cheaper to assure that equipment and crew perform properly during a transfer operation than being caught violating the law and forced to pay a penalty in the case of being monitored. A higher probability of being monitored raises the expected cost and thus induces greater "ex-ante" efforts by ship owners to decrease the chances of an oil spill. Therefore, we assume that the expected volume of oil spilled by a ship decreases with increases in the probability of monitoring. We also expect ship owners to correct first the defects that are cheapest, so that, the expected volume of oil spilled decreases at a rate in absolute value as monitoring increases. Thus,

$$V_T^T = \frac{\partial EV^T(P_T f_H)}{\partial P_T} < 0. \tag{3.5}$$

and

$$V_{TT}^T = \frac{\partial^2 EV^T(P_T, f_H)}{\partial P_T^2} > 0. \quad (3.6)$$

Since increasing monitoring affects all ships of a given type "ex-ante", we refer to this as the public good effect of the probability of monitoring.

Monitoring transfer operations also has a private good effect. We also assume that the expected volume of oil spilled by a ship that is actually monitored is smaller than the expected volume of oil spilled if the ship is not monitored. We denote by  $\alpha_T$  the proportion by which a tanker's expected volume ("ex-ante") of oil spilled is reduced if the tanker is monitored. Therefore,  $\alpha_T$  represents the private good effect of actually monitoring a transfer operation. We assume that  $\alpha_T$  is the same for all tankers. From the assumptions made above, the expected volume of oil spilled decreases by  $-(1 - \alpha_T) EV^T < 0$  if the tanker is actually monitored. The decrease in the expected volume of oil spilled due to actually monitoring a transfer operation is larger, the lower the probability of being monitored. From the agency's point of view, there is more to be gained from actually monitoring a tanker if the probability of being monitored is low  $-(1 - \alpha_T) EV_T^T > 0$ . Thus the expected volume of oil spilled also decreases at a decreasing rate in absolute value with the private good effect of the probability of monitoring.

Effort spent on harbour patrols is the second type of measure available to the agency to reduce the volume of oil spilled. It is also a public good type measure, but a distinction has to be made between the public good effect of the probability of monitoring and the frequency of harbour patrols. Harbour patrols have a broader public good effect. Since they affect all types of ships in the harbour, we assume that increases in the frequency of harbour patrols increases the level of pollution prevention measures taken by the crew in charge of the vessel. It is assumed that the expected volume of oil spilled of both tankers and barges decreases at a decreasing rate in absolute value with harbour patrols, thus

$$V_H^T = \frac{\partial EV^T(P_T, f_H)}{\partial f_H} < 0, \quad V_{HH}^T = \frac{\partial^2 EV^T(P_T, f_H)}{\partial f_H^2} > 0 \quad (3.7)$$

for the cases of tankers, and

$$V_{HH}^B = \frac{\partial EV^B(P_B, f_H)}{\partial f_H} < 0, \quad V_{HH}^B = \frac{\partial^2 EV^B(P_B, f_H)}{\partial f_H^2} > 0 \quad (3.8)$$

for the barges case.

Oil spills may occur when a ship is moving in a harbour or during oil transfer operations. Equipment failures in the two cases are different, though there is some overlap. We assume the crew of a ship reacts to increases in the level of harbour patrols by increasing the level of care devoted to operations performed when the ship is moving in the harbour. If the probability of being monitored increases, the efforts are concentrated on the type of equipment and skills used during transfer operations. These differences lead to a trade off in the effectiveness of different types of monitoring. We assume that each enforced policy measure affects most intensively that type of pollution prevention equipment and skills used during the operations at which it is aimed; thus we assume:

$$\left| \frac{\partial^2 EV^T(P_T, f_H)}{\partial P_T^2} \right| > \left| \frac{\partial^2 EV^T(P_T, f_H)}{\partial f_H \partial P_T} \right| \quad (3.9)$$

and

$$\left| \frac{\partial^2 EV^B(P_B, f_H)}{\partial P_B^2} \right| > \left| \frac{\partial^2 EV^B(P_B, f_H)}{\partial f_H \partial P_B} \right| \quad (3.10)$$

This does not imply that increases in the level of harbour patrols do not affect the effect of the probability of monitoring transfer operations; we allow for some cross effects. Consider the public good effect of the probability of monitoring transfer operations. In this case transfer operations, together with harbour patrols, can be thought as complementary inputs in the welfare (clean water) production process. Increasing one type of measure increases the marginal benefit that can be obtained from the other. Harbour patrols enhance the public good effect of monitoring transfer operations. For example, increases in the harbour patrol frequency increases the reduction of expected volume of oil spilled caused by monitoring transfer operations. The reduction in the expected volume of oil spilled due to increasing the probability of monitoring transfer operations increases with harbour patrols. Thus, it is assumed that

$$\frac{\partial EV^T(P_T, f_H)}{\partial P_T \partial f_H} < 0, \quad \text{and} \quad \frac{\partial EV^B(P_B, f_H)}{\partial P_B \partial f_H} < 0 \quad (3.11)$$

for tankers and barges, respectively.

The higher the level of harbour patrols the less is to be gained from monitoring

transfer operations. The higher the frequency of harbour patrols, the more is the care taken by ship owners and the fewer are the equipment failures left to be discovered during a transfer operation. The marginal benefit of actually monitoring a transfer operation  $-(1-\alpha_T)EV_T < 0$ , decreases with harbour patrols  $-(1-\alpha_T)EV_H^T > 0$ . An increase in the frequency of harbour patrols has opposite effects in the private and in the public good effects of monitoring transfer operations. We assume, however, that the overall effect of monitoring transfer operations increases with increases in the frequency of harbour patrols.

#### 4 OPTIMAL ALLOCATION OF ENFORCEMENT EFFORT

The agency chooses the number of each type of ship to be monitored and the frequency of harbour patrols. We assume that there is no transfer operation that last longer than the period chosen to allocate effort.<sup>4</sup> The agency's problem can be stated as choosing the number of tankers and barges to be monitored and the man-hours devoted to harbour patrols to minimize the volume of oil spilled during a period of time  $(0, \tau]$  given the man power resources available. Thus, define  $EW(m_T, m_B, f_H; \tau | n_T, n_B; \mathcal{E})$  to be the expected volume of oil spilled in the harbour during a period  $(0, \tau]$  given: (i)  $n_T$  tankers and  $n_B$  barges transferring oil in the harbour during that period, (ii)  $m_T$  of the tankers' and  $m_B$  of the barges' transfer operations are monitored (iii)  $f_H$  is the level of harbour patrolling, and (iv) the long run level of enforcement effort is  $\mathcal{E}$ . Therefore, the expected volume of oil spilled in the harbour equals<sup>5</sup>:

$$\min_{m_T, m_B, f_H} \quad EW = (n_T - m_T(1 - \alpha_T)P_T)EV^T(P_T, f_H) + (n_B - m_B(1 - \alpha_B)P_B)EV^B(P_B, f_H).$$

$$s.t. \quad e(\tau) = h_T m_T + h_B m_B + f_H$$

$$P_T = \frac{m_T}{n_T} \quad \text{and} \quad P_B = \frac{m_B}{n_B}$$

$$0 \leq P_T \leq 1 \quad \text{and} \quad 0 \leq P_B \leq 1$$

where:  $h_T, h_B$  are the number of hours spent by the agency in monitoring a tanker and a barge respectively and  $e(\tau)$  is the number of hours available for enforcement effort.

Assuming an interior solution<sup>6</sup>, the first order conditions require that the optimal number of tankers and barges monitored transfer operations (i.e.  $m_T, m_B$ ) and man-hours allocated to harbour patrols  $f_H$  satisfy:

$$d_T \frac{EV_T^T(P_T, f_H)}{h_T n_T} - (1 - \alpha_T) \frac{EV^T(P_T, f_H)}{h_T} = d_B \frac{EV_B^B(P_B, f_H)}{h_B n_B} - (1 - \alpha_B) \frac{EV^B(P_B, f_H)}{h_B} \quad (4.1)$$

$$d_T \frac{EV_T^T(P_T, f_H)}{h_T n_T} - (1 - \alpha_T) \frac{EV^T(P_T, f_H)}{h_T} = d_T EV_H^T(P_T, f_H) + d_B EV_H^B(P_B, f_H). \quad (4.2)$$

$$h_T m_T + h_B m_B + f_H = e(\tau) \quad (4.3)$$

Condition (4.1) is the efficiency condition across monitoring transfer operations of different types of ships. Efficiency requires that the ratio between marginal social benefit and marginal cost of monitoring transfer operation be equal for all ship types. Condition (4.2) is the efficiency condition across monitoring transfer operations and harbour patrols. It is the classical public good efficiency condition where the marginal benefit of monitoring tankers transfer operations equals the marginal benefit of the public good, harbour patrols. Harbour patrols affect all ships in the harbour so the marginal benefit of harbour patrols is the sum of the marginal individual benefits,  $d_T$  tankers and  $d_B^7$  barges, in the harbour. Monitoring transfer operations only affects one type of ships, tankers in this case. Equivalently this condition could have been expressed in terms of the marginal benefit of barges<sup>8</sup>. Notice that monitoring transfer operations is both a public and a private good, but it only affects a concrete type of ships, it is like a local public good. The public good effect of the probability of monitoring is represented by  $d_T EV_T^T / n_T h_T$ . A unit increase in the number of tankers monitored  $m_T$  decreases the expected volume of oil spilled by  $EV_T^T / n_T$ , and it affects the  $d_T$  tankers in the harbour. The larger the number of a type of ship in the harbour, the larger the public good effect of monitoring that type of ship. But in the case of monitoring transfer operations what causes the public good effect is the probability of being monitored, decreases as the number of tankers increases in the harbour. So in allocating effort to monitoring transfer operations the agency has to take into account that the larger the number of tankers in the harbour the more ships are affected by the policy but also the smaller the probability of being monitored. The private good effect is represented by  $(1 - \alpha_T) EV^T (P_T f_H) / h_T$ . It represents the amount by which the expected volume of oil spilled by tankers that have actually been monitored decreases. It completes the effect of monitoring transfer operations. Condition (4.3) is the agency budget constraint<sup>9</sup>.

## 5 COMPARATIVE STATIC RESULTS AND POLICY MEASURES

We may now list and discuss a number of comparative static results showing how optimal policy measures change when there is an increase in the number of tankers in the harbour.

$$\frac{\partial m_B}{\partial n_T} = \frac{1}{D} \left[ -P_T EW_{TT} EW_{HH} + h_T \left( EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right) \frac{EW_{TT}}{h_T^2} \right] < 0. \quad (5.1)$$

$$\frac{\partial m_T}{\partial n_T} = \frac{h_B}{D} \left[ P_T \frac{EW_{TT}}{h_T} \left( EW_{HH} - 2 \frac{EW_{HB}}{h_B} + \frac{EW_{BB}}{h_B^2} \right) + \left( EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right) \frac{EW_{BB}}{h_B^2} \right]. \quad (5.2)$$

$$\frac{\partial f_H}{\partial n_T} = \left[ -\frac{h_T h_B}{D} \right] \left[ P_T \frac{EW_{TT} EW_{BB}}{h_T h_B^2} + \left( EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right) \left[ \frac{EW_{TT}}{h_T^2} + \frac{EW_{BB}}{h_B^2} \right] \right]. \quad (5.3)$$

where<sup>10</sup>:

$$D = h_T h_B EW_{HH} \left( \frac{EW_{TT}}{h_T^2} + \frac{EW_{BB}}{h_B^2} \right) - 2 \left( \frac{EW_{TT} EW_{HB}}{h_T h_B} + \frac{EW_{BB} EW_{HT}}{h_B h_T} \right) + \frac{EW_{TT} EW_{BB}}{h_T h_B} > 0. \quad (5.4)$$

If the number of tankers increases in the harbour, the marginal benefit of harbour patrols and marginal benefit of monitoring transfer operations increase. The marginal benefit of a public good is equal to the sum of the ship marginal benefits affected by the public good. There are more ships affected by harbour patrols so the marginal benefit of harbour patrols increases. As the number of tankers affected by monitoring tanker transfer operations increases so the marginal benefit of the probability of tanker monitoring increases.

The only policy measure by which marginal benefit is not affected is monitoring barges transfer operations, therefore to restore equality among the marginal benefits of each type of measure the marginal benefit of monitoring barges should increase. A decrease in the number of barges monitored increases the marginal benefits of this policy measure. So the optimal number of barges to be monitored decreases as the number of tankers increase in the harbour.

Increasing the number of tankers monitored decreases the marginal benefit of monitoring tankers. But the marginal benefits of harbour patrols increases<sup>11</sup>. So the number of harbour patrols should be increased in order to decrease the marginal benefit of harbour patrols. But the total amount of enforcement effort remains constant, the resources that before were devoted to monitoring barges transfer operations have to be allocated, both to monitoring tankers transfer operations and harbour patrols. How these resources should be



assessed depends on how marginal benefit of harbour patrols and the marginal benefit of monitoring tankers transfer operations increases with the number of tankers in the harbour  $n_T$ .

**Proposition 1:** If increases in the number of tankers in the harbour increases the marginal benefit of monitoring tanker transfer operations more than the marginal benefit of harbour patrols then  $\partial m_T / \partial n_T > 0$ . That is, a sufficient but not necessary condition for  $\partial m_T / \partial n_T > 0$  is:

$$\left| P_T \frac{EW_{TT}}{h_T} \right| \geq \left| EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right| \quad (5.5)$$

where:

$$\frac{\partial EW_T}{\partial n_T} = -P_T \frac{EW_{TT}}{h_T} < 0.$$

$$\frac{\partial EW_H}{\partial n_T} = EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} < 0.$$

**Proof:** From equation (5.2)

$$\begin{aligned} \frac{\partial m_T}{\partial n_T} &= \frac{h_B}{D} \left[ P_T \frac{EW_{TT}}{h_T} \left( EW_{HH} - 2 \frac{EW_{HB}}{h_B} + \frac{EW_{BB}}{h_B^2} \right) + \left( EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right) \frac{EW_{BB}}{h_B^2} \right] \\ &= \frac{h_B}{D} \left[ P_T \frac{EW_{TT}}{h_T} \left( EW_{HH} - 2 \frac{EW_{HB}}{h_B} \right) + \left( \frac{P_T EW_{TT}}{h_T} + \left( EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right) \right) \frac{EW_{BB}}{h_B^2} \right] \end{aligned} \quad (5.6)$$

Then and given assumptions made above:

$$\frac{\partial m_T}{\partial n_T} > 0 \text{ if } \left| P_T \frac{EW_{TT}}{h_T} \right| \geq \left| EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right| \quad \text{Q.E.D.}$$

**Corollary 1:** A necessary condition for  $\frac{\partial m_T}{\partial n_T} < 0$  is:

$$\left| P_T \frac{EW_{TT}}{h_T} \right| < \left| EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right| \quad (5.7)$$

**Proof:** The first term of expression (5.6) is always positive so for  $\frac{\partial m_T}{\partial n_T} < 0$  it is necessary but not sufficient that the above condition is satisfied.

**Proposition 2:** If the number of tankers increases in the harbour and the increase in the marginal benefit of monitoring tanker transfer operations is smaller than the increase in the marginal benefit of harbour patrols then  $\frac{\partial f_H}{\partial n_T} > 0$ . That is, a sufficient but not necessary condition for  $\frac{\partial f_H}{\partial n_T} > 0$  is that condition (5.7) is satisfied (or 5.5 not satisfied).

**Proof:** From equation (5.3) we have:

$$\begin{aligned} \frac{\partial f_H}{\partial n_T} = & \left[ -\frac{h_T h_B}{D} \right] \left[ P_T \frac{EW_{TT}}{h_T} \frac{EW_{BB}}{h_B^2} + \left( EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right) \left[ \frac{EW_{TT}}{h_T^2} + \frac{EW_{BB}}{h_B^2} \right] \right] \\ & \left[ -\frac{h_T h_B}{D} \right] \left[ P_T \frac{EW_{TT}}{h_T} \left( EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right) + \left[ \frac{EW_{TT}}{h_T} + \left( EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right) \right] \frac{EW_{BB}}{h_B^2} \right] \end{aligned} \quad (5.8)$$

Given the assumptions made above

$$\frac{\partial f_H}{\partial n_T} > 0 \text{ if } \left| P_T \frac{EW_{TT}}{h_T} \right| < \left| EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right| \quad \text{Q.E.D.}$$

**Corollary 2:** A necessary but not sufficient condition for  $\frac{\partial f_H}{\partial n_T} < 0$  is:

$$\left| P_T \frac{EW_{TT}}{h_T} \right| > \left| EV_H^T - P_T EV_{HT}^T \frac{d_T}{n_T} \right|$$

**Proof:** The first term of expression (5.8) is always positive so for  $\frac{\partial f_H}{\partial n_T} < 0$  it is

necessary that the above condition is satisfied. Q.E.D.

If condition (5.5) is satisfied then the optimal policy rule is to increase the number of monitored tankers and decrease the number of monitored barges when the number of tankers increases in the harbour. Also condition (5.5) is necessary for  $\partial f_H / \partial n_T < 0$ . Therefore, it is likely that if the marginal benefit of monitoring tanker transfer operations increases more than the marginal benefit of harbour patrols the number of harbour patrols should be reduced. Condition (5.5) is more likely to be satisfied the more sensitive the marginal expected volume of oil spilled is to monitoring transfer operations and the larger the rate at which the marginal volume of oil spilled decreases with an increase in monitoring transfer operations.

If condition (5.5) is not satisfied and the number of tankers increases in the harbour then the agency has to increase the number of harbour patrols. Thus, the optimal policy rule is to decrease the number of monitored barges and to increase the number of harbour patrols.

The difference between the increases in the marginal benefit of monitoring transfer operations and harbour patrols when the number of tankers increases in the harbour is an empirical result. Increases in the number of ships in a harbour decreases the probability of being monitored, so both, the marginal benefit of the private and public good effect of the probability of monitoring increases. Increases in the number of ships increases the marginal benefit of harbour patrols, the marginal benefit of harbour patrols depends also on the probability of being monitored. Decreases in the probability of monitoring has a negative effect on the marginal benefit of harbour patrols.

## 6 CONCLUSIONS

The stochastic model developed here allows us to see how each step of the spilling process is affected by each policy measure and to compare the relative efficiency of different measures in reducing spills. We show that efficiency requires that the marginal social benefit of monitoring a transfer operation be equal for all ship types. And also, it is necessary that the marginal benefit of monitoring transfer operations equal the marginal benefit of harbour patrols.

The comparative static results show that the optimal number of barges to be monitored decreases as the number of tankers increase in the harbour. The resources that were devoted to monitor barges transfer operations should be allocated, both to monitor tanker transfer operations and harbour patrols. How these resources should be assessed depends on how the marginal benefit of harbour patrols and the marginal benefit of monitoring tanker transfer operations increases with the number of tankers in the harbour. If increases in the number of tankers in the harbour increases the marginal benefit of monitoring tanker transfer operations more than the marginal benefit of harbour patrols then the number of monitored tankers should be increased. Also if it is the marginal benefit of harbour patrols that is larger then the number of harbours patrols should be increased. Together with estimation of these parameters the model allow us to predict among other, the expected number of oil spills per ship during a transfer operation, and the expected volume of oil spilled.

This model can be used for other types of environmental issues where the arrival of pollution is stochastic in nature such as, in general, transportation or handling of hazardous wastes. Note that the model can be generalized to different types of processes. We can define a process not only by type of ship but also by other characteristics like type of operation that the ship was performing when the spill occurred, and cause of the spill. The more precise the description of a process the better it allows us to allocate effort to minimize a specific type of spill. For example, we assume that the damage function is a linear function of the model, it allows us to assume that minimizing the expected volume of oil spilled is equivalent to minimize social damage. But this assumption can be seen as a limitation of the model if damage increases at an increasing rate with spill size more effort should be allocated to avoid large spills. The model allows us to solve this limitation if we can associate a type of process to a spill size. We conclude saying that looking at pollution arrivals as a combination of stochastic process can allow the pollution prevention agency to allocate the

**Pollution prevention measures to minimize the more harmful processes.**

## Footnotes

<sup>1</sup> We do not consider that a minimum volume of oil is necessary for a positive social damage. Taking into account such a consideration implies that spills made up to a certain volume are harmless to the environment and that the same spills can cause different level of damage depending on when they are done, and how many spills (and their size) were done before them.

We assume that the goal of the agency is to minimize the volume of oil spilled in a given period. We do not take into account when the spills were done. Therefore, the damage function depends on the final amount that is being dumped, instead of being an increasing step function that increases with each oil spill.

This formulation, assuming that the damage during a period is the sum of the damage caused by each spill done during that period allows for larger spills to have proportionally larger cost. But this possibility is ruled out next by assuming that the individual damage is a linear function of the quantity spilled.

<sup>2</sup> Notice that for each type of oil there is a constant  $a$ , it depends on the damage that each type of oil causes to the environment. We are going to analyze the allocation of enforcement effort without taking into account the differences in damage caused by different types of oil. If these differences are relevant, the agency should take into account the type of oil of each ship carrying in allocating its enforcement effort. But we do not look at this problem, we assume that all the ships considered carry the same type of oil.

<sup>3</sup>  $\alpha_T$  is a constant proportion of the volume of oil spilled by a tanker before being monitored. With a constant proportion we represent that the same type of actions can be taken by any tanker once monitored, but the effect of monitoring, the reduction on the volume of oil spilled, is proportional to the "ex-ante" pollution volume. Thus, the reduction of oil spilled is smaller if ship owners have undertaken the necessary long-run pollution prevention measures.

<sup>4</sup> If we had assumed that the number of ships changes during the period of time that the agency allocates effort, the model would have had to take into account: First how to allocate effort across periods of time with different numbers of ships of the same type, and second the agency optimal allocation of effort during period  $t$  would have to be conditioned on the number of ships in the harbour that have been monitored during period  $t-1$  and are still in the harbour. But these problems do not add any insight to the optimal allocation of effort across different types of ships.

<sup>5</sup> Notice that the agency is using the steady state expected volume of oil spilled per unit of time. We assume that this is the unit of time chosen by the agency to allocate its enforcement effort. Notice also that the harbour expected volume of oil spilled is not a steady state function, it depends on the number of ships in the harbour and in their type.

<sup>6</sup> We can get a boundary solution if the probability of monitoring has no public good effect. In that case the marginal benefit of monitoring a transfer operation includes only the private good effect that it is constant for each ship monitored.

$$W_T^T = \frac{\partial W^T}{\partial m_T} = (1-\alpha_T)V^T(f_H)$$

$$W_B^B = \frac{\partial W^B}{\partial m_B} = (1-\alpha_B)V^B(f_H)$$

The marginal benefits of monitoring tanker transfer operations differ from the marginal benefit obtained from monitoring barges transfer operation, besides in the unlikely case that both marginal benefit are equal the marginal benefit equality condition across policy measures cannot be satisfied. The marginal benefit of the public good harbour patrols will equate only one of the marginal benefits of monitoring transfer operations. So if the marginal benefit of tankers is smaller than the marginal benefit of barges:

$$(1-\alpha_T)\frac{V^T(f_H)}{h_T} = d_T V_H^T(f_H) + d_B V_H^B(f_H).$$

And no enforcement effort will be allocated to monitor barges.

<sup>7</sup> Notice that the vessel's marginal benefit of monitoring transfer operations is multiplied by  $d_T$  ( $d_B$  in the case of barges) instead of  $n_T$ , the total number of tankers in the harbour.  $d_T$  also represents the total number of tankers in the harbour, the only difference is that  $d_T$  takes into account that  $m_T$  of them have been monitored. That is,

$$d_T = (n_T - (1-\alpha_T)m_T) = (n_T - m_T) + \alpha_T m_T$$

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$$d_B \frac{V_B^B(P_B f_H)}{h_B n_B} - (1-\alpha_B) \frac{V^B(P_B f_H)}{h_B} = d_T V_H^T(P_T f_H) + d_B V_H^B(P_B f_H).$$

<sup>9</sup> The necessary conditions for a minimum are satisfied as we assumed that  $-V^T (1-(1-\alpha_T))$  was a concave function and the sum of concave functions is also concave:

$$W_{TT} > 0$$

$$W_{TT} W_{BB} > 0$$

$$W_{TT} \left( \frac{1}{2} W_{HH} W_{BB} - W_{BB}^2 \right) + W_{BB} \left( \frac{1}{2} W_{HH} W_{TT} - W_{TT}^2 \right) > 0.$$

For these to be true it is sufficient that:

$$\left| \frac{W_{BB}}{h_B} \right| > |W_{HB}| \quad \left| \frac{1}{2} W_{HH} \right| > \left| \frac{W_{HB}}{h_B} \right|$$

$$\left| \frac{W_{TT}}{h_T} \right| > |W_{HT}| \quad \left| \frac{1}{2} W_{HH} \right| > \left| \frac{W_{HT}}{h_T} \right|$$

$$\begin{aligned}
D &= h_T h_B W_{HH} \left( \frac{W_{TT}}{h_T^2} + \frac{W_{BB}}{h_B^2} \right) - 2 \left( \frac{W_{TT}}{h_T} W_{HB} + \frac{W_{BB}}{h_B} W_{HT} \right) \\
&\quad + \frac{W_{TT} W_{BB}}{h_T h_B} - h_T h_B \left( \frac{W_{HT}}{h_T} - \frac{W_{HB}}{h_B} \right)^2 = \\
h_B &= \left( \frac{1}{2} W_{HH} \frac{W_{TT}}{h_T} - \frac{W_{HT}^2}{h_T} \right) + h_T \left( \frac{1}{2} W_{HH} \frac{W_{BB}}{h_B} - \frac{W_{HB}^2}{h_B} \right) \\
&\quad + \frac{W_{TT}}{h_T} \left( \frac{1}{2} W_{HH} h_B - W_{HB} \right) + \frac{W_{BB}}{h_B} \left( \frac{1}{2} W_{HH} h_T - W_{HT} \right) \\
&\quad - \left( \frac{W_{TT}}{h_T} W_{HB} + \frac{W_{BB}}{h_B} W_{HT} \right) + \frac{W_{TT} W_{BB}}{h_T h_B} + 2W_{HT} W_{HB} > 0.
\end{aligned}$$

<sup>11</sup> The marginal benefit of harbour patrols can decrease when the number of tankers in the harbour increases. The marginal benefit of harbour patrols increases with  $n_T$  if

$|V_H^T| < |P_T V_{HT}^T d_T/n_T|$  We will not consider this case, but it should be noted that in this

case:  $\frac{\partial m_T}{\partial n_T} > 0$ .



## APPENDIX 1

The appendix proceeds in modelling a pollution accident as a Poisson process which is split in various parts.

### A.1 SHIP ARRIVAL PROCESS

There are several types of ships arriving in a harbour. We assume that the types of ship arrive according to a Poisson process with different rates. Thus, the interarrival times for vessels of type  $i$  are an independent negative exponential random variable with parameter  $\zeta_i$ , having density:

$$f_i(t) = \zeta_i e^{-\zeta_i t} \quad t \geq 0, \quad (\text{A.1})$$

where  $\zeta_i$  is the arrival rate of a ship type  $i$ , that is, the expected number of ships type  $i$  that arrive in a harbour per unit of time.

The combination of independence and exponentially distributed arrival times implies that the system has no memory, the arrival of one ship will not depend on the time elapsed since the arrival of the last ship.

Modelling the length of stay a ship in a harbour implies also modelling its service time (the time spent unloading the cargo). We assume that service times are also independently and negative exponentially distributed, with parameter  $\delta_i$ . Thus, the density of the time till completion of service is given by:

$$f_i(t) = \delta_i e^{-\delta_i t} \quad t \geq 0. \quad (\text{A.2})$$

where  $\delta_i$  is the serving rate for a ship type  $i$ , that is, the expected number of ships type  $i$  that are served in a harbour per unit of time.

The number of ships in the harbour changes with time. For every  $t \geq 0$  let  $N_i(t)$  be the random variable that indicates the number of ships of type  $i$  in the harbour. We are interested in the steady state behaviour of the system, the best description of the long run behaviour. Assuming that ships are served immediately on arrival, the steady state density is a Poisson density with parameter  $\rho_i$ . See Karlin and Taylor (1975). That is,

$$\lim_{t \rightarrow \infty} P|N_i(t)=n_i| = \Phi_i(n_i) \quad (\text{A.3})$$

where:

$$\Phi_i(n_i) = \frac{\rho_i^{n_i} e^{-\rho_i}}{n_i!} \quad n_i \geq 0. \quad (\text{A.4})$$

An immediate consequence of this is that the steady state average number of ships in the harbour is,

$$\lim_{t \rightarrow \infty} E(N_i(t)) = \sum_{n_i=0}^{\infty} n_i \Phi_i(n_i) = \rho_i. \quad (\text{A.5})$$

note that  $\rho_i = \frac{\zeta_i}{\delta_i}$  for all ships type  $i$ .

For  $t \geq 0$  let  $N(t) = \sum_{i=0}^{\infty} N_i(t)$  be the random variable giving the total number of ships in the harbour at time  $t$ . Given that the stochastic processes  $|N_i(t): t \geq 0|$  for all  $i = 1, 2, \dots, m$  are independent, the steady state density of total number of ships in the harbour is also a Poisson density with parameter  $\rho = \sum_{i=1}^m \rho_i$ . That is  $\lim_{t \rightarrow \infty} P|N(t)=n|$  is  $\Phi(n)$  defined by:

$$\Phi(n) = \frac{\rho^n e^{-\rho}}{n!} \quad n \geq 0. \quad (\text{A.6})$$

## A.2 SPILL ARRIVAL PROCESS

Whether or not a spill occurs depends on physical characteristics of the vessel, the size and training of the crew and the shore personnel, as well as on the enforcement in the harbour. We reduce this complicated process into two competing effects: the probability that a spill occurs during any given instant that the ship is in harbour; and the time the ship spends in harbour. More specifically, the ship will stay in harbour for the random amount of time  $\tau_s$  (this service time is distributed as a negative exponential with parameter  $\delta_i$  as above), and during each instant of time,  $dt$ , spent in harbour there is a probability  $\lambda_i dt$  of a spill occurring. We assume that  $\lambda_i$  depends on a vector of parameters  $\Delta_i$  including the level of enforcement, the training of the crew, year in which the ship was built, etc. But for now on we leave this dependence implicit. Letting  $|S_i(t):t \geq 0|$  be a Poisson process with parameter  $\lambda_i$ , the random number of spills that occur by time  $t$  after the arrival of a ship is  $S_i(t)$  if  $t \leq \tau_s$  and  $S_i(\tau_s)$  if  $t \geq \tau_s$ . (Equivalently,  $S_i(t \wedge \tau_s)$  where  $a \wedge b = \min(a, b)$ ).

We recall the definition of a Poisson process:  $|S_i(t):t \geq 0|$  is a (standard) Poisson process with parameter  $\lambda_i$  if every realization is right continuous,  $S_i(0) = 0$ , and for any finite collection,  $\{0 \leq t_1 \leq t_2 \leq \dots \leq t_k\}$  the random variables  $S_i(t_k) - S_i(t_{k-1}), \dots, S_i(t_2) - S_i(t_1)$  are independent Poisson with parameters  $\{\lambda_i(t_k - t_{k-1}), \dots, \lambda_i(t_2 - t_1)\}$ .

The assumption that the spill arrival process is Poisson implies that given the  $(j-1)$ 'th spill, the waiting time till the  $j$ 'th spill follows a negative exponential with parameter  $\lambda_i$ . That is,

$$f(\tau_w) = \lambda_i e^{-\lambda_i \tau_w} \quad (\text{A.7})$$

When combined with the assumption of an independent negative exponential service time, the assumption that the spill arrival process is Poisson has the following implication: given that  $t$  has elapsed since the ship entered harbour, and independent of what has happened since the ship entered the harbour, the waiting time until either the next spill or the ship's departure is distributed as the minimum of a negative exponential with parameter  $\lambda_i$  and a negative exponential with parameter  $\delta_i$ . Thus, a negative exponential with parameter  $\delta_i + \lambda_i$ .

**Proposition 1:** Under the assumptions just given, the probability that a ship leaves harbour without a spill occurring is  $\frac{\delta_i}{\delta_i + \lambda_i}$ .

**Proof:** Let  $\tau_a$  and  $\tau_s$  be independent negative exponential random variables with parameters  $\lambda_i$  and  $\delta_i$ . The probability that a ship leaves harbour without a spill occurring is  $P(\tau_a > \tau_s)$ , the probability that the arrival time till next spill is longer than the serving time.

$$\begin{aligned}
 P(\tau_a > \tau_s) &= \int_0^{\infty} \int_x^{\infty} \lambda_i e^{-\lambda_i y} \delta_i e^{-\delta_i x} dx dy = \int_0^{\infty} \delta_i e^{-\delta_i x} \int_x^{\infty} e^{-\lambda_i y} dy dx \\
 &= \int_0^{\infty} -\delta_i e^{-(\delta_i + \lambda_i)x} dx = \\
 &= \frac{\delta_i}{\delta_i + \lambda_i} [-e^{-(\delta_i + \lambda_i)x}]_0^{\infty} \\
 &= \frac{\delta_i}{\delta_i + \lambda_i}.
 \end{aligned}$$

Q.E.D.

### A.3 SPILL ARRIVAL DURING A PERIOD OF TIME

The more ships there are in harbour, the more likely a spill is. The rest of this section describes an arrival process for spills that is dependent on the intensity of the harbours traffic. As can be seen, the arrival rate of spills is random, depending on the number of ships in harbour. The resultant arrival process for oil spills is called a doubly stochastic Poisson process. We derive its major properties.

The simplest characterization of the oil spill arrival process is given by specifying the (random) distribution of the independent interarrival times. Recall that  $N_i(t)$  is the number of ships of type  $i$  in harbour at time  $t$ . At every  $t \geq 0$ , the distribution of the waiting time until the next spill from a ship of type  $i$ ,  $\tau_i$ , is given by  $P(\tau_i > t+s) = \exp(-\lambda_i \int_t^{t+s} N_i(x) dx), s \geq 0$ . Thus, if  $N_i = n_i$  over the interval  $[t, t+s)$  then the waiting time during this interval is a negative exponential with parameter  $n_i \lambda_i$ .

Let the  $m$ 'th quarter,  $T_m$ , be the interval  $[mT, (m+1)T], T > 0$ . Let  $Z_{im}$  be the number of spills from ships type  $i$  during  $T_m$ . We are interested in the steady state behaviour of  $Z_{im}$ .

**Proposition 2:** For every  $z_i \geq 0, \lim_{m \rightarrow \infty} P\{Z_{im} = z_i\} = \sum_{n_i=0}^{\infty} \frac{(n_i \lambda_i T)^{z_i} e^{-n_i \lambda_i T}}{z_i!} \Phi(n_i)$  where as

before  $\Phi(n_i) = \lim_{t \rightarrow \infty} P\{N_i(t) = n_i\} = \frac{\rho_i^{n_i} e^{-\rho_i}}{n_i!}$ , and  $\rho_i = \zeta_i / \delta_i$

**Proof:** Note that  $\Phi(\cdot)$ , giving the steady state distribution of  $N_i$ , is unchanged if we replace  $\delta_i$  and  $\zeta_i$  by  $\delta_i / \kappa$  and  $\zeta_i / \kappa$  for any  $\kappa \geq 0$ .

Letting  $C(m, n_i)$  be the event  $\{N_i(T_m) = n_i\}$  the Proposition can be rewritten as

$$P\{Z_{im} = z_i\} = \sum_{n_i=0}^{\infty} P\{Z_{im} = z_i | C(m, n_i)\} \cdot \Phi(n_i) \quad \text{where} \quad \Phi(n_i) = \lim_{t \rightarrow \infty} P\{N_i(t) = n_i\}.$$

$$\text{Let} \quad B = \sum_{n_i=0}^{\infty} P\{Z_{im} = z_i | C(m, n_i)\} \cdot \Phi(n_i)$$

Pick arbitrary  $\epsilon > 0$ . We must demonstrate the existence of an  $M$  s.t. for all  $m \geq M$

$$|P\{Z_{im} = z_i\} - B(m)| < \epsilon. \quad (\text{A.8})$$

Let  $S(m, n_i)$  be the event  $\{Z_{im} = z_i\} \cap \{N_i(mT) = n_i\} \cap C(m, n_i)$  and  $T(m, n_i)$  be the event  $\{Z_{im} = z_i\} \cap \{N_i(mT) = n_i\} \cap \Omega \setminus C(m, n_i)$ . The event  $\{Z_{im} = z_i\}$  is the disjoint union of the

$S(m, n_i)$  and the  $T(m, n_i)$  so that

$$P\{Z_{im} = z_i\} = \sum_{n_i=0}^{\infty} P\{S(m, n_i)\} + \sum_{n_i=0}^{\infty} P\{T(m, n_i)\}. \quad (\text{A.9})$$

For any  $\kappa$ , if the arrival and service processes have parameters  $\delta_i/\kappa$  and  $\zeta_i/\kappa$  then,

$$\sum_{n_i=0}^{\infty} P\{T(m, n_i)\} < 1 - e^{-\kappa(\delta_i + \zeta_i)/\kappa}. \quad (\text{A.10})$$

Pick  $\kappa$  sufficiently large that  $1 - e^{-\kappa(\delta_i + \zeta_i)/\kappa} < \epsilon/3$ . For such a  $\kappa$  and for all  $m$  and  $n_i$ ,  $P\{C(m, n_i)\} = P\{N_i(mT) = n_i\} \cdot e^{-\kappa(\delta_i + \zeta_i)/\kappa}$ . Since  $\{N_i(mT) = n_i\} \subset C(m, n_i)$ , this gives us:

$$\begin{aligned} P\{S(m, n_i)\} &= P\{Z_{im} = z_i | C(m, n_i)\} \cdot P\{C(m, n_i)\} \\ &= P\{Z_{im} = z_i | C(m, n_i)\} \cdot P\{N_i(mT) = n_i\} \cdot e^{-\kappa(\delta_i + \zeta_i)/\kappa}. \end{aligned} \quad (\text{A.11})$$

Combining equations A.9-A.11 setting  $q = e^{-\kappa(\delta_i + \zeta_i)/\kappa}$ , and

$$A(m) = \sum_{n_i=0}^{\infty} P\{Z_{im} = z_i | C(m, n_i)\} \cdot P\{N_i(mT) = n_i\} \quad \text{we find that}$$

$$P\{Z_{im} = z_i\} = q \cdot A(m) + p, \quad (\text{A.12})$$

for some  $0 < p < \epsilon/3$ . Let  $B(m) = \sum_{n_i=0}^{\infty} P\{Z_{im} = z_i | C(m, n_i)\} \cdot \Phi(n_i)$ . Because

$\lim_{n_i \rightarrow \infty} P\{N_i(mT) = n_i\} = \Phi(n_i)$  for every  $n_i$  we can pick  $M$  sufficiently large so that for all  $m \geq M$  we have:

$$|A(m) - B(m)| \leq \sum_{n_i} |P\{N_i(mT) = n_i\} - \Phi(n_i)| \epsilon/3. \quad (\text{A.13})$$

For all such  $m$  we have

$$|P\{Z_{im} = z_i\} - B(m)| = |q \cdot A(m) + p - B(m)| \leq q \cdot |A(m) - B(m)| + (1 - q)B + p < \epsilon/3 + \epsilon/3 + \epsilon/3 = \epsilon. \quad (\text{A.14})$$

Comparing (A.14) with (A.8) we see that the proof is complete. Q.E.D.

The last Proposition shows that the sequence of random variables,  $Z_{im}$ , converges weakly to  $Z_i^*$  having the given distribution. We now turn to the expected value and variance of  $Z_i^*$ .

**Proposition 3:**  $E(Z_i^*) = \lambda_i T \rho_i$  and  $Var(Z_i^*) = \rho_i \lambda_i T (1 + \lambda_i T)$ .

**Proof:** Using the notation from the previous Proposition we see that  $E(Z_{im} | C(m, n_i)) = n_i \lambda_i T$ . Repeating the approximation arguments nearly verbatim we see that

$$E(Z_i^*) = \lim_{m \rightarrow \infty} E(Z_{im}) = \sum_{n_i=0}^{\infty} n_i \lambda_i T \Phi(n_i) = \lambda_i T \sum_{n_i=0}^{\infty} n_i \Phi(n_i) = \lambda_i T \rho_i.$$

Recall that if  $\{A_n : n \geq 0\}$  is a partition of  $\Omega$  then for any random variable,  $X$ , having a second moment  $E(X^2) = \sum_{n=0}^{\infty} P(A_n) \cdot E(X^2 | A_n)$ . Again repeating the approximation arguments, we have  $P(C(m, n_i)) E(Z_{im}^2 | C(m, n_i))$  converging to  $\Phi(n_i) [(n_i \lambda_i T)^2 + n_i \lambda_i T]$  and  $\{C(m, n_i) : n_i \geq 0\}$  being arbitrarily close to a partition as  $m, \kappa \rightarrow \infty$ . From these we conclude that  $E(Z_i^{*2}) = (\lambda_i T)^2 (\rho_i^2 + \rho_i) + \lambda_i T \rho_i$  so that  $var(Z_i^*) = \rho_i \lambda_i T (1 + \lambda_i T)$ . Q.E.D.

#### A.4 SEVERAL PROCESSES IN A HARBOUR

Until now we have been looking at a single process  $i$ . Now suppose we want to know the total number of spills that occur in a harbour in a given period of time. Usually more than one type of process takes place in a harbour over a period of time. The characteristics of the ship, the causes of the spill and the ship's operating environment, among other variables define a process. We assume that the number of spills from different processes are independent random variables. Let  $\tau_i$  be the waiting time until the next spill from any ship. For every  $t, s \geq 0$ ,  $P\{\tau_i > t+s\} = \exp\{-\sum_i \lambda_i \int_t^{t+s} N_i(x) dx\}$ . Thus if  $N_i = n_i$  for all  $i=1,2,\dots,m$  over the interval  $[t, t+s)$  the waiting time during this interval is a negative exponential with parameter  $\sum_{i=1}^h n_i \lambda_i$ .

Let, as before, the  $m$ 'th quarter,  $T_m$ , be the interval  $[mT, (m+1)T)$ ,  $T > 0$ . Let  $Z_m$  be the number of spills for any type of ship during  $T_m$ . Thus  $Z_m = \sum_{i=1}^h Z_{im}$ . We are interested in the steady state behaviour of  $Z_m$ . Proposition 3 shows that the sequence of random variables  $Z_{im}$  converges weakly to  $Z_i^*$ , for each ship type  $i$ . Define  $Z^* = \sum_{i=1}^h Z_i^*$ . Recall that, if  $Z_{im}$  converges weakly to  $Z_i^*$  then independence implies  $\sum_{i=1}^h Z_{im}$  converges weakly to  $\sum_{i=1}^h Z_i^*$ . That is,  $Z_m$  converges weakly to  $Z^*$ . Thus,

$$P\{Z_m = z\} = \lim_{m \rightarrow \infty} P\{\sum_{i=1}^h Z_i^* = z\}.$$

**Proposition 4:**  $E(Z) = \sum_{i=1}^h \lambda_i T \rho_i$  and  $Var(Z) = \sum_{i=1}^h \rho_i \lambda_i T (1 + \lambda_i T)$ .

**Proof:**

The expectation of a sum is the sum of the expectations, and, for independent random variables, the variance of a sum of the variances. Q.E.D.



## A.5 SPILL SIZE DISTRIBUTION

To this point, the process determining the size of oil spills has been ignored. We assume that the physical characteristics of the vessels and the levels of fines and enforcement effort affect both the spill frequency and the spill size distribution function. The spill size distribution adds a second dimension to the characterizations of the stochastic processes of oil spills.

We assume that spill sizes from ship type  $i$  are independent and identically distributed with a distribution function  $G_i(\cdot)$ . Again  $G_i$  depends on a vector of parameters  $\Delta_i$  including level of fines, enforcement effort, skills of the crew among other parameters, but we leave this dependence implicit for now. Let  $X_{i0}=0$  and  $(X_{in})_{n \in \mathcal{N}}$  be a collection of independent random variables with distribution function  $G_i$ . The random variable  $\theta_{i,m} = \sum_{n=0}^{Z_{im}} X_{in}$  gives the distribution of the total amount of oil spilled by ships of type  $i$  during the  $m$ 'th quarter. The random variable  $\theta_m = \sum_{i=1} \theta_{i,m}$  gives the distribution of the total amount of oil spilled from any source during the  $m$ 'th quarter. The corresponding steady state distributions are  $\theta_i^* = \sum_{n=0}^{Z_i^*} X_{in}$  and  $\theta^* = \sum_i \theta_i^*$ .

## APPENDIX 2

### SPILL SIZE DENSITY FUNCTION

Define  $f_i(X_i, \Theta_i \mid t \in (0, \tau])$  as the spill size distribution of a ship type  $i$  during a period of time, (i.e. during a transfer operation). We can think of each time that a ship does a transfer operation as a random experiment. (The spill arrival process being Poisson which implies that given the  $(j-1)$ th spill, the waiting time till the  $j$ -th spill follows a negative exponential with parameter  $\lambda_i$ . The transfer done by ships of the same type are repetitions of the same experiment.  $f_i(X_i, \Theta_i \mid t \in (0, \tau])$  is the probability that a ship type  $i$  does a spill of size  $X_i$  during its transfer operation in the harbour.  $X_i$  is the spill size random variable,  $\Theta_i$  represents the vector of parameters that affect this distribution function, and  $(0, t]$  the length of time needed to complete the operation.

$$F(0 \leq X_i \leq \Psi; \Theta_i \mid t \in (0, \tau]) = F(X_i = 0; \Theta_i \mid t \in (0, \tau]) + F(0 < X_i \leq \Psi; \Theta_i \mid t \in (0, \tau]).$$

Since  $X_i = 0$  if and only if there is no spill during period  $(0, \tau]$ , That is:

$$F_i(X_i=0; \Theta_i \mid t \in (0, \tau]) + P_i(S_i(t)=0 \mid t \in (0, \tau]) = e^{-\lambda_i t}$$

and

$$\begin{aligned} F_i(0 < X_i \leq \Psi; \Theta_i \mid t \in (0, \tau]) &= G_i(0 < X_i \leq \Psi; \Delta_i \mid S_i(t) > 0) P_i(S_i(t) > 0 \mid t \in (0, \tau]) = \\ &= G_i(0 < X_i \leq \Psi; \Delta_i \mid S_i(t) > 0) (1 - e^{-\lambda_i t}) \end{aligned}$$

Then the spill size distribution function of a ship type  $i$  during a transfer of length  $(0, t]$  can be expressed as:

$$F_i(0 \leq X_i \leq \Psi; \Theta_i \mid t \in (0, \tau]) = \begin{cases} e^{-\lambda_i t} & \text{if } X_i = 0 \\ (1 - e^{-\lambda_i t}) G_i(0 < X_i \leq \Psi; \Delta_i \mid S_i(t) > 0) & \text{if } 0 < X_i \leq \Psi \end{cases}$$

Notice that this density function is conditioned on the period  $t$ , the length of time that it takes the ship to complete the transfer operation. Different ships with different length of

stay, even if they are of the same type, have different conditional spill size density functions. The steady state density function of spill size give us the spill size density function for any length of time. So we are interested in the steady state marginal density function of spill size. The unconditional joint density function of spill size and time for a ship type  $i$ , is equal to:

$$f_i(X_i; \Theta_i; t) = f_i(X_i; \Theta_i | t \in (0, \tau]) f_i(t).$$

Thus,

$$f_i(X_i; t; \Theta_i) = \begin{cases} e^{-\lambda_i t} \delta_i e^{-\delta_i t} & \text{if } X_i = 0 \\ (1 - e^{-\lambda_i t}) (\delta_i e^{-\delta_i t}) g_i(0 < X_i \leq \Psi; \Delta_i | S_i(t) > 0) & \text{if } 0 < X_i \leq T \end{cases}$$

So the marginal spill size density function will be equal to:

$$f_i(0 < X_i \leq \Psi; \Theta_i) = \int_0^\tau f_i(X_i; \Theta_i; t) dt = \int_0^\tau f_i(X_i; \Theta_i | t) f_i(t) dt$$

In the case of  $X_i = 0$  the marginal density function of spill size is equal to:

$$\begin{aligned} f_i(X_i=0; \Theta_i) &= \int_0^\tau f_i(X_i=0; \Theta_i | t) f_i(t) dt = \int_0^\tau (e^{-\lambda_i t} \delta_i e^{-\delta_i t}) dt \\ &= \int_0^\tau \delta_i e^{-(\lambda_i + \delta_i)t} dt = \left[ -\frac{\delta_i}{\lambda_i + \delta_i} e^{-(\lambda_i + \delta_i)t} \right]_0^\tau \\ &= \frac{\delta_i}{\lambda_i + \delta_i} (1 - e^{-(\lambda_i + \delta_i)\tau}). \end{aligned}$$

And in the case of  $0 < X_i \leq \Psi$  the marginal density function is:

$$\begin{aligned} f_i(0 < X_i \leq \Psi; \Theta_i) &= \int_0^\tau f_i(0 < X_i \leq \Psi; \Theta_i; t > 0) \delta_i e^{-\delta_i t} dt = \\ &= \int_0^\tau g_i(0 < X_i \leq \Psi; \Delta_i | S_i(t) > 0) (1 - e^{-\lambda_i t}) \delta_i e^{-\delta_i t} dt. \end{aligned}$$

$$\begin{aligned}
&= g_i(0 < X_i \leq \Psi; \Delta_i \mid S_i(t) > 0) \int_0^\tau (1 - e^{-\lambda t}) \delta_i e^{-\delta t} dt = \\
&= g_i(0 < X_i \leq \Psi; \Delta_i \mid S_i(t) > 0) \left( -e^{-\delta t} \Big|_0^\tau + \frac{\delta_i}{\lambda_i + \delta_i} e^{-(\lambda_i + \delta_i)t} \Big|_0^\tau \right) \\
&= g_i(0 < X_i \leq \Psi; \Delta_i \mid S_i(t) > 0) \left( \frac{\lambda_i}{\lambda_i + \delta_i} + \left( e^{-\delta \tau} \left( \frac{\delta_i}{\delta_i + \lambda_i} e^{-\lambda_i \tau} - 1 \right) \right) \right)
\end{aligned}$$

Notice that, in steady state;

$$\lim_{\tau \rightarrow \infty} e^{-(\lambda_i + \delta)\tau} \rightarrow 0$$

and

$$\lim_{\tau \rightarrow \infty} \left( e^{-\delta \tau} \left( \frac{\delta_i}{\delta_i + \lambda_i} e^{-\lambda_i \tau} - 1 \right) \right) \rightarrow 0$$

So that:

$$f_i(0 \leq X_i \leq \Psi; \Theta_i) = \begin{cases} \delta_i / \delta_i + \lambda_i & \text{if } X_i = 0 \\ (\lambda_i / \delta_i + \lambda_i) g_i(0 < X_i \leq \Psi; \Delta_i \mid S_i(t) > 0) & \text{if } 0 < X_i \leq T \end{cases}$$

where  $\delta_i / \lambda_i + \delta_i$  is the probability that a ship finish a transfer operation without a spill occurring. This spill size density function does not depend on the actual length of stay in a harbour but on the steady state probability of a ship type  $i$  leaving the harbour without spilling. It is the steady state spill size density function per unit of time. The period can be chosen such that it coincides with the period during which the agency allocates enforcement effort.

### APPENDIX 3

#### RANDOM VOLUME OF OIL SPILLED

To assume that  $V_{TH}^T < 0$  and  $V_{BH}^B < 0$  for tankers and barges respectively, it implies that the less  $P_T$  is used in order to attain the same level of clean water the use of  $f_H$  has to increase at an increasing rate.

Let  $-V^T(P_T, f_H)(1-(1-\alpha_T)P_T)$  be the benefit (volume of clean water produced) attained with policy instrument  $P_T$  and  $f_H$ . Then the second order necessary conditions for concavity imply:

$$(V_{TT}^T - 2(1-\alpha_T)V_{TT}^T)V_{HH}^T d_T - (V_{TH}^T d_T + (1-\alpha_T)V_H^T) > 0.$$

If  $V_{HT}^T < 0$  then

$$|V_{TT}^T V_{HH}^T| > |V_{HT}^T d_T + (1-\alpha_T)V_H^T|^2.$$

because

$$|V_{HT}^T| > |V_{HT}^T d_T + (1-\alpha_T)V_H^T|.$$

If  $V_{HT}^T < 0$  then for strict concavity we need:

$$|V_{TT}^T V_{HH}^T d_T| > |V_{HH}^T d_T^2 + (1-\alpha_T)^2 V_H^T| \text{ and}$$

$$|V_{HH}^T V_T^T| > |V_{HT}^T V_H^T|$$

Notice that we need to make some assumptions about the first derivatives in this case in order to satisfy the second order necessary conditions.

## APPENDIX 4

### EFFECTS OF THE POLLUTION PREVENTION MEASURES AT THE HARBOUR LEVEL

The assumptions we made are in terms of an individual ship; now we look at the implications of these assumptions on the expected total volume of oil spilled in the harbour by all ships. Define  $EW(m_T; m_B; f_H; \tau | n_T; n_B; \mathcal{E})$  to be the expected volume of oil spilled in the harbour during a period  $(0, \tau]$  given: i)  $n_T$  tankers and  $n_B$  barges transferring oil in the harbour during that period, ii)  $m_T$  of the tankers' and  $m_B$  of the barges' transfer operations are monitored, iii)  $f_H$  is the level of harbour patrolling, and iv) the long run level of enforcement effort is  $\mathcal{E}$ . Therefore, the expected volume of oil spilled in a harbour equals:

$$EW = (n_T - m_T(1-\alpha_T)P_T) EV^T(P_T; f_H | \mathcal{E}) + (n_B - m_B(1-\alpha_B)P_B) EV^B(P_B; f_H | \mathcal{E})$$

Recall from (3.5) and (3.6) that the vessel expected spilled volume decreases at a decreasing rate with the private effect of monitoring a transfer operation. Also the vessel expected spilled volume decreases at a decreasing rate with the public good effect of the provability of monitoring. The public good effect of the probability of monitoring at the harbour level equals the sum of the public good effects on each of the tankers in the harbour. Therefore, the expected volume of oil spilled in a harbour decreases at a decreasing rate with monitoring oil transfer operations. That is,

$$EW_T = -(1-\alpha_T)EV^T(P_T; f_H) + d_T/n_T EV_{TT}^T(P_T; f_H) < 0 \quad \text{and,}$$

$$EW_{TT} = -2(1-\alpha_T)EV_{TT}^T(P_T; f_H) + d_T/n_T EV_{TTT}^T(P_T; f_H) > 0.$$

Equivalently for the case of barges:

$$EW_B = -(1-\alpha_B)EV^B(P_B; f_H) + d_B/n_B EV_{BB}^B(P_B; f_H) < 0, \quad \text{and}$$

$$EW_{BB} = 2(1-\alpha_B)EV_{BB}^B(P_B; f_H) + d_B/n_B EV_{BBB}^B(P_B; f_H) > 0.$$

In the case of harbour patrols the vessel expected spilled volume decreases at a

decreasing rate in absolute value, therefore, the reduction on the volume of oil spilled in the harbour decreases with harbour patrols at a decreasing rate. Thus,

$$EW_H = EV_H^T d_T + EV_H^B d_B < 0,$$

$$EW_{HH} = EV_{HH}^T d_T + EV_{HH}^B d_B > 0.$$

From (3.11) it is implied that the overall effect of monitoring transfer operations increases with harbour patrols. Thus:

$$EW_{TH} = -(1-\alpha_T)EV_H^T + \frac{d_T EV_{TH}^T}{n_T} < 0.$$

in the tankers case. And for barges:

$$EW_{BH} = -(1-\alpha_B)EV_H^B + \frac{d_B EV_{BH}^B}{n_B} < 0.$$

We assume that  $EW_{HT} - EW_{HB} = 0$ , that is the effect of harbour patrols on the marginal benefit of monitoring tankers and barges is the same.

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