



Center for  
International Climate  
and Environmental  
Research - Oslo

Working Paper 1997:5

# Climate agreements: Optimal taxation of fossil fuels and the distribution of costs and benefits across countries

*Bjart Holtsmark*



University of Oslo

ISSN: 0804-452X



Climate agreements:  
Optimal taxation of fossil fuels and the distribution of  
costs and benefits across countries<sup>\*</sup>

by

*Bjart Holtsmark*<sup>\*\*</sup>

**Abstract**

The paper analyses governments' response to a climate agreement that commits themselves to reducing their emissions of CO<sub>2</sub>. A formula for optimal taxation of fossil fuels in open economies subject both to an emission constraint and a public budget constraint is developed. The applied theory captures how national governments' behaviors are sensitive to the size of the benefits from revenue recycling and how these benefits adjust the distribution of abatement costs. The empirical part of the paper illustrates the significance of the participating countries' current and potential fossil fuel taxation schemes and their role in the fossil fuel markets.

JEL classification: H21, Q48.

Key words: Climate agreements; CO<sub>2</sub> taxes; Marginal costs of public funds; Terms of trade.

---

<sup>\*</sup> Comments from Agnar Sandmo, Asbjørn Aaheim, Cathrine Hagem, and Asbjørn Torvanger to different draft versions of the paper are highly appreciated. Erik S. Sørensen (ECON-Paris) has generously provided data on fossil fuel taxes in the OECD countries.

<sup>\*\*</sup> Center for International Climate and Environmental Research - Oslo (CICERO),  
Address: Box 1129 Blindern, N-0317 Oslo, Norway. Telephone: 47 22 85 87 59. Telefax: 47 22 85 87 51  
E-mail: b.j.holtsmark@cicero.uio.no



## 1. Introduction

The paper analyses optimal taxation of fossil fuel consumption in countries subject to both a CO<sub>2</sub>-emission constraint and a public budget constraint firstly from a theoretical point of view. An optimal taxation rule is developed that extends and adjusts the taxation rule in Sandmo (1975) taking into account the countries market-power in the fossil fuel markets. As in Sandmo (1975), environmental considerations are included in the formula, although in a more indirect manner: While Sandmo analyzed the case with a negative external effect connected to a number of goods, I replace the external effects of the fossil fuels with a CO<sub>2</sub>-emission constraint.

Secondly, the paper applies the developed optimal taxation rule in an numerical model that is calibrated to the world economy as in 1993. In the model the countries are linked together through their relations to the fossil fuel markets. The OECD-countries are assumed to react simultaneously to a climate agreement with flat rate CO<sub>2</sub> emission reduction commitments.

Each of the OECD countries is incorporated into the model by national welfare functions. The variables in these functions are the fossil fuel taxes, the production and consumption of fossil fuels, and the amount of public revenue generated by fossil fuel taxation. Public revenues from fossil fuel taxation are included in the welfare functions in order to capture how national governments' behaviors are sensitive to the size of the benefits from revenue recycling and how these benefits adjust the distribution of abatement costs across countries.

The empirical part of the paper illustrates how the distribution of gains and losses among the participating countries in the climate agreements are sensitive to the different countries' links to the fossil fuel markets and their current and potential fossil fuel taxation schemes. The national governments are assumed to redesign their fossil fuel taxation schemes in the light of the climate agreement and the other governments' reactions to the agreement. The model incorporates to what extent resource rents are transferred from fossil fuel exporting countries to fossil fuel importing countries when the governments act strategically, taking benefits from revenue recycling into account. Furthermore, the structures of the countries' energy demand, prior tax distortions and the size of the marginal excess burden of taxation in the different countries are important factors behind the models' estimates of the simultaneous abatement costs.

Although there is a considerable amount of literature on the costs of combating greenhouse gas emissions, surprisingly, few of the studies take into account the gains from revenue recycling and benefits or losses from changes in terms-of-trade, cf. Ekins (1995). Some

examples of model studies taking terms-of-trade effects into account are Burniaux, Martin, Nicoletti and Oliveira Martins (1992), DFAT and ABARE (1995), Rosendahl (1994), and Berg et al. (1996). None of these studies analyzes the benefits of revenue recycling. Several other studies, for example Jorgensen and Wilcoxon (1993) and Håkonsen (1995), emphasize on the other hand the importance of taking revenue recycling into account, but ignore the terms-of-trade effects of several countries implementing climate policies at the same time (the simultaneous abatement costs). The present study provides some indications about the importance of including both terms-of-trade effects and benefits from revenue recycling when economic implications of climate agreements are analyzed.

The theoretical approach in this paper is somewhat similar to the approach used in Golombek, Hagem and Hoel (1995) and Golombek and Bråten (1994). The matter under discussion is however quite different. Golombek et al. (1994) and (1995) analyze how taxes on fossil fuels should be designed in a group of countries that cooperate and have committed themselves to reducing *global* emissions of CO<sub>2</sub>. The purpose of the present paper is to analyze a climate agreement where the OECD countries are committed to reduce their *national* emissions, and where these countries do *not* co-ordinate their actions. Contrary to the approach in Golombek et al. (1995) it is assumed that each of the OECD countries maximizes their *national* welfare under a *national* emission reduction constraint.

Another crucial difference from the analysis of Golombek et al. (1994) and Golombek et al. (1995) is the inclusion of benefits from revenue recycling as an explanatory factor. The two mentioned articles did not take into account how the benefits from revenue recycling are likely to alter the fossil fuel taxes and the measured welfare effects, while the importance of such benefits is emphasized in this paper.

It should be emphasized that the model used in this paper is a partial and static one, and the damage costs from climate change are not incorporated into the model. Dynamic aspects of the countries' climate policies, as emphasized by for example Nordhaus and Yang (1996), are therefore not taken into account in the present analysis. Some relevant structural characteristics of the national economies that are emphasized by other studies, as for example DFAT and ABARE (1995) and Burniaux et al. (1992), are also ignored. In contrast to the mentioned studies the present paper, however, analyses in further detail the countries' fossil fuel taxation policies under the implementation of a climate agreement, and to what extent these taxation policies influence the distribution of costs and benefits of an agreement. Unlike the mentioned studies, the present paper directs the focus towards the links between a possible climate agreement and both the current and potential fossil fuel taxation policies in the light of public budget constraints.

The fossil fuel prices are endogenously determined within regional and global fossil fuel markets. In the present version of the theoretical and numerical models these markets are assumed to be competitive with price taking behavior also at the supply side. This is of course a simplifying assumption. A future improvement of the model could be to let one or more groups of the fossil fuel producers take their market power into consideration.

The paper is organized into four sections. The following section provides a theoretical foundation for predictions about possible changes in fossil fuel taxation in countries that are assumed to be committed to reducing their emissions of CO<sub>2</sub> from fossil fuel combustion. In the third section some economic impacts of climate agreements are analyzed numerically assuming that a Nash-equilibrium is established. Finally, conclusions are presented in section 4.

## 2. Optimal taxation of fossil fuels and climate agreements

Let us define a welfare indicator, or payoff function, of country  $n$  as the produced quantity of a macro good  $z_n$ , that also serves as the numeraire good, minus the costs from indigenous production and net import of oil, coal and gas:

$$W_n = z_n - \sum_{i=1}^3 c_{in}(x_{in}) - \sum_{i \in I_r} (p_i y_{in} - p_i x_{in}), \quad \forall n \in N, \quad (1)$$

$$z_n = g_n(y_{1n}, y_{2n}, y_{3n}) + m(R_{On}) \quad (2)$$

$y_{1n}, y_{2n}, y_{3n}$  and  $x_{1n}, x_{2n}, x_{3n}$  are total consumed and produced quantities of oil, coal and gas in country  $n$  respectively measured in units of their carbon content.  $p_1, p_2$  and  $p_3$  are the prices of oil, coal and gas in the world markets respectively.<sup>1</sup>  $N$  is the set of countries.  $c_{in}(x_{in})$  is the cost related to indigenous production of fuel  $i$ . The third term at the right hand side of (1) represents the net import bill.

Equation (2) specifies how the production of the numeraire is accomplished by input of fossil fuels, and how the efficiency of this production process is sensitive to the level the total revenue collected from the taxation of other goods and services than fossil fuels represented by  $R_{On}$ . This variable is included in the production function of the numeraire as an additively separable variable. This is done in order to enable the model to incorporate that economic

---

<sup>1</sup> Due to the high transportation costs the empirical model splits the world in three regional natural gas markets; one in North America, a second in Europe including Eastern Europe and Russia and a third in the Pacific region. In the theoretical analysis for simplicity only one global gas market is assumed.

activity in general is likely to be the more inefficient the higher are the tax rates in the economy, cf. Ballard and Fullerton (1992) among others. The sum of collected revenue from other goods and services than fossil fuels is used as an approximation for the general level of the tax rates exclusive the fossil fuel taxes. This is an important feature of the model because we want to embody benefits from recycling of the revenues generated by fossil fuel taxation. An inverse relationship between the fossil fuel taxes and the sum of collected revenue from other taxes is established by assuming a public budget constraint. We have that:

$$R_{On} = R_n - R_{Fn}. \quad (3)$$

where  $R_n$  is total revenue from taxation in country  $n$ , while  $R_{Fn}$  is revenue from taxation of production and consumption of fossil fuels. The public budget constraint requires that  $R_n \leq R_n^0$ . For simplicity we assume that this constraint is always binding.

The first order derivative in (2) with respect to  $R_{On}$  is assumed to be negative and should be interpreted as an incorporation of the marginal excess burden (MEB) of taxation. The efficiency loss from taxation of production and consumption of fossil fuels is incorporated into the model in a direct and more elaborated manner, because these tax rates are included specifically.

## 2.1 Private sector behaviour

It is now important to notice that there are assumed to be many producers of the numeraire good and that the production function in (1) consequently should be seen as an aggregated function. It is assumed that the fossil fuel producers are price takers in the fossil fuel markets. It is also assumed that the producers of the numeraire good each consider themselves as too small to be able to significantly affect the size of  $R_{On}$ . This does not mean that we rule out the possibility that each producer of the numeraire good is fully aware of the relationship between the public budget constraint, revenues generated from fossil fuels and the level of efficiency in the economy in general (represented by the variable  $R_{On}$ ). It is only an assumption about their strategic behavior. It is then reasonable to assume that the consumed and produced quantities of the fossil fuels in country  $n$  are determined by the following set of first order conditions:



$$\frac{\partial \bar{g}_n(y_{1n}, y_{2n}, y_{3n})}{\partial y_{in}} = p_i + t_{cin}, \quad (4)$$

$$\frac{\partial c_m(x_m)}{\partial x_m} = p_i, \quad i = 1, 2, 3, \forall n \in N,$$

where  $t_{cin}$  is tax on the consumption of fossil fuel  $i$  in country  $n$ .<sup>2</sup> These equations define the consumed quantities as functions of the prices and taxes. Consequently we could define the demand and supply functions:

$$\left. \begin{aligned} y_m &= D_m(P_{1n}, P_{2n}, P_{3n}) \\ x_m &= S_m(p_i) \end{aligned} \right\} i = 1, 2, 3, \quad (5)$$

where  $P_m = p_i + t_{cin}$ .

We define the profit functions of the four production sectors of country  $n$ :

$$\begin{aligned} v_n(P_{1n}, P_{2n}, P_{3n}, R_{Fn}) &= g_n(D_{1n}(\cdot), D_{2n}(\cdot), D_{3n}(\cdot)) + m(R_n^0 - R_{Fn}) - \sum_{i=1}^3 (P_{in}) D_{in}(\cdot) \\ \Pi_m(p_i) &= p_i S_m(p_i) - c_m(S_m(p_i)), \quad i = 1, 2, 3. \end{aligned} \quad (6)$$

## 2.2 Market equilibrium

The equilibrium conditions of the fossil fuel markets are:

$$\sum_{n \in N} D_{in}(p_1 + t_{c1n}, p_2 + t_{c2n}, p_3 + t_{c3n}) = \sum_{n \in N} S_{in}(p_i), \quad i = 1, 2, 3, \quad (7)$$

Hence the prices could be defined as functions of the set of domestic tax rates:

$$p_i = f_i(t_{c11}, t_{c21}, t_{c31}, \dots, t_{c1N}, t_{c2N}, t_{c3N}), \quad i = 1, 2, 3. \quad (8)$$

---

<sup>2</sup> For simplicity we ignore that taxes on production could play a role in the countries climate policy. For a discussion on taxes on production in relation to climate agreements, see Golombek et al. (1995).

### 2.3 Public sector behaviour

Let us assume that a climate agreement commits a number of countries to reduce their emissions of CO<sub>2</sub> to a certain level  $E_n$  and that these countries design efficient systems for taxation of fossil fuels in order to achieve a cost-effective fulfillment of their commitments. Different equilibrium concepts could be used as starting point here. We use the Nash-equilibrium as the equilibrium concept, i.e. the equilibrium where the different governments maximize their pay off functions taking all the other governments' actions as given. Although both the producers of the numeraire good and the fossil fuel producers are assumed to be price takers in the fossil fuel markets, the national governments on the other hand are assumed to take into account that their fossil fuel taxes affect the fossil fuel prices. The Lagrangian to each single country's maximization problem is then:

$$L_n(t_{c1n}, t_{c2n}, t_{c3n}, \lambda_n) = v_n(P_{1n}, P_{2n}, P_{3n}, R_{Fn}) + \sum_{i=1}^3 \Pi_m(p_i) + R_{Fn} - \lambda_n \left( \sum_{i=1}^3 y_{in} - E_n \right) \quad (9)$$

Using Hotellings lemma, cf. for example Varian (1984), which states that  $\partial v_n / \partial P_{jn} = -y_{jn}$ , and  $\partial \Pi_n / \partial p_j = x_{jn}$  and introducing the notation  $v_{Rn} = \partial v_n / \partial R_{Fn}$ , the first order derivatives of the Lagrangian are:

$$\frac{dL_n}{dt_{ckn}} = -y_{kn} - \sum_{j=1}^3 y_{kn} \frac{\partial f_j}{\partial t_{ckn}} + (1 + v_{Rn}) \frac{dR_{Fn}}{dt_{ckn}} + \sum_{j=1}^3 x_{jn} \frac{\partial f_j}{\partial t_{ckn}} + \lambda_n \left( \sum_{j=1}^3 \sum_{i=1}^3 \frac{\partial D_{jn}}{\partial P_{in}} \left( \frac{\partial f_i}{\partial t_{ckn}} + \delta_{ik} \right) \right) \quad (10)$$

for  $k=1,2,3$  and where  $\delta_{ij}=1$  if  $i=j$ ,  $\delta_{ij}=0$  if  $i \neq j$ . We have that:

$$\frac{dR_{Fn}}{dt_{ckn}} = y_{kn} + \sum_{j=1}^3 t_{cjn} \sum_{i=1}^3 \frac{\partial D_{jn}}{\partial P_{in}} \left( \frac{\partial f_i}{\partial t_{ckn}} + \delta_{ik} \right) \quad (11)$$

Then we have:

$$\begin{aligned} \frac{dL_n}{dt_{ckn}} &= v_{Rn} y_{kn} + \sum_{j=1}^3 (x_{jn} - y_{jn}) \frac{\partial f_j}{\partial t_{ckn}} + (1 + v_{Rn}) \sum_{i=1}^3 \left( \left( \frac{\partial f_i}{\partial t_{ckn}} + \delta_{ik} \right) \sum_{j=1}^3 t_{cjn} \frac{\partial D_{jn}}{\partial P_{in}} \right) \\ &\quad + \lambda_n \sum_{i=1}^3 \left( \left( \frac{\partial f_i}{\partial t_{ckn}} + \delta_{ik} \right) \sum_{j=1}^3 \frac{\partial D_{jn}}{\partial P_{in}} \right) \end{aligned} \quad (12)$$

The first order conditions could therefore be written as follows:

$$\sum_{j=1}^3 \left( t_{cjn} - \frac{\lambda_n}{1+v_{Rn}} \right) \sum_{i=1}^3 \frac{\partial D_{jn}}{\partial P_{in}} \left( \frac{\partial f_i}{\partial t_{ckn}} + \delta_{ik} \right) = \frac{-v_{Rn} y_{kn} + \sum_{j=1}^3 (y_{jn} - x_{jn}) \frac{\partial f_j}{\partial t_{ckn}}}{1+v_{Rn}} \quad (13)$$

Let us then define:

$$\theta_{kjn} = \sum_{i=1}^3 \frac{\partial D_{jn}}{\partial P_{in}} \left( \frac{\partial f_i}{\partial t_{ckn}} + \delta_{ik} \right). \quad (14)$$

The first order conditions could be transformed to matrix format:

$$\begin{bmatrix} \theta_{11n} & \theta_{12n} & \theta_{13n} \\ \theta_{21n} & \theta_{22n} & \theta_{23n} \\ \theta_{31n} & \theta_{32n} & \theta_{33n} \end{bmatrix} \begin{bmatrix} t_{c1n} - \frac{\lambda_n}{1+v_{Rn}} \\ t_{c2n} - \frac{\lambda_n}{1+v_{Rn}} \\ t_{c3n} - \frac{\lambda_n}{1+v_{Rn}} \end{bmatrix} = \begin{bmatrix} \frac{1}{1+v_{Rn}} \left\{ \sum_{i=1}^3 \left( (y_{in} - x_{in}) \frac{\partial f_i}{\partial t_{c1n}} \right) - v_{Rn} y_{1n} \right\} \\ \frac{1}{1+v_{Rn}} \left\{ \sum_{i=1}^3 \left( (y_{in} - x_{in}) \frac{\partial f_i}{\partial t_{c2n}} \right) - v_{Rn} y_{2n} \right\} \\ \frac{1}{1+v_{Rn}} \left\{ \sum_{i=1}^3 \left( (y_{in} - x_{in}) \frac{\partial f_i}{\partial t_{c3n}} \right) - v_{Rn} y_{3n} \right\} \end{bmatrix}. \quad (15)$$

This defines an equation system to the determination of  $t_{1n}$ ,  $t_{2n}$ , and  $t_{3n}$ . Let us now define  $J_n$  as the determinant of the matrix at the left in the equation system above and  $J_{jn}$  as the co-factor of element  $ji$  in this matrix. We then have an implicit taxation rule of a country with a binding emission constraint taking the other countries' behavior as given:

$$t_{ckn} = \frac{1}{1+v_{Rn}} \sum_{j=1}^3 \left( \left\{ \sum_{i=1}^3 (y_{in} - x_{in}) \frac{\partial f_i}{\partial t_{ckn}} \right\} \frac{J_{jkn}}{J_n} \right) - \frac{v_{Rn}}{1+v_{Rn}} \left( \frac{\sum_{j=1}^3 y_{jn} J_{jkn}}{J_n} \right) + \frac{\lambda_n}{1+v_{Rn}}. \quad (16)$$

This solution has its parallel in the optimal solution of the second best problem in Sandmo (1975) where the tax is a weighted average of two terms. The second term at the right hand side of (16) corresponds to the first term in Sandmo's formula, equation (23), and it represents the efficiency requirements familiar from the theory of optimal taxation in the presence of a public budget constraint. We see that if  $v_{Rn}=0$ , corresponding to the MEB being equal to zero, this term disappears. This corresponds to the case where the budget constraint is non-binding in Sandmo's analysis. The third term in (16), corresponding to Sandmo's second term, has its origin in the emission constraint and scales the taxes upwards according

to the emission coefficients. In Sandmo's model this term has its origin in an externality. This externality has its parallel in the emission constraint formulated here.

With high marginal costs of public funds the third term is relative small while the second term is high. In countries with high MEB the tax rates on fossil fuels could therefore differ substantially even though the actual country has implemented an efficient climate policy.

If the emission constraint is non-binding, i.e. where the taxation of fossil fuels from solely fiscal purposes is enough to accomplish the emission targets,  $\lambda_n$  is zero and the third term at the right hand side of (16) disappears.

The first term on the right hand side of (16) has no parallel in Sandmo's formula because trade was not included in his theoretical model. This term, however, tells us how the governments should adjust the fossil fuel taxes in order to take advantage of market power in the fossil fuel markets. This term disappears either if the country under consideration has zero net export of fossil fuels or if the prices in the fossil fuel markets are not affected by its fossil fuel taxes, or both.

### **3. Empirical illustrations**

The previous section derived the general properties of fossil fuel taxes in a country maximizing its net income under an emission constraint. This section provides some empirical illustrations of this optimal taxation rule.

We assume that the countries committed to reduce their emissions of CO<sub>2</sub> are the countries listed in Annex II in the Climate Convention<sup>3</sup>. We are treating the EU<sup>4</sup> as a single party. The other parties, or groups of parties, with emission reduction commitments are consequently Australasia<sup>5</sup>, Canada, Japan, Norway, Turkey and the USA. We analyze climate agreements that commit these parties and group of parties to equal percentage emission reductions.

We shall make the simplifying assumption that the countries that are not committed to reduce their emissions of CO<sub>2</sub> do not change their fossil fuel taxes or take other actions as a result of the climate agreement. Furthermore we assume that the fossil fuel markets are competitive.<sup>6</sup> The emission reductions are assumed to be brought about by the introduction of efficient taxation policies as in the Nash-equilibrium described in the previous section.

---

<sup>3</sup> This is identical to OECD as in 1992.

<sup>4</sup> Actually the aggregate of Switzerland and the EU.

<sup>5</sup> The aggregate of Australia and New Zealand.

<sup>6</sup> The assumption that the markets for oil and gas are competitive is of course a simplification. It is likely that several producers of these fuels acts strategically. How the numerical results of this and the next chapter might be altered if there are producers acting strategically could be the topic of further research.

There are global markets for oil and coal, while there due to high transportation costs are assumed to be three regional markets for gas, one in North America, one in Europe including Eastern Europe and Russia, and one in the Pacific region.

Both the demand curves and the supply curves are linear due to quadratic production- and cost functions. The level and slope of the curves are determined by the supply elasticity and by the point in the price plus tax-production spaces in 1993. I have followed Golombek et al. (1994) by assuming supply elasticities of coal equal to 4.0. Such relative high supply elasticities lead to small carbon leakage compared to several other studies such as Pezzey (1992) who used the model in Whalley and Wigle (1991). As in Golombek et al. (1995) the supply elasticities of oil and gas are equal to 0.75.

*Table 1 Assumed demand elasticities in the reference situation*

	$e_{11}$	$e_{12}$	$e_{13}$	$e_{21}$	$e_{22}$	$e_{23}$	$e_{31}$	$e_{32}$	$e_{33}$
USA	-0.50	0.03	0.03	0.10	-0.50	0.17	0.10	0.19	-0.50
Canada	-0.50	0.01	0.05	0.10	-0.51	0.15	0.19	0.07	-0.51
Mexico	-0.50	0.00	0.02	0.00	-0.50	0.03	0.11	0.01	-0.50
EU and Switzerland	-0.50	0.02	0.02	0.14	-0.50	0.15	0.10	0.11	-0.51
Norway	-0.50	0.00	0.00	0.00	-0.50	0.02	0.01	0.01	-0.50
Turkey	-0.50	0.00	0.00	0.02	-0.50	0.01	0.01	0.04	-0.50
Eastern Europe	-0.50	0.09	0.08	0.10	-0.51	0.18	0.05	0.10	-0.50
Australasia	-0.50	0.03	0.03	0.10	-0.50	0.08	0.15	0.15	-0.50
Japan	-0.50	0.01	0.01	0.07	-0.50	0.10	0.10	0.15	-0.50
ROW	-0.50	0.07	0.05	0.10	-0.50	0.05	0.15	0.11	-0.50

As in Golombek et al. (1994) and Golombek et al. (1995) the demand for fuels is derived from quadratic utility functions. The parameters of the utility functions are determined such that all direct price elasticities are -0.5. This is based on a view that the direct elasticities of -0.9 chosen in Golombek et al. (1994) and (1995) seems somewhat high even though we are considering long run elasticities, cf. Smith, Hall and Kyer (1995). The assumed cross price elasticities are listed in Table 1.

The public revenue is incorporated as a linearly separable variable in the production function with the derivative as a starting point set to 0.4.<sup>7</sup> This corresponds to assuming that the marginal excess burden of taxation is at this level in all the OECD countries or group of countries.

Table 2 *Production, consumption and taxation of fossil fuels in the base year (1993).*

	Production (Mtoe)			Consumption (Mtoe)			Taxes (USD/toe)*		
	Oil	Coal	Gas	Oil	Coal	Gas	Oil	Coal	Gas
USA	404.3	482.9	431.7	769.6	469.2	481.9	58.3	0.0	0.0
Canada	103.6	37.5	112.5	76.4	24.1	61.2	102.2	0.0	0.0
Mexico	158.7	3.1	24.2	87.6	4.1	25.2	-	-	-
EU	130.1	153.9	158.0	586.8	243.7	254.0	229.1	5.4	25.3
Norway	117.6	0.2	24.2	8.3	0.9	2.5	214.8	93.1	131.8
Turkey	4.0	11.6	0.2	27.7	16.3	4.2	166.3	0.0	0.0
E. Europe	415.6	402.8	665.6	328.3	383.0	587.3	-	-	-
A-NZ	29.7	120.7	25.3	38.7	38.8	19.3	155.4	0.0	0.0
Japan	0.9	4.0	1.9	255.9	76.8	47.7	117.6	0.0	0.2
ROW	1814.6	903.4	419.2	999.8	863.1	379.4	-	-	-

\* The sources are OECD (1995) and BP (1995) as far as production and consumption are concerned. The estimated average tax rates are taken from ECON (1995), which presents average fossil fuel taxes in the OECD countries from 1980 to 1994. The tax rates presented there are based on weighting energy taxes by product and sector. The information on taxes is based on IEA Energy Prices and Taxes. The information on taxes has been supplemented with EU's oil price statistics, 'Oil Bulletin' and with direct contact with national administrations. The weights are based on 'Basic Energy Statistics'. The Basic Energy Statistics have been supplemented with oil industry information and EU statistics on the use of leaded and unleaded gasoline and on the breakdown of heavy fuel oil according to sulphur content (relevant for countries differentiating heavy fuel oil taxes according to sulphur content). The calculation of the average taxation by sector takes into account the exempted use of energy within the sector. Concerning gasoline the taxes are for premium gasoline. Taxes for leaded and unleaded gasoline (where relevant) have been weighted with the consumption of the two qualities. For countries differentiating the tax between high and low sulphur, taxes are represented by the tax on the typical quality in industry and power generation.

Jorgensen and Yun (1993) have estimated the MEB to be 38 cents pr. dollar in the USA. However, estimates of the MEB vary substantially between different studies and are the subject of professional dispute, cf. Ballard and Fullerton (1992) and Brendemoen and Vennemo (1996). Hence sensitivity analysis will be carried out with respect to this assumption. Although the chosen value of the MEB might be high in the case of the US, it might be more appropriate for the European economies where marginal tax rates in general are higher than in the US.

The model framework does not allow taking variations in domestic fossil fuel taxes across sectors fully into account. The average tax rates of 1993 used in calibration are based on ECON (1995), cf Table 2.

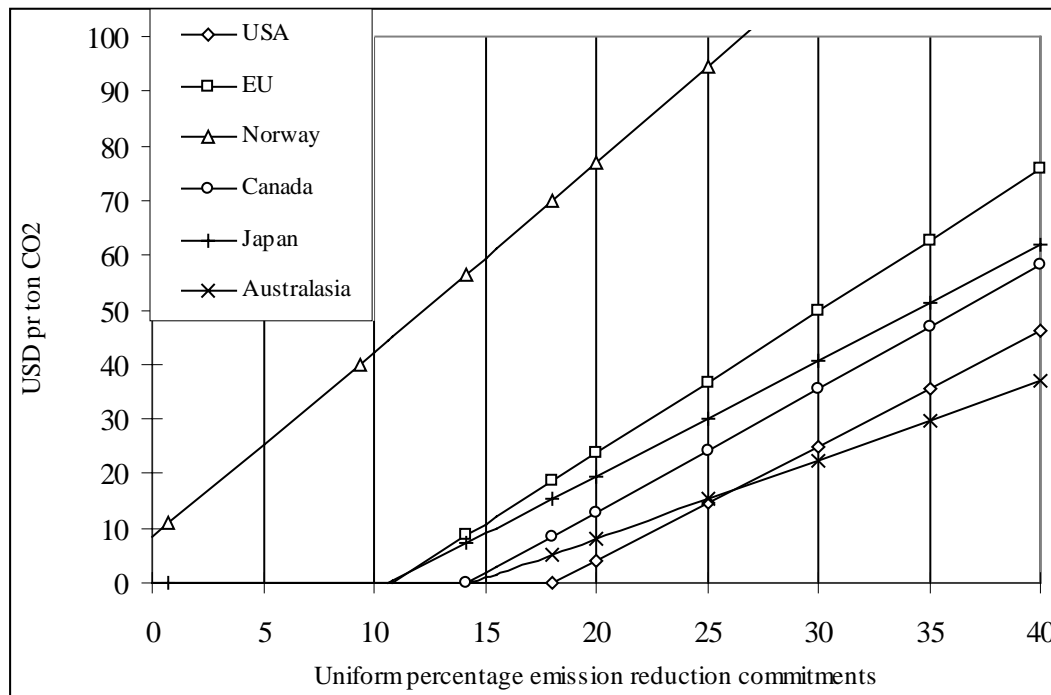
Table 2 also presents the numbers for production and consumption of fossil fuels in the OECD countries and group of countries. Norway is an exceptional case because of this country's large production of natural gas and oil relative to its consumption. Compared to the other OECD countries Norway is therefore a vulnerable party as far as repercussions

<sup>7</sup> Assuming the MEB to be a constant is of course a simplifying approximation. It is likely to be an inverse relationship between the revenue generated from fossil fuel taxes and the MEB of other taxes.

affecting the fossil fuel markets are concerned. The negative effects on the Norwegian economy from declining fossil fuel prices will be further exaggerated since the fossil fuel production in Norway, through taxation of resource rent and direct public ownership, represents a substantial part of the total public revenue. The model is able to take these elements of the Norwegian costs into consideration by the specification of a tax on profit in indigenous fossil fuel production.

Let us then turn to a numerical illustration of some possible consequences of a climate agreement committing the OECD countries to reduce their emissions of CO<sub>2</sub> on a flat rate basis. To understand the results it is important to keep in mind that we assume that the emission reduction commitments trigger a complete redesign of the national fossil fuel taxation patterns towards efficiency.

Figure 1 Shadow prices of the emission constraints.



With the numerical choice of the MEB=0.4 together with the efficiency rules built into the model it predicts substantial changes in fossil fuel taxes even if the emission constraints are non-binding. This redesign of the fossil fuel taxation patterns induces significant emission reductions. According to the model simulations we are in other words faced with ‘no regret’ options. The sizes of these no regret emission reductions are evident from Figure 1 in which the shadow prices of the emission constraints are presented. The largest no regret option is found in the USA where just the implementation of an efficient system of fossil fuel taxes

would reduce the CO<sub>2</sub>-emission by 17.1%, according to this numerical example. This is of course an example of a result that is very sensitive to the chosen functional forms and to the many empirical assumptions made. If for example the MEB instead is set to 0.15 the estimated no regret emission reduction in the United States is reduced to 7.6%. The corresponding results of the other countries are also sensitive to the assumed value of the MEB: The no regret emission reduction in Australasia<sup>8</sup>, which is estimated to be 14.5% in the case with the MEB set to 0.4, is for example reduced to 4.4% if the MEB instead is set to 0.15.

Figure 1 also shows that the estimated marginal abatement costs are higher in the EU than in Japan. This is partly due to the fact that the fossil fuel consumption is less carbon intensive in EU (a smaller share of coal and a bigger share of gas). At least as far as EU is concerned the somewhat higher fossil fuel taxes in the reference situation are factors of explanation. Due to the less carbon intensive fossil fuel consumption in Canada compared to the consumption patterns in the USA and Australasia, the marginal abatement costs are somewhat higher in Canada relative to the USA and Australasia. In Norway there are on the other hand no ‘no regret’ options according to the model simulation due to the high fossil fuel taxes in the reference situation in this country. This result could be questioned because of the abundant supply of cheap hydropower in this country that probably has lead to inefficient use of electricity. With the chosen model structure the ‘no regret’ options connected to more efficient use of the supplied electricity in this country and a corresponding reduction in fossil fuel consumption is not taken into account.<sup>9</sup>

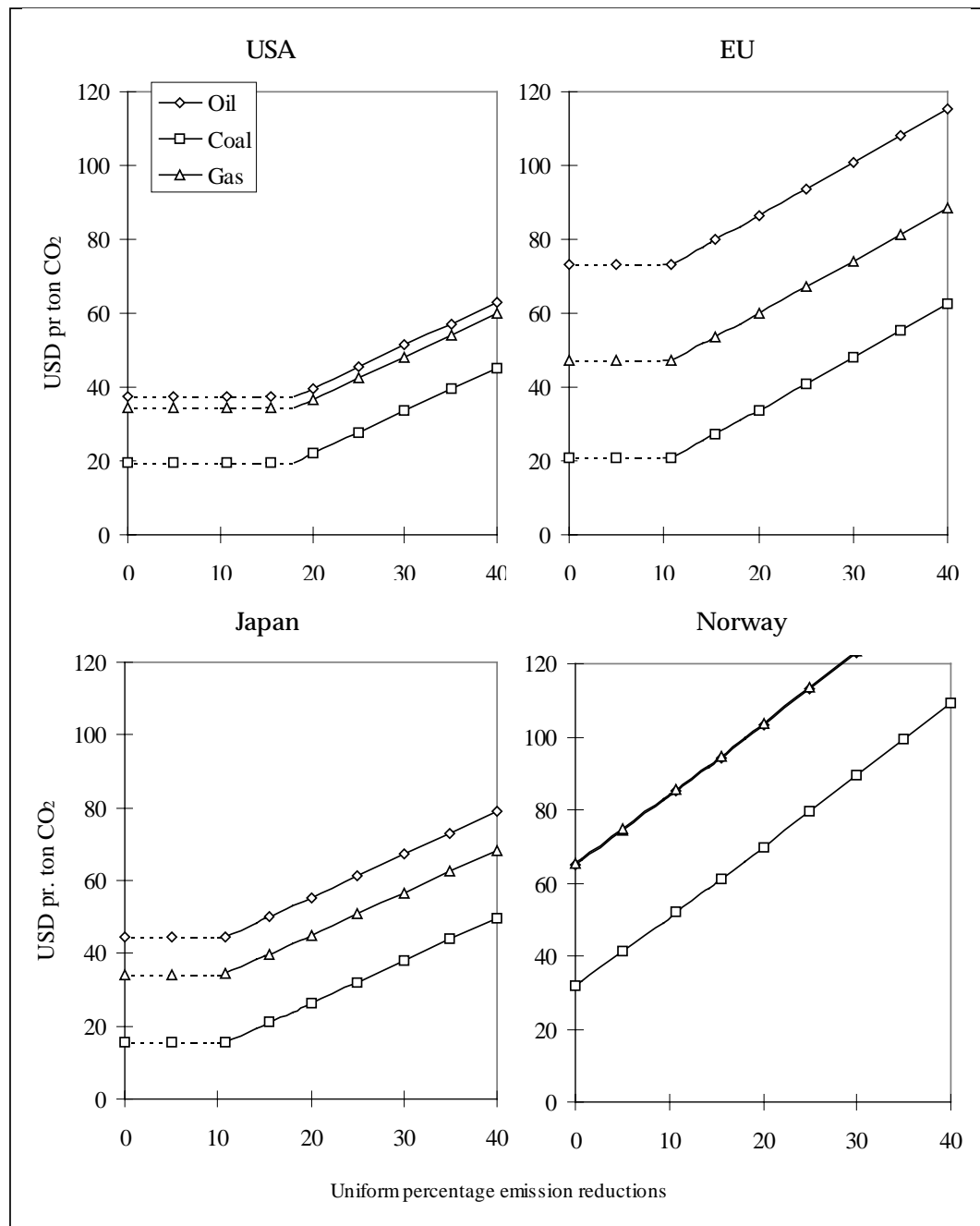
---

<sup>8</sup> Australia and New Zealand.

<sup>9</sup> A possible improvement of the model would be to include other energy sources, as hydro power and nuclear, in the production function of the numeraire good in (1). Then the model would have been able to capture the special situation of countries with intensive use of such ‘fuels’.



Figure 2 Fossil fuel taxes due to efficient implementation of climate agreements.

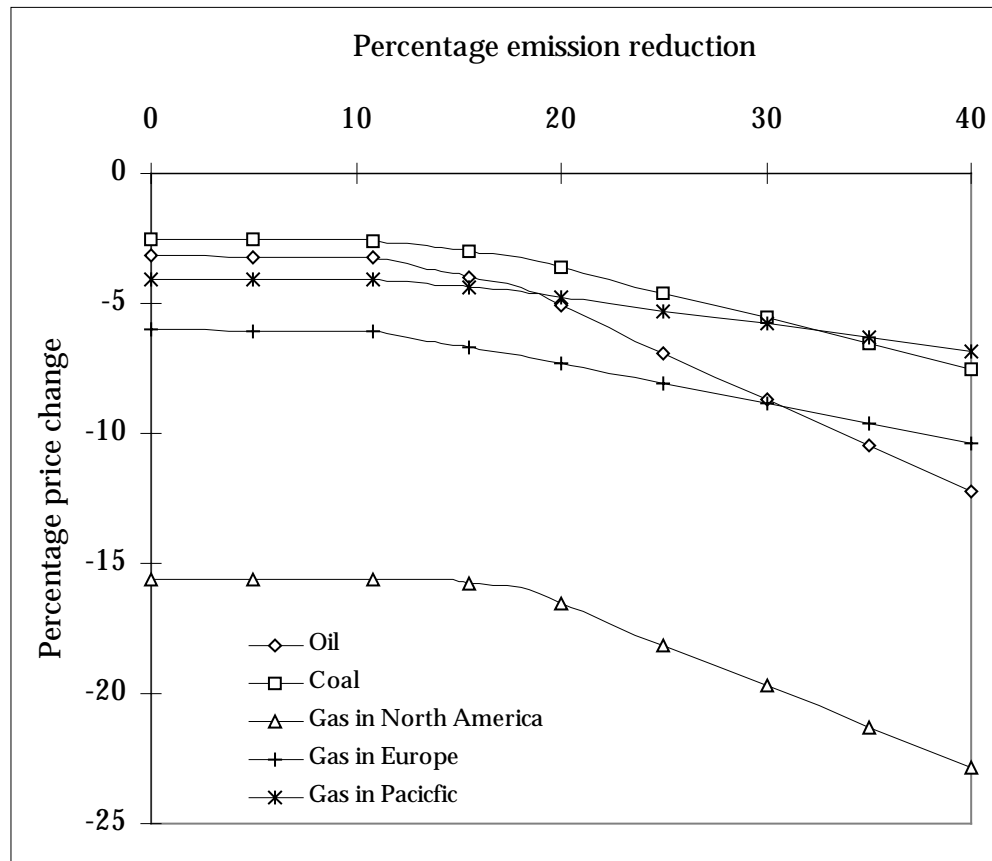


\* The curves are broken where the emission constraints are non-binding.

The development of the fossil fuel taxes along the emission reduction path is presented in Figure 2. The emission levels where the different countries' emission constraints start to be binding are apparent. When the emission constraints turn to be binding (the shadow price changes from zero value to a positive value, i.e.  $\lambda_n > 0$ ), an element proportional to the shadow price enters additively to the fossil fuel taxes, in agreement with equation (16). Because the chosen functional forms, with linear marginal cost and demand functions, lead to

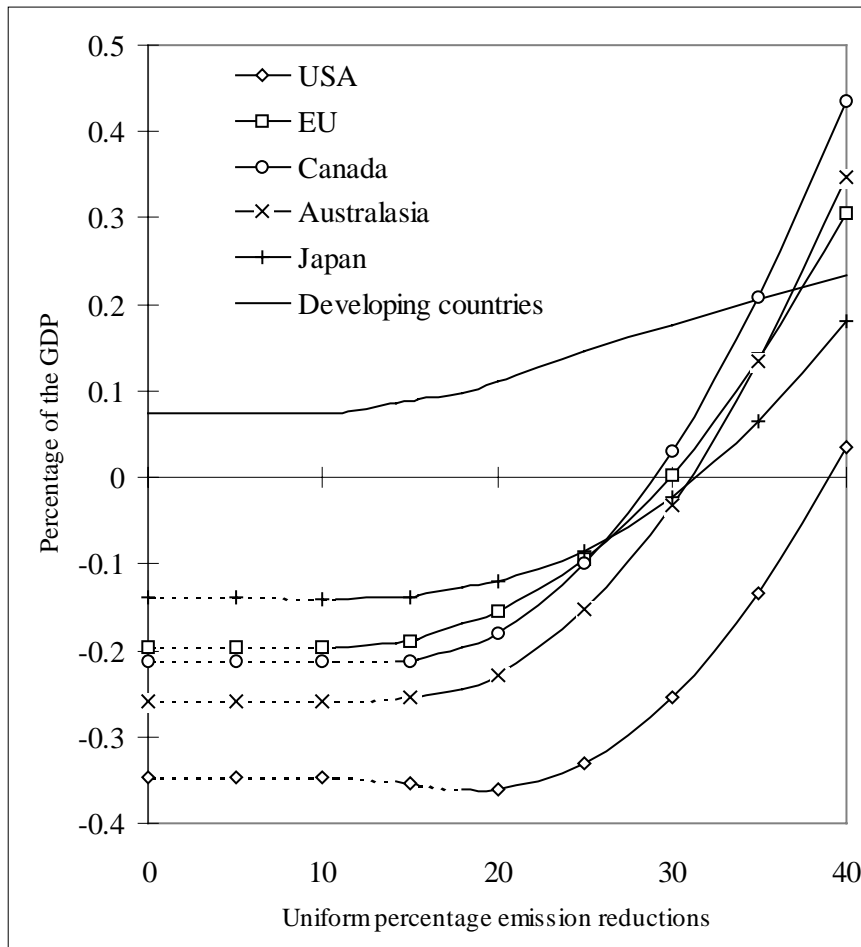
shadow prices increasing linearly with respect to increasing emission reduction commitments, the fossil fuel taxes also show such a response.

Figure 3 Price changes after cost-efficient implementation of climate agreement commitments.



The induced price changes in the fossil fuel markets are displayed in Figure 3. The redesign of the fossil fuel taxes in the OECD-countries induced by the models' efficiency requirements is followed by falling prices in the fossil fuel markets. The price drop is particularly large in the North-American gas market. This result is due to the fact that this market mainly is limited to countries with emission reduction commitments, while the other fossil fuel markets are not. The price drop caused by a higher tax on natural gas in for example EU will be weakened by increased demand for gas in Eastern Europe and Russia. This offsetting effect is considerably weaker when the USA imposes higher tax on natural gas in the North-American market, due to the relative small impact of the increased demand for gas in Canada as the gas price declines. The resource rents transferred from Canada to the USA, as a result of the tax increase in the USA, do, however, also partly explain this result.

Figure 4 Income losses. Climate agreements with flat rate emission reductions.\*



\* Except in the case of ROW, where there are no emission constraints, the curves are broken where the emission constraints are non-binding.

The distribution of net costs is summarized in Figure 4. The numbers concerning Norway are not included in this figure because it would require a scale of the vertical axis inappropriate to the other countries. The cost curve of Norway is in the neighborhood of 0.7 on the vertical axes when the emission reduction is in the range 0 - 10%. Thereafter the curve rises sharply and reaches 3.5% of the GDP at 40% emission reduction commitments. These income losses are caused by the price fall in the oil market and in the market for natural gas in Europe that follow from the introduction of efficient fossil fuel taxation in the OECD-countries and the increased fossil fuel taxes as emission reduction commitments are increased.

According to the model simulation the USA experience net gains as long as the emission reduction commitments are below 37%. This is due to several circumstances, among other things the considerable net import of oil and gas. The estimated price fall in the North American gas market is important in that respect. Another factor to explain this result is of

course the relative low fossil fuel taxes in the reference situation that mean small direct abatement costs, especially when benefits from revenue recycling are considered. According to the simulation the USA could reduce its emissions by 17% simply by the introduction of a more efficiently designed fossil fuel taxation system. One should at this point remark that the assumed MEB in USA equal to 0.4 could be high taking the relatively low marginal taxes in the US into consideration. If the MEB is reduced to 0.3 in the US only, while kept unaltered in the other countries, the cost curve of the US is slightly above the cost curve of Australasia, for emission reduction commitments below 15% and slightly above the cost curve of Japan when the emission reductions are larger than 35%.

The somewhat elevated cost curve of the EU to some extent reflects the high fossil fuel taxes in the reference situation, which also is reflected in the shadow price of the emission constraint, cf. Figure 1. Other factors, such as the carbon intensity in the reference situation, also play a role here.

Canada has a relatively steep cost curve according to the simulation. This is due to a terms-of-trade loss and high domestic abatement costs due to low coal consumption and high gas consumption in the reference situation.

The developing countries are treated as an aggregate in the model. According to the simulation described so far this group will experience an income loss when a climate agreement of the type discussed here is implemented. This is due to the terms of trade loss following from decreasing fossil fuel prices and reduced export of oil and coal to the developed countries.

As pointed out above, the point estimate of the MEB set to 0.4, is a crucial assumption in the analysis. It is therefore important to take into account that this estimate is especially uncertain. Consequently, sensitivity analyses with respect to this assumption are presented in Appendix B.

#### **4. Concluding comments**

A possible outcome of the climate negotiations is an agreement that commits at least the Annex II countries of the Climate Convention to certain CO<sub>2</sub>-emission reduction commitments. The purpose of the paper is to shed light on the countries rational reactions to such an agreement. The study is part of a project with the aim to understand the countries' positions in the climate negotiations and to make predictions about likely negotiation results. Pivotal in the paper is the rule developed in section 2 for optimal taxation of fossil fuel

consumption in a country subject to a CO<sub>2</sub>-emission constraint. A basic point here is the model framework that enables us to take into account the marginal excess burden of taxation and the benefits from revenue recycling. Furthermore, the governments are assumed to take into account that other governments are committed to take action and are mutually affected by these actions through their links to the regional and global fossil fuel markets. Their degrees of market power in the fossil fuel markets are also assumed to be taken into consideration. The taxation rule expands and adjusts the taxation rule developed in Sandmo (1975).

The formula was applied in some numerical examples presenting estimates for different countries, and group of countries of total costs of a climate agreement. This was done in a model framework where the national governments are assumed to take both benefits from revenue recycling, prior distortions from fossil fuel taxes and strategic behavior in relation to the fossil fuel markets into account.

The total costs of a climate agreement are estimated to be negative or small for limited emission reductions, but to increase rapidly when the emission reductions exceed certain limits. This, however, mainly represents a confirmation of results of other model studies. One contribution of the empirical part of the present paper is nevertheless to present these simultaneous abatement cost curves taking among other factors both benefits from revenue recycling and the governments' market power in the fossil fuel markets into account.

The sensitivity analyses performed, and presented in further detail in Appendix B, show that knowledge about the magnitude of the different countries MEB are likely to be essentially when studying the different governments' interests in the climate negotiations. In future work with the theoretical and empirical framework applied in the present paper an improved empirical basis for the size of the different countries MEB should therefore be emphasized. This is however a difficult task, because the empirical basis for the estimated marginal excess burden of taxation is relatively weak and the subject of professional dispute.

## References

- Ballard, Charles L. and Fullerton, Don: Distortionary Taxes and the Provision of Public Goods. *Journal of Economic Perspectives* 6, 117-131, 1992.
- Berg, Elin, Kverndokk, Snorre and Rosendahl, Knut Einar: *Market Power, International CO<sub>2</sub> Taxation and Petroleum Wealth*. Discussion Paper no. 170 Research Department, Statistics Norway, 1996.
- Brendemoen, Anne, and Vennemo, Haakon: The Marginal Cost of Public Funds in the Presence of Environmental Externalities. *Scandinavian Journal of Economics* 98, 405-422. 1996.
- British Petroleum: *Statistical Review of World Energy*, June, 1995.
- Burniaux, Jean-Marc, Martin, John P., Nicoletti, Guiseppe and Oliveira Martins, Joaquim: *The Costs of Reducing CO<sub>2</sub> Emissions. Evidence from GREEN*. Working Paper No. 115, Economics Department, OECD, Paris, 1992.
- DFAT and ABARE: *Global climate change - Economic dimensions of a co-operative international policy response beyond 2000*. Australian Bureau of Agricultural and Resource Economics and Department of Foreign Affairs and Trade, Canberra, 1995.
- ECON: *Energy Taxes in the OECD*. ECON-report no. 332, 1995.
- Ekins, Paul: Rethinking the Costs Related to Global Warming. A Survey of the Issues. *Environmental and Resource Economics* 6: 231-277, 1995.
- Golombek, Rolf and Bråten, Jan: Incomplete International Climate Agreements. Optimal Carbon Taxes, Market Failures and Welfare Effects. *The Energy Journal* 15, 141-165, 1994.
- Golombek, Rolf, Hagem, Cathrine and Hoel, Michael: Efficient Incomplete International Climate Agreements. *Resource and Energy Economics* 17, 25-46, 1995.
- Håkonsen, Lars: *Optimal commodity taxation with binding CO<sub>2</sub> restriction*. Discussion Paper 5/95, Norwegian School of Economics and Business Administration, 1995.
- IEA: *Energy Prices and Taxes*. OECD/IEA, Paris, 1995.
- Jorgensen, Dale W. and Wilcoxon, Peter: Reducing US Carbon Emissions. An Econometric General Equilibrium Assessment. *Resource and Energy Economics* 15, 7-15, 1993.
- Jorgenson, Dale W. and Yun, K.: The Excess Burden of Taxation in the US, in Alberto Heimler and D. Meulders (eds.) *Empirical approaches to fiscal policy modelling*. International Studies in Economic Modelling, no. 13. London, pages 9-24, 1993.
- Nordhaus, William D. and Yang, Zili: A Regional Dynamic General-Equilibrium Model of Alternative Climate-Change Strategies. *The American Economic Review* 86. 741-765, 1996.

- OECD: *Energy Balance of OECD-countries 1992-1993*. OECD/IEA, Paris, 1995.
- Pezzey, John: Analysis of unilateral CO<sub>2</sub> control in the European Community. *The Energy Journal* 13, 159-172, 1992.
- Rosendahl, Knut Einar: *Carbon Taxes and the Petroleum Wealth*. Discussion Paper 128, Statistics Norway, 1994.
- Sandmo, Agnar: Optimal taxation in the presence of externalities. *Swedish Journal of Economics* 77, 86-98, 1975.
- Smith, Clare, Hall, Stephen, and Mabey, Nick: Econometric modelling of international carbon tax regimes. *Energy Economics* 17, 133-146, 1995.
- Torvanger, Asbjørn, Terje Berntsen, Jan Fuglestedt, Bjart Holtmark, Lasse Ringius, Asbjørn Aaheim: *Exploring Distribution of Commitments - A Follow-up to the Berlin Mandate*, CICERO Report 3, 1996.
- Varian, Hal: *Microeconomic Analysis*, New York - London, 1984.
- Whalley, John and Wigle, Randall: The International Incidence of Carbon Taxes. In Rudiger W. Dornbush and James M. Poterba eds.: *Global Warming: Economic Policy Responses*, Cambridge, 1991.



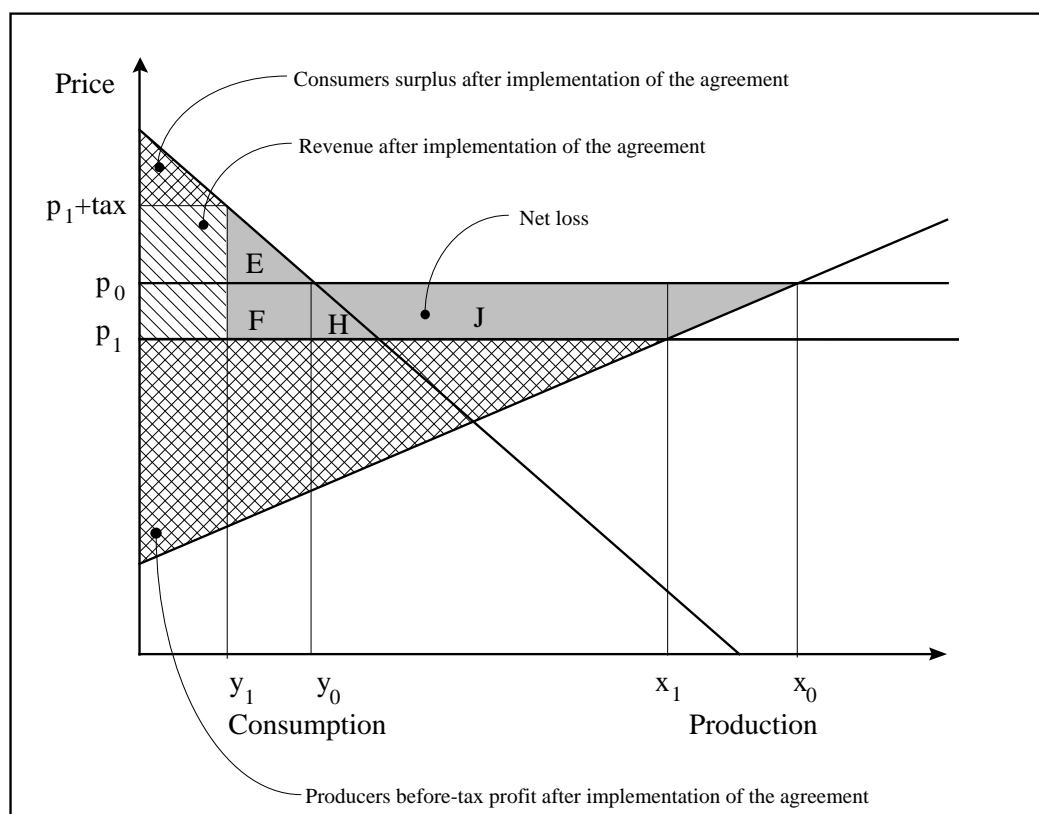


## Appendix A The distribution of costs and benefits

This appendix provides a theoretical overview of some costs and benefits that will follow from a climate agreement. The appendix might be useful to economists not familiar with economics related to climate agreements.

Assume that there is only one type of fossil fuel. The world market price is  $p_0$  in an equilibrium established before the implementation of a climate agreement. We also assume that the climate agreement commits a number of countries to reduce their *demand* for the fossil fuel. Suppliers of the fossil fuel are on the other hand not committed to take any specific actions to reduce the supply. For simplicity, assume that a rising supply curve characterizes the supply of the fossil fuel to the world market.

*Figure A.1 The costs and benefits from a climate agreement in the case of a net exporter of fossil fuels.*



Consider a country with a net export of fossil fuels, but which is so small that we can ignore its power in the fossil fuel market. Figure A.1 captures the situation of the country under consideration. The horizontal line at the price level  $p_0$  should be interpreted as the non-domestic supply of the fossil fuel before the implementation of the climate agreement. For simplicity let us assume that there are no taxes on production or consumption of fossil fuel in this country before the climate agreement is implemented. The upward sloping line represents

this country's total production of fossil fuels at different price levels, while the downward sloping line represents the corresponding domestic demand. Consequently, the domestic consumption of the fossil fuel is  $y_0$ , while the indigenous production is  $x_0$ .

The net export is then the distance  $(x_0 - y_0)$ . If the supply curve represents the domestic marginal cost curve, the area below this curve is an indicator of the extraction costs. Because the producers' total income is  $p_0x_0$ , the triangle limited by the vertical axis, the price line  $p_0$  and the supply curve represents the producers' before-tax profit. Correspondingly, the area limited by the price level ( $p_0$ ), the vertical axis and the demand curve represents the consumers' surplus. Ultimately, the sum of these two triangles could be used as an indicator of this country's net benefit from consumption and production of fossil fuels before the implementation of the climate agreement.

Because the supply curve is upward sloping the demand reduction brought about by the climate agreement will cause a fall in the price of fossil fuels on the world market.<sup>1</sup> Hence, in the equilibrium established after the implementation of the climate agreement, the fossil fuel price is  $p_1$ . The consumers' surplus is reduced to the upper crosshatched triangle and the producers' profit is reduced to the lower crosshatched area. The hatched rectangle represents the public revenue from the fossil fuel taxation. Thus, the country's total net benefit from the production of fossil fuels after the implementation of the climate agreement is reduced to the sum of the hatched and crosshatched areas. The sum of the areas E, F, and H represents the dead-weight loss of the fossil fuel tax and the area J represents this country's net income loss from a lower fossil fuel price in the world market.<sup>2</sup> Hence, the climate agreement has caused a net income loss to this country, with the loss corresponding to the size of the shaded areas (E+F+H+J).

Figure A.2 describes the corresponding case of a net importing country that is also without significant market power.<sup>3</sup> The net benefit from the production and consumption of fossil fuels in this country is the sum of the hatched and cross-hatched areas plus the triangle I, where the triangle *below* the price line  $p_0$  represents the producers' profit and the triangle above represents the consumers' surplus.

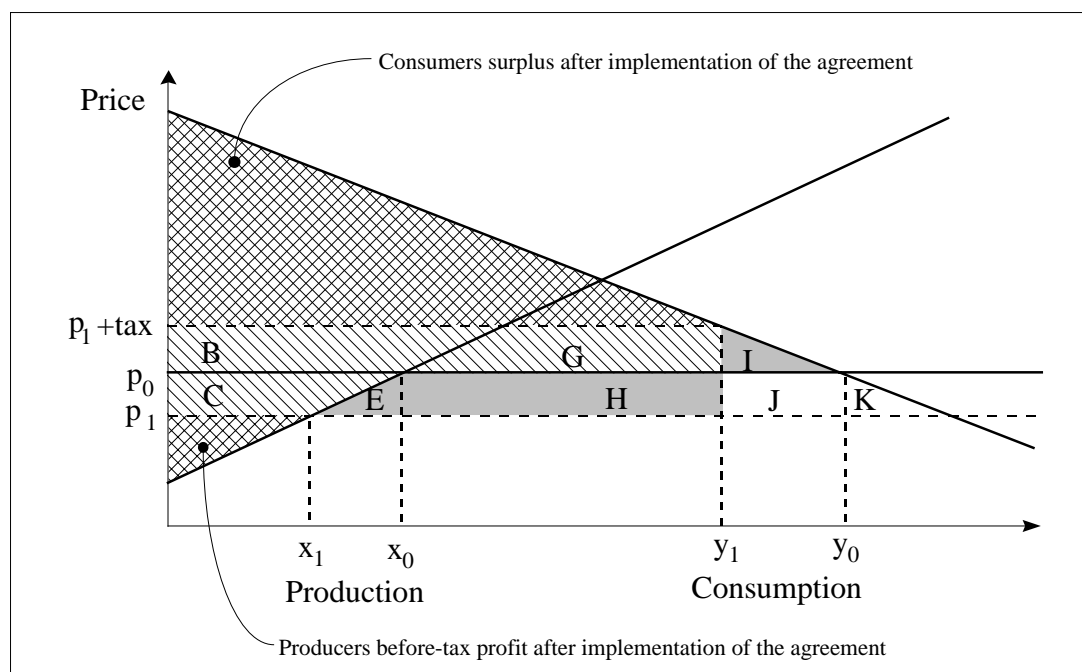
---

<sup>1</sup> If the suppliers are behaving strategically and are forward looking, it is not quite obvious that such a climate agreement will cause an immediate price fall. For a further discussion, see Rosendahl (1994) and Berg et al. (1996).

<sup>2</sup> It is of course definitely not clear that this fossil fuel tax causes a dead-weight-loss because the tax is introduced to correct for an externality. Hence, if the tax is equal to the a discounted marginal damage cost, the introduction of the fossil fuel tax removes a dead-weight loss rather than introducing one. From a short-sighted, national point of view the term 'dead-weight-loss' could, however, be used as an approximation, cf. the advantages of being a free rider while other countries reduce their greenhouse gas emissions.

<sup>3</sup> The scales of the axes in Figure A.1 and Figure A.2 should be interpreted as different.

Figure A.2 An illustration of the costs and benefits from a climate agreement in the case of a fossil fuel net importer without significant market power.



Let us now assume that this country is committed to reducing its consumption of fossil fuels from  $y_0$  to  $y_1$ . As in the case of the net exporter of fossil fuels this is brought about by the introduction of a tax on consumption. Contrary to the case of the net-exporting country it is not clear whether the importing country will experience a net gain or loss from the implementation of the climate agreement. Due to the price decline in the market for fossil fuels the producers' profit is reduced to the lower crosshatched triangle, and the consumers' surplus is reduced to the upper crosshatched triangle. However, we should include the rectangle containing B, C, E, G, and H on the income side because it represents the revenue from taxation of fossil fuel consumption. Consequently, whether the country will experience a net gain or a net loss depends on whether the area I is larger than the sum of E and H. If the price falls and the net import is relatively large, we are talking about a substantial terms-of-trade gain and E and H will be large. On the other hand, if the domestic demand for fossil fuels is relatively inelastic, for example due to few possibilities for substitution, the area I will be large.

In the above comments to Figure A.1 and Figure A.2 some important components in the complete set of costs and benefits of a climate agreement are ignored in order to simplify the discussion. In the numbered paragraphs below some comments on these components are given:

1. According to the previous discussion, increased dead-weight loss should be expected as a cost element of a climate policy. However, this is a result of the simplifying assumption that there is only one type of fossil fuel. In reality there are several, ranging from coal to gas, and these are again applied in several different sectors and in different qualities. There are however few reasons for assuming that the fossil fuel taxes are efficiently designed in the first place.<sup>4</sup> In the OECD the typical taxation pattern currently is characterised by heavily taxation of petrol while most other oil products and coal and gas are not as heavily taxed, if taxed at all. In that case, the total dead-weight loss will not necessarily be increased when a cost-effective climate policy is implemented. Examples of this are presented in section 3.
2. If the emission reduction commitments are met by the use of fossil fuel taxes, public revenue will be affected. Increased revenue enables the governments to reduce other taxes ('recycle' the revenue) and, consequently, reduce the dead-weight loss from traditional taxation. Consequently, the revenue generated by the implementation of a climate policy should be seen as an element that reduces the costs of the climate policy. The higher the marginal excess burdens of taxation in general, the more weight should be given to the revenue generating effect.<sup>5</sup> As an example, let us reconsider Figure A.2 in the case where the excess burden of taxation is 0.5. The implementation of the climate policy in this country generates an sum of revenue equal to the size of the rectangle containing B+C+G+E+H, which we denote R. This revenue could be 'recycled' in order to reduce other distortionary taxes in the economy. Under our assumptions such a tax reduction would increase the efficiency in the economy and thereby the total value of the production by half the rectangle R. This means that the country will experience a net loss only if the area I minus the area E+H is larger than half of the rectangle R.
3. The implementation of a climate policy in the OECD countries will not only alter the terms-of-trade in the fossil fuel markets. The increased energy prices (paid by consumers) will be reflected in increased prices of products produced by intensive use of energy; for example, iron, steel, and non-ferrous metals such as aluminium. These effects, together with general changes in demand and supply patterns caused by the relative price changes,

---

<sup>4</sup> In this case "efficiently" is interpreted as what is efficient from a narrow, short-term national point of view, ignoring the climate change externality of fossil fuel combustion.

<sup>5</sup> Taxes distort behaviour of households and firms. With the exception of taxes that correct for external effects such as environmental harm, such distortions generate lower efficiency and consequently reduced national income. The marginal excess burden of taxation (MEB) is a short expression for costs in the form of reduced national income from a marginal increase of public revenue brought about by increased taxes. There is a vast amount of literature on MEB with the estimates varying between 0.0-1.0. One half is a relative high point estimate according to the literature. See for example Ballard et al. (1985) and Jorgensen and Wilcoxon (1993). The MEB of taxation is equal to marginal costs of public funds minus 1.

will alter terms-of-trade in several other directions than those mentioned above. Some countries will experience net terms-of-trade improvements from this, while others will experience deteriorated terms-of-trade. These secondary terms-of-trade effects further modify the burden sharing consequences in relation to the simplified schemes used in the illustration above.

In addition to these three points it should also be underlined that the levels of the fossil fuel taxes in the reference situation are fundamental for the magnitude of the costs of the implementation of a country's commitments. The importance of the level of fossil fuel taxes in the reference situation could be illustrated by a more formal presentation of the model used in Figure A.1 and Figure A.2. Let us assume that a welfare indicator of a country with no indigenous production is written:

$$W(y) = u(y) + f(ty) - py \quad (\text{A.1})$$

where  $y$  is fossil fuel consumption,  $t$  is the tax rate and  $p$  is the world market price of the fossil fuel. The first term at the right hand side represents the gross welfare from consumption of fossil fuels, the second term the excess benefits from revenue recycling, the third term represents the import bill of the country.

Assume furthermore that the consumers are price takers and that the following standard condition is satisfied:

$$u'(y) = p + t \quad (\text{A.2})$$

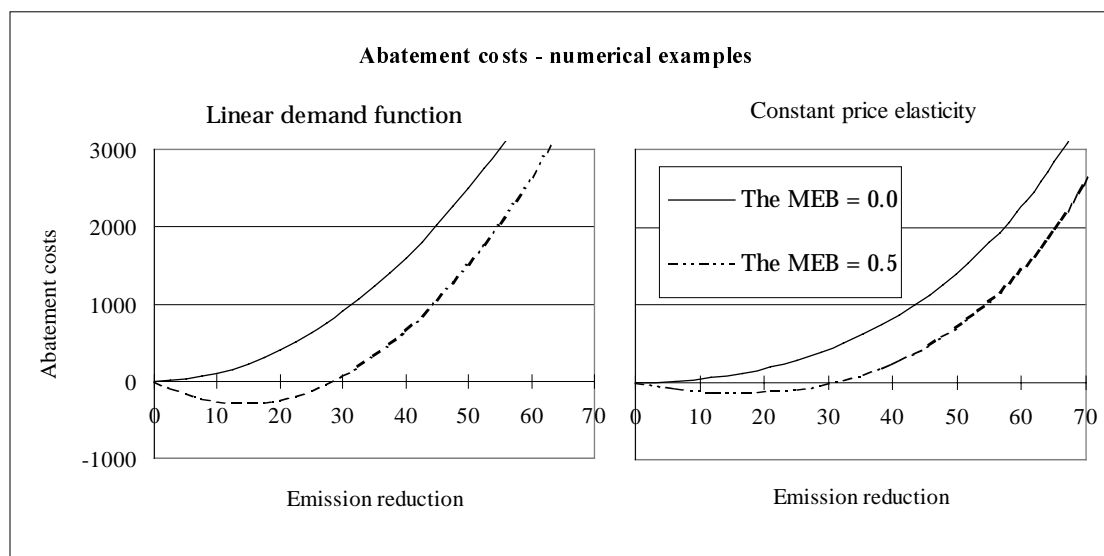
Let us now assume that the amount of fossil fuels consumed is determined by an international climate agreement. Then the tax rate  $t$  is a function of  $y$  and it follows from (2):

$$\frac{dt}{dy} = u''(y) \quad (\text{A.3})$$

Let us for the sake of the discussion assume that the price of the fossil fuel,  $p$ , is invariant to the emission abatement. We define the emission abatement  $A$  as the difference between the fossil fuel consumption in a reference situation (denoted  $y_0$ ) and  $y$ , that is  $A = y_0 - y$ . Furthermore we define the abatement cost function  $C(A) = W(y_0) - W(y_0 - A)$ . Using both (A.2) and (A.3) the marginal abatement cost could be expressed as:

$$C'(A) = t + [t + (y_0 - A)u''(y_0 - A)]f'(y_0 - A) \quad (\text{A.4})$$

Figure A.3 Two numerical examples to illustrate abatement cost patterns. The left diagram is based on a linear demand function and a price elasticity of  $-0.5$  in the reference situation. The right diagram is based on a demand function with a constant price elasticity of  $-1.3$ .



If the Marginal Excess Burden of taxation (MEB) is zero, that is, if  $f'(y) = 0$ , then the marginal abatement cost is simply equal to the fossil fuel tax rate  $t$ , cf. (A.4). If, on the other hand  $f'(y) > 0$ , the marginal abatement costs are affected by the revenue changes brought about by the fossil fuel tax increases, cf. the second term at the right hand side of (A.4). The abatement cost function is illustrated by two numerical examples in Figure A.3. The demand functions are derived from two quite different utility functions in order to illustrate the more general rule that the marginal increase in dead-weight loss from increasing tax rate is higher, the higher the tax is in the first place.

In both the two numerical examples illustrated in Figure A.3, the function  $f(ty)$  in (A.1) is assumed to be linear with  $f'(ty) = 0$  in the cases with the solid curves and  $f'(ty) = 0.5$  in the cases with broken curves.

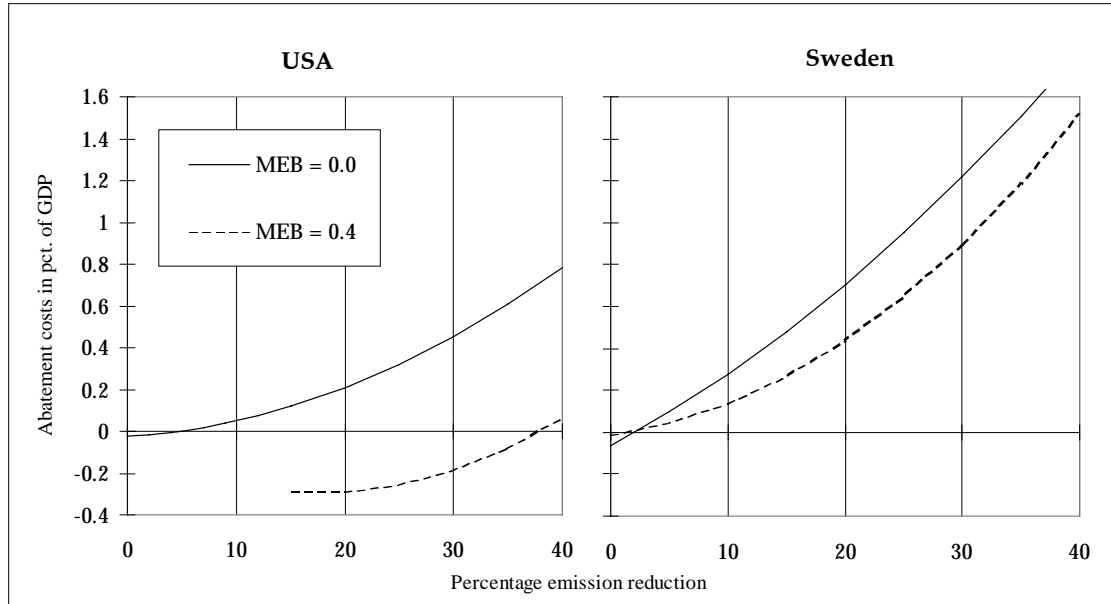
In the left diagram the utility function is assumed to be:

$$u(y) = \alpha_1 y - \alpha_2 y^2, \quad \alpha_i > 0, \quad i = 1, 2. \quad (\text{A.5})$$

From (A.2) we then have the following linear demand function:

$$y = \frac{\alpha_1 - (p+t)}{2\alpha_2} \quad (\text{A.6})$$

Figure A.4 Abatement costs as a percentage of GDP in the USA and Sweden assuming unilateral actions.



It is assumed that  $\alpha_1 = 300$  and  $\alpha_2 = 1$ . The fossil fuel price  $p$  is kept constant at 100.

In the right diagram the utility function is assumed to be:

$$u(y) = \beta y^\alpha, \beta > 0, 0 < \alpha < 1. \quad (\text{A.7})$$

From (A.2) we then have the following demand function with constant demand elasticity:

$$y = \left( \frac{(p+t)}{\beta\alpha} \right)^{\frac{1}{\alpha-1}} \quad (\text{A.8})$$

It is assumed that  $\alpha = 0.23077$  and  $\beta = 14971.1$ . The fossil fuel price  $p$  is kept constant at 100. The demand elasticity is equal to  $1/(\alpha - 1)$ .

The modifications made in point 1 and 2 could also be illustrated by numerical examples produced by simulations of a disaggregated version of the model used in section 3, cf. Torvanger et al. (1996). We use this model to estimate the costs that Sweden and the USA will experience if they take unilateral actions to reduce their CO<sub>2</sub> emissions. It is assumed that the fossil fuel taxation system in connection to this action is redesigned in an efficient way and that these taxes are set at a level necessary to reach the specified emission reduction

target.<sup>6</sup> The results are presented in Figure A.4. Let us first have a look at the solid lines that represent the case where the MEB is assumed to be zero, i.e. the welfare loss when benefits from revenue recycling are assumed not to occur. The simulations indicate that the USA could reduce its emissions of CO<sub>2</sub> by about 5% without any welfare loss because more efficiently designed fossil fuel taxes would give rise to some benefits. The corresponding figure in the case of Sweden is only about 2%. The more rapid increase in the welfare loss in Sweden is due to the high tax level in this country in the first place that gives rise to a higher marginal dead-weight loss, cf. the comments to Figure A.3.

The broken lines represent the welfare losses in the case where the MEB is assumed to be 0.4, i.e. substantial benefits from revenue recycling are assumed to follow if the climate policies give rise to increased public revenue. Notice that the broken line in the left chart starts at 15%. Hence, according to these numerical examples just the implementation of an efficient taxation of fossil fuels under the specified assumptions means a 15% reduction of emissions of CO<sub>2</sub> in the USA. Further reductions in the USA imply reduced welfare compared to the efficient taxation situation, but the emissions could be reduced by slightly more than 35% in the USA without any net loss. The situation is quite contrary in Sweden where the marginal abatement costs are high due the relatively efficiently designed and high fossil fuel taxes in the reference situation. For further details about this numerical example, cf. Torvanger et al. (1996).

Summarized, the numerical examples presented in Figure A.4, illustrate first of all how sensitive estimated abatement costs are to assumptions made about the size of the benefits from revenue recycling. Secondly, they illustrate the importance of the level of the fossil fuel taxes in the reference situation.

---

<sup>6</sup> The countries maximise their welfare function subject to emission constraints. It should be underlined that the model used is developed in order to analyse and compare the *relative* abatement costs in different countries, and how such cost estimates are sensitive to different empirical assumptions. The absolute abatement cost estimates presented here in isolation should therefore be interpreted with care.



## Appendix B Sensitivity analysis

Figure B.1 Sensitivity of marginal abatement costs (the shadow prices of the emission constraints) with respect to different estimates of the MEB. A climate agreement with 30% flat rate emission reductions.

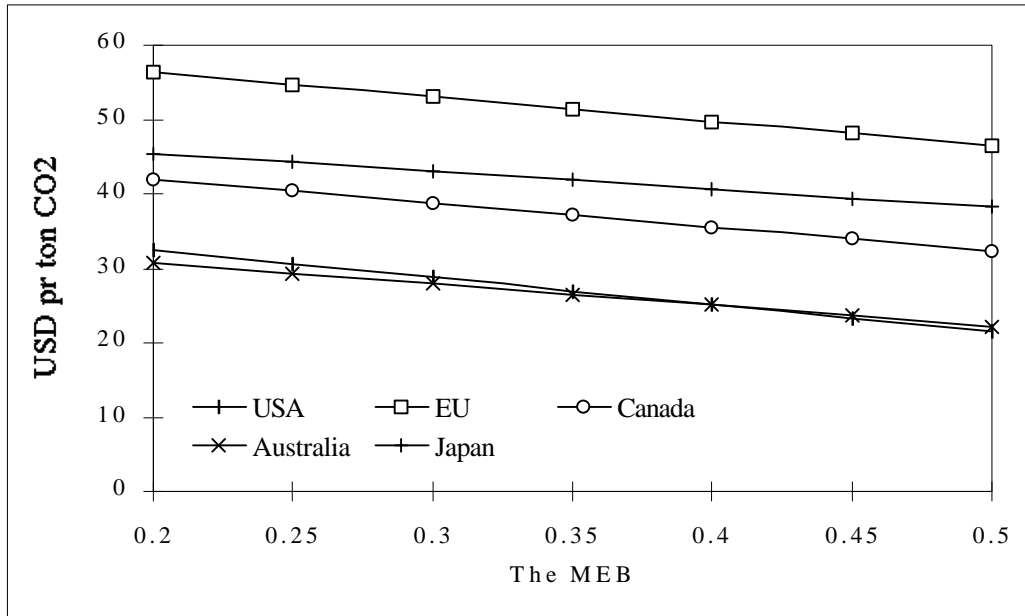
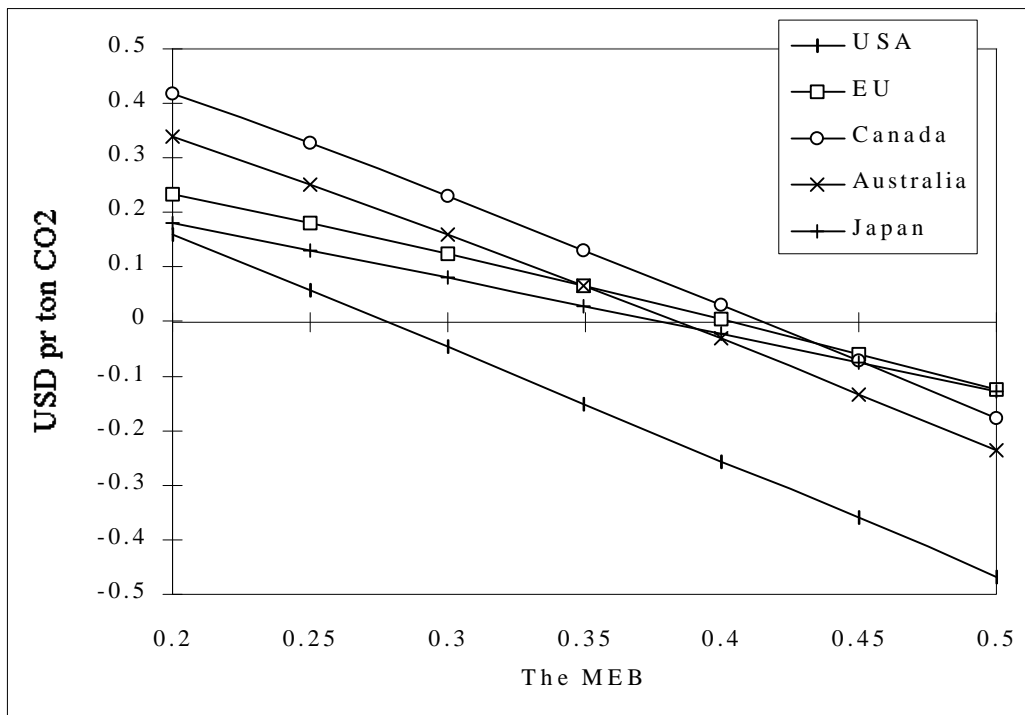


Figure B.2 Sensitivity of income losses with respect to different estimates of the MEB. A climate agreement with 30% flat rate emission reductions.



As pointed out above, the point estimate of the MEB set to 0.4 in the numerical examples presented in section 3, is a crucial assumption in the analysis. It is therefore important to take

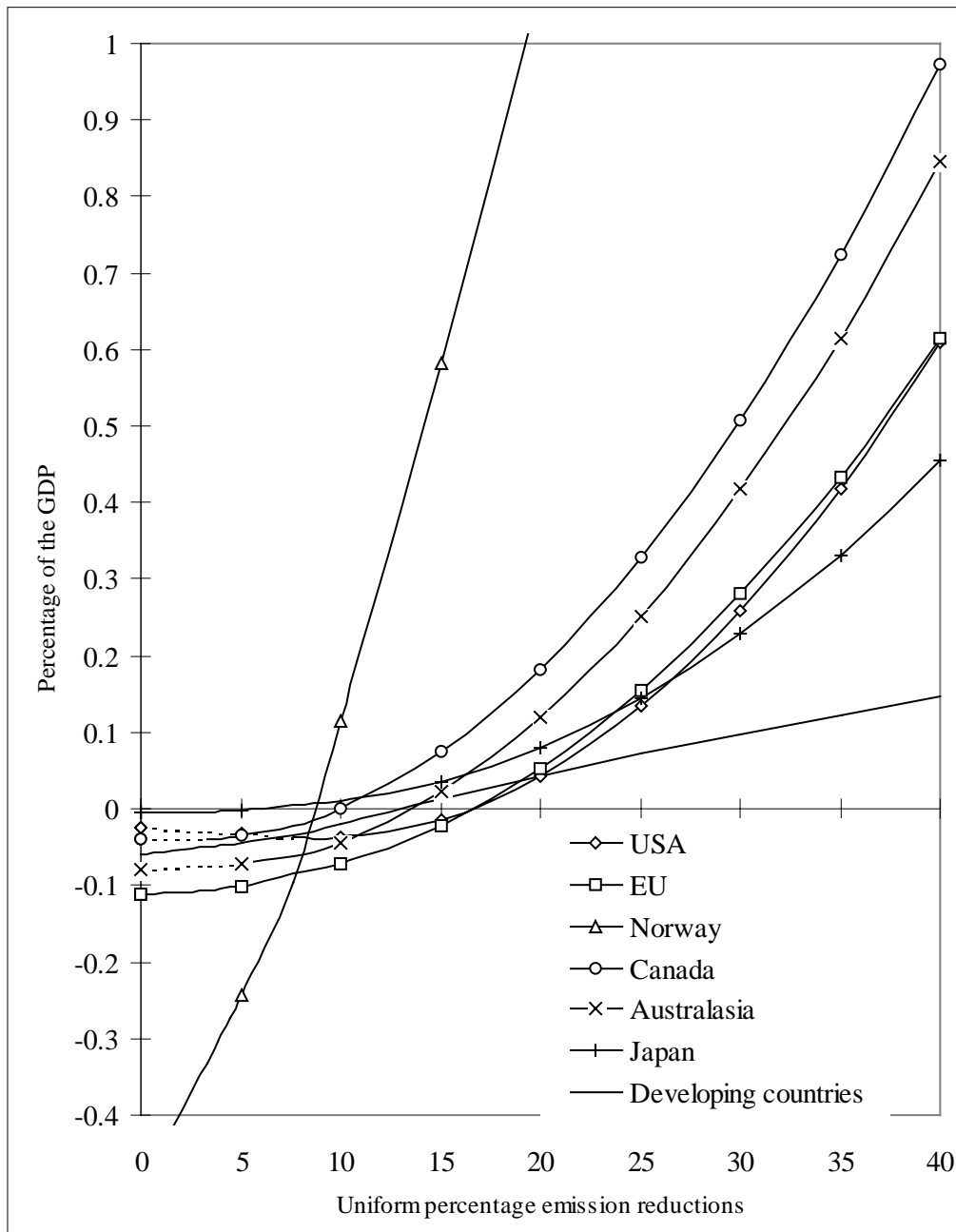
into account that this estimate is especially uncertain. Consequently, in this appendix a sensitivity analysis is performed with respect to the size of the MEB in the case where the emission reductions are 30%, cf. figure B.1 and B.2.

The estimated marginal abatement costs are increased when the assumed size of the MEB is reduced. Both figure B.1 and B.2 confirms that the size of the assumed MEB is important. However, we could notice that marginal abatement costs are less sensitive compared to total abatement costs. This result is explained by the fact that *marginal* public revenue increase from increased fossil fuel taxes might be limited at the chosen emission reduction level of 30% even though the *total* increase in public revenue generated by the climate policy is considerable.

As part of the sensitivity analysis figure B.3 reproduces the total abatement costs presented in figure 8 in the case where the MEB is 0.15. If figure B.3 is compared to figure 8, we will have at more complete impression to what extent the total abatement costs are altered when the assumed MEB is reduced significantly. The abatement cost curves are substantially elevated in almost all countries. Furthermore the estimated size of the no regret emission reductions are reduced substantially in the USA, Australasia and in Canada or even completely eliminated in Japan and EU.

Figure B.3 illustrates also the vulnerability of the Norwegian economy to other countries' reaction to a climate agreement. The position of the Norwegian cost curve is perhaps not quite straightforward to understand. The reason why this curve is below the vertical axis at low levels of emission reductions, is related to the assumption that the OECD-countries are introducing efficiently designed fossil fuel taxation systems as an immediate reaction to any climate agreement analyzed. At small emission reduction levels and a relatively low value of the MEB, generally speaking, this leads to increased taxes on coal, while the oil taxes are reduced. Consequently the world market price of oil is increased and the Norwegian terms of trade are improved.

Figure B.3 Income losses in the case where the MEB is 0.15. Climate agreements with flat rate emission reductions.



## Appendix C Detailed description of the simulation model

In this appendix the parametric form of the model used in section 3 is specified. The starting point is the production function of the numeraire good for each country or region that has the following functional form:

$$z_n = g_n(y_{1n}, y_{2n}, y_{3n}) = \sum_{i=1}^3 \alpha_{in} y_{in} + \sum_{i=1}^3 \alpha_{iin} y_{in}^2 + \alpha_{12n} y_{1n} y_{2n} + \alpha_{13n} y_{1n} y_{3n} + \alpha_{23n} y_{2n} y_{3n} + m(R_{0n}) \quad (\text{C.1})$$

The three first order conditions constitute the following equation system, cf. equation (3):

$$\begin{bmatrix} 2\alpha_{11n} & \alpha_{12n} & \alpha_{13n} \\ \alpha_{12n} & 2\alpha_{22n} & \alpha_{23n} \\ \alpha_{13n} & \alpha_{23n} & 2\alpha_{33n} \end{bmatrix} \begin{bmatrix} y_{1n} \\ y_{2n} \\ y_{3n} \end{bmatrix} = \begin{bmatrix} P_{1n} - \alpha_{1n} \\ P_{2n} - \alpha_{2n} \\ P_{3n} - \alpha_{3n} \end{bmatrix} \quad (\text{C.2})$$

Define  $H_n$  as determinant of the transpose of the Jacobi matrix of the above equation system, and  $H_{ijn}$  as the corresponding co-factor of element  $ij$  in  $H_n$ . Define:

$$\begin{aligned} a_{i0n} &= -\sum_{j=1}^3 \alpha_{jn} \frac{H_{jin}}{H_n}, \\ a_{i1n} &= \frac{H_{1in}}{H_n}, \\ a_{i2n} &= \frac{H_{2in}}{H_n}, \\ a_{i3n} &= \frac{H_{3in}}{H_n}. \end{aligned} \quad (\text{C.3})$$

The demand for fossil fuels in country  $n$  is:

$$y_{in} = D_{in}(P_{1n}, P_{2n}, P_{3n}) = a_{i0n} + a_{i1n} P_{1n} + a_{i2n} P_{2n} + a_{i3n} P_{3n}, \quad i = 1, 2, 3 \quad (\text{C.4})$$

The cost functions in fossil fuel production are specified as follows:

$$c_{in}(x_{in}) = \beta_{i0n} x_{in} + \beta_{i1n} x_{in}^2 \quad (\text{C.5})$$

In section 2 taxes on fossil fuel production were not included for simplicity. In the empirical model we need to include both taxes on production ( $t_{pin}$ ) and consumption ( $t_{cin}$ ) as well as a tax on profit in fossil fuel extraction ( $t_{\pi in}$ ). Assuming that the producers are price-takers we have the supply functions:

$$\begin{aligned} x_{in} &= s_{in} + b_{in}(p_i - t_{pin}), \quad i = 1, 2, \\ x_{3n} &= s_{3n} + b_{3n}(p_{3r} - t_{p3n}) \end{aligned} \quad (C.6)$$

## Market equilibrium

The equilibrium conditions of the oil and coal markets:

$$\sum_{n \in N} y_{in} = \sum_{n \in N} x_{in}, \quad i = 1, 2. \quad (C.7)$$

which are transformed to:

$$\sum_{n \in N} ((a_{11n} - b_{1n})p_1 + a_{12n}p_2) + \sum_{r \in R} \sum_{n \in r} (a_{13n}p_{3r}) = \sum_{n \in N} \left( s_{1n} - a_{1n} - b_{1n}t_{p1n} - \sum_{s=1,2,3} a_{1sn}t_{csn} \right) \quad (C.8)$$

$$\sum_{n \in N} (a_{21n}p_1 + (a_{22n} - b_{2n})p_2) + \sum_{r \in R} \sum_{n \in r} (a_{23n}p_{3r}) = \sum_{n \in N} \left( s_{2n} - a_{2n} - b_{2n}t_{p2n} - \sum_{s=1,2,3} a_{2sn}t_{csn} \right),$$

There is one equilibrium conditions to each of the three, regional gas markets:

$$\sum_{n \in r} y_{3n} = \sum_{n \in r} x_{3n}, \quad r \in \{A, E, P\} \quad (C.9)$$

where  $A$  represents North America,  $E$  is Europe and  $P$  the Pasific region. The equilibrium conditions of the gas markets could be written as follows:

$$\sum_{n \in r} a_{3n} + \sum_{n \in r} \sum_{s=1,2} a_{3sn}(p_s + t_{csn}) + \sum_{n \in r} a_{33n}(p_{3r} + t_{c3n}) = \sum_{n \in r} s_{3n} + \sum_{n \in r} b_{3n}(p_{3r} - t_{p3n}) \quad (C.10)$$

and are further transformed to:

$$\sum_{n \in r} a_{31n}p_1 + \sum_{n \in r} a_{32n}p_2 + \sum_{n \in r} (a_{33n} - b_{3n})p_{3r} = \sum_{n \in r} \left( s_{3n} - a_{3n} - b_{3n}t_{p3n} - \sum_{s=1,2,3} a_{3sn}t_{csn} \right) \quad (C.11)$$

Let us then define

$$\varphi_{fs} = \sum_{n \in N} (a_{fsn} - \delta_{fs}b_{fn}), \quad f = 1, 2, s=1, 2, \quad (C.12)$$

$$\varphi_{f3r} = \sum_{n \in r} a_{f3n}, \quad f = 1, 2, r = A, E, P, \quad (\text{C.13})$$

$$\varphi_{3rs} = \sum_{n \in r} (a_{3sn} - \delta_{3s} b_{3n}), \quad \forall r = A, E, P, \quad s = 1, 2, 3 \quad (\text{C.14})$$

$$\varepsilon_f = \sum_{n \in N} \left( s_{fn} - a_{fn} - b_{fn} t_{pfn} - \sum_{s=1,2,3} a_{fsn} t_{csn} \right), \quad f=1, 2 \quad (\text{C.15})$$

$$\varepsilon_{3r} = \sum_{n \in r} \left( s_{3n} - a_{3n} - b_{3n} t_{p3n} - \sum_{s=1,2,3} a_{3sn} t_{csn} \right), \quad r=A, E, P, \quad (\text{C.16})$$

The equilibrium conditions of the oil, coal and gas markets could then be written:

$$\varphi_{11} p_1 + \varphi_{12} p_2 + \sum_{r \in M} \varphi_{13r} p_{3r} = \varepsilon_1 \quad (\text{C.17})$$

$$\varphi_{21} p_1 + \varphi_{22} p_2 + \sum_{r \in M} \varphi_{23r} p_{3r} = \varepsilon_2 \quad (\text{C.18})$$

$$\varphi_{3r1} p_1 + \varphi_{3r2} p_2 + \varphi_{3r3} p_{3r} = \varepsilon_{3r}, \quad \forall r \in \{A, E, P\} \quad (\text{C.19})$$

The simultaneous system of equilibrium conditions could therefore be written:

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13A} & \varphi_{13E} & \varphi_{13P} \\ \varphi_{21} & \varphi_{22} & \varphi_{23A} & \varphi_{23E} & \varphi_{23P} \\ \varphi_{3A1} & \varphi_{3A2} & \varphi_{3A3} & 0 & 0 \\ \varphi_{3E1} & \varphi_{3E2} & 0 & \varphi_{3E3} & 0 \\ \varphi_{3P1} & \varphi_{3P2} & 0 & 0 & \varphi_{3P3} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_{3A} \\ p_{3E} \\ p_{3P} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{3A} \\ \varepsilon_{3E} \\ \varepsilon_{3P} \end{bmatrix} \quad (\text{C.20})$$

This equation system is linear and determines implicitly the prices in the five fossil fuel markets as linear functions of the tax rates in the countries (and regions), cf equations (8) and (9).

## The emission constraint

The emission constraint of country  $n$  is:

$$\sum_{j=1}^3 q_j y_{jn} = E_n \quad (\text{C.21})$$

where  $q_i$  is the carbon intensity of fossil fuel  $i$ . (If the fossil fuels are measured according to their CO<sub>2</sub> content,  $q_i = 1$ ,  $i=1,2,3$ .) Inserting the demand functions defined in (C.4) gives:

$$\sum_{j=1}^3 q_j a_{jn} + \sum_{j=1}^3 \sum_{i=1}^3 q_j a_{jin} p_i + \sum_{j=1}^3 \sum_{i=1}^3 q_j a_{jin} t_{ci} = E_n \quad (\text{C.22})$$

Reformulations give:

$$\sum_{j=1}^3 q_j a_{jn} + \sum_{i=1}^3 \left( \sum_{j=1}^3 q_j a_{jin} \right) p_i + \sum_{i=1}^3 \left( \sum_{j=1}^3 q_j a_{jin} \right) t_{cin} = E_n \quad (\text{C.23})$$

Define:

$$\xi_{0n} = E_n - \sum_{j=1}^3 q_j a_{jn} \quad \text{and} \quad \xi_{in} = \sum_{j=1}^3 q_j a_{jin} \quad (\text{C.24})$$

The emission constraints could then be written:

$$\sum_{i=1}^3 \xi_{in} p_i + \sum_{i=1}^3 \xi_{in} t_{cin} = \xi_{0n} \quad (\text{C.25})$$

### The fossil fuel prices and tax rates and shadow prices of the emission constraints in the Nash-equilibrium

The Lagrangian of the government in a country faced with an emission constraint is (cf. eq. (9) where the tax rate on profit was omitted for simplicity):

$$L_m = v_m(\cdot) + \sum_{i=1}^3 (1 - t_{\Gamma m}) \Pi_{mi}(\cdot) + R_{Fm} - \lambda_m \left( \sum_{i=1}^3 q_i y_{im} - E_m \right), \quad (\text{C.26})$$

The public revenue is:

$$R_{Fm} = \sum_{i=1}^3 t_{cim} y_{im} + \sum_{i=1}^3 t_{pim} x_{im} + t_{\Gamma m} \sum_{i=1}^3 \Pi_{im} \quad (\text{C.27})$$

For later use we will need:

$$\begin{aligned} \frac{dR_{Fm}}{dt_{ckm}} &= y_{km} + \sum_{j=1}^3 t_{cjm} \left\{ \sum_{i=1}^3 \frac{\partial y_{jm}}{\partial P_{im}} \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \right\} + \sum_{j=1}^3 t_{pjm} \frac{\partial x_{jm}}{\partial p_j} \frac{\partial p_j}{\partial t_{ckm}} + t_{\Gamma m} \sum_{j=1}^3 \frac{\partial \Pi_{jm}}{\partial p_j} \frac{\partial p_j}{\partial t_{ckm}} \\ &= y_{km} + \sum_{i=1}^3 \left\{ \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ikm} \right) \sum_{j=1}^3 t_{cjm} \frac{\partial y_{jm}}{\partial P_{im}} \right\} + \sum_{j=1}^3 t_{pjm} \frac{\partial x_{jm}}{\partial p_j} \frac{\partial p_j}{\partial t_{ckm}} + t_{\Gamma m} \sum_{j=1}^3 \frac{\partial \Pi_{jm}}{\partial p_j} \frac{\partial p_j}{\partial t_{ckm}} \end{aligned} \quad (\text{C.28})$$

The first order derivatives of the Lagrangian, with respect to the user tax on fossil fuel  $k$  in country  $m$ , is:

$$\begin{aligned} \frac{\partial L_m}{\partial t_{ckm}} &= \frac{\partial v_m}{\partial P_{km}} + \sum_{j=1}^3 \frac{\partial v_m}{\partial P_{jm}} \frac{\partial \bar{p}_j}{\partial t_{ckm}} + (1 + v_{mR}) \frac{dR_{Fm}}{dt_{ckm}} + (1 - t_{\Gamma m}) \sum_{j=1}^3 \frac{\partial \Pi_{jm}}{\partial p_j} \frac{\partial \bar{p}_j}{\partial t_{ckm}} \\ &\quad - \lambda_m \sum_{j=1}^3 \left( q_j \sum_{i=1}^3 \frac{\partial y_{jm}}{\partial P_{im}} \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \right) \end{aligned} \quad (\text{C.29})$$

We use (C.28) and Hotellings lemma, cf. Varian (1984), stating that  $\partial v_m / \partial P_{jm} = -y_{jm}$  and that  $\partial \Pi_{jm} / \partial p_j = x_{jm}$ . Then we have:

$$\begin{aligned} \frac{\partial L_m}{\partial t_{ckm}} &= \sum_{j=1}^3 \left( (1 - t_{\Gamma m}) x_{jm} - y_{jm} \right) \frac{\partial \bar{p}_j}{\partial t_{ckm}} + v_{mR} y_{km} \\ &\quad + (1 + v_{mR}) \left\{ \sum_{i=1}^3 \left\{ \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \sum_{j=1}^3 t_{cjm} \frac{\partial y_{jm}}{\partial P_{im}} \right\} + \sum_{j=1}^3 t_{pjm} \frac{\partial x_{jm}}{\partial p_j} \frac{\partial p_j}{\partial t_{ckm}} + t_{\Gamma m} \sum_{j=1}^3 x_{jm} \frac{\partial p_j}{\partial t_{ckm}} \right\} \\ &\quad - \lambda_m \sum_{i=1}^3 \left\{ \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \sum_{j=1}^3 q_j \frac{\partial y_{jm}}{\partial P_{im}} \right\} \end{aligned} \quad (\text{C.30})$$

for  $k=1,2,3$  and where  $\delta_{ij}=1$  if  $i=j$ ,  $\delta_{ij}=0$  if  $i \neq j$ .



$$\begin{aligned}
\frac{\partial L_m}{\partial t_{ckm}} &= \sum_{j=1}^3 \left( (1 + v_{mR} t_{\Gamma m}) x_{jm} - y_{jm} \right) \frac{\bar{\partial} p_j}{\partial t_{ckm}} + v_{mR} y_{km} \\
&+ (1 + v_{mR}) \left\{ \sum_{i=1}^3 \left[ \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \sum_{j=1}^3 t_{cjm} \frac{\partial y_{jm}}{\partial P_{im}} \right] + \sum_{j=1}^3 t_{pjm} \frac{\partial x_{jm}}{\partial p_j} \frac{\partial p_j}{\partial t_{ckm}} \right\} \\
&- \lambda_m \sum_{i=1}^3 \left[ \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \sum_{j=1}^3 q_j \frac{\partial y_{jm}}{\partial P_{im}} \right]
\end{aligned} \tag{C.31}$$

$$\begin{aligned}
\frac{\partial L_m}{\partial t_{ckm}} &= \sum_{j=1}^3 \left( (1 + v_{mR} t_{\Gamma m}) x_{jm} - y_{jm} \right) \frac{\bar{\partial} p_j}{\partial t_{ckm}} + v_{mR} y_{km} \\
&+ (1 + v_{mR}) \left\{ \sum_{j=1}^3 t_{cjm} \sum_{i=1}^3 \frac{\partial y_{jm}}{\partial P_{im}} \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \right\} + (1 + v_{mR}) \left( \sum_{j=1}^3 t_{pjm} \frac{\partial x_{jm}}{\partial p_j} \frac{\partial p_j}{\partial t_{ckm}} \right) \\
&- \lambda_m \left[ \sum_{j=1}^3 q_j \sum_{i=1}^3 \frac{\partial y_{jm}}{\partial P_{im}} \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \right]
\end{aligned} \tag{C.32}$$

$$\begin{aligned}
\frac{\partial L_m}{\partial t_{ckm}} &= \sum_{j=1}^3 \left( (1 + v_{mR} t_{\Gamma m}) x_{jm} - y_{jm} \right) \frac{\bar{\partial} p_j}{\partial t_{ckm}} + v_{mR} y_{km} \\
&+ \sum_{j=1}^3 \left\{ \left( (1 + v_{mR}) t_{cjm} - \lambda_m q_j \right) \sum_{i=1}^3 \frac{\partial y_{jm}}{\partial P_{im}} \left( \frac{\partial p_i}{\partial t_{ckm}} + \delta_{ik} \right) \right\} \\
&+ (1 + v_{mR}) \left( \sum_{j=1}^3 t_{pjm} \frac{\partial x_{jm}}{\partial p_j} \frac{\partial p_j}{\partial t_{ckm}} \right)
\end{aligned} \tag{C.33}$$

Let us then define  $\mu_{ikm} = \bar{\partial} p_i / \bar{\partial} t_{ckm}$ , which obviously is a constant, cf. (C.20), and recalling that  $\partial y_{im} / \partial P_{jm} = a_{ijm}$  and that  $\bar{\partial} x_{im} / \bar{\partial} p_i = b_{im}$ . Then we have:

$$\begin{aligned}
\frac{\bar{\partial} L_m}{\partial t_{ckm}} &= \sum_{j=1}^3 \left( (1 + v_{mR} t_{\Gamma m}) x_{jm} - y_{jm} \right) \mu_{jkm} + v_{mR} y_{km} \\
&+ \sum_{j=1}^3 \left\{ \left( (1 + v_{mR}) t_{cjm} - \lambda_m q_j \right) \sum_{i=1}^3 a_{jim} (\mu_{ikm} + \delta_{ik}) \right\} \\
&+ (1 + v_{mR}) \left( \sum_{j=1}^3 t_{pjm} b_{jm} \mu_{jkm} \right)
\end{aligned} \tag{C.34}$$

Define  $\sigma_{jkm} = \sum_{i=1}^3 a_{jim} (\mu_{ikm} + \delta_{ik})$ . Then we could write:

$$\begin{aligned}
\frac{dL}{dt_{ck}} &= \sum_{j=1}^3 \left( (1 + v_{mR} t_{\Gamma m}) \mu_{jkm} x_{jm} \right) - \sum_{j=1}^3 \left( \mu_{jkm} - \delta_{jk} v_{mR} \right) y_{jm} \\
&+ \sum_{j=1}^3 \left\{ \sigma_{jkm} \left( (1 + v_{mR}) t_{cjm} - q_j \lambda_m \right) \right\} \\
&+ \sum_{j=1}^3 (1 + v_{mR}) b_{jm} \mu_{jkm} t_{pjm}
\end{aligned} \tag{C.35}$$

Recalling that  $y_{im} = a_{im} + \sum_s a_{ism} (p_s + t_{csm})$ , and  $x_{im} = s_i + b_{im} (p_i - t_{pim})$ ,

then we have:

$$\begin{aligned}
\frac{\partial L}{\partial t_{ck}} &= \sum_{j=1}^3 \left( (1 + v_{mR} t_{\Gamma m}) \mu_{jkm} (s_{jm} + b_{jm} p_j - b_{jm} t_{pjm}) \right) \\
&- \sum_{j=1}^3 \left\{ \left( \mu_{jkm} - \delta_{jk} v_{mR} \right) \left( a_{jm} + \sum_{s=1}^3 a_{jsm} p_s + \sum_{s=1}^3 a_{jsm} t_{csm} \right) \right\} \\
&+ \sum_{j=1}^3 \left\{ \sigma_{jkm} \left( (1 + v_{mR}) t_{cjm} - q_j \lambda_m \right) \right\} \\
&+ \sum_{j=1}^3 (1 + v_{mR}) b_{jm} \mu_{jkm} t_{pjm}
\end{aligned} \tag{C.36}$$

$$\begin{aligned}
\frac{\partial L_m}{\partial t_{ckm}} &= \sum_{j=1}^3 (1 + v_{mR} t_{\Gamma m}) \mu_{jkm} s_{jm} + \sum_{j=1}^3 (1 + v_{mR} t_{\Gamma m}) \mu_{jkm} b_{jm} p_j \\
&- \sum_{j=1}^3 (1 + v_{mR} t_{\Gamma m}) \mu_{jkm} b_{jm} t_{pjm} \\
&- \sum_{j=1}^3 \left( \mu_{jkm} - \delta_{jk} v_{mR} \right) a_{jm} \\
&- \sum_{j=1}^3 \left[ \sum_{s=1}^3 \left( \mu_{skm} - \delta_{sk} v_{mR} \right) a_{sjm} \right] p_j \\
&- \sum_{j=1}^3 \left[ \sum_{s=1}^3 \left( \mu_{skm} - \delta_{sk} v_{mR} \right) a_{sjm} \right] t_{cjm} \\
&+ \sum_{j=1}^3 \sigma_{jkm} (1 + v_{mR}) t_{cjm} - \sum_{j=1}^3 \sigma_{jkm} q_j \lambda_m \\
&+ \sum_{j=1}^3 (1 + v_{mR}) b_{jm} \mu_{jkm} t_{pjm}
\end{aligned} \tag{C.37}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_m}{\partial t_{ckm}} &= \sum_{j=1}^3 \left[ (1 + v_{mR} t_{\Pi m}) \mu_{jkm} b_{jm} - \sum_{s=1}^3 (\mu_{skm} - \delta_{sk} v_{mR}) a_{sjm} \right] p_j \\
&\quad - \sum_{j=1}^3 \sigma_{jkm} q_j \lambda_m \\
&\quad + \sum_{j=1}^3 \left[ \sigma_{jkm} (1 + v_{mR}) - \sum_{s=1}^3 (\mu_{skm} - \delta_{sk} v_{mR}) a_{sjm} \right] t_{cjm} \\
&\quad + \left( - \sum_{j=1}^3 (1 + v_{mR} t_{\Pi m}) \mu_{jkm} b_{jm} + \sum_{j=1}^3 (1 + v_{mR}) b_{jm} \mu_{jkm} \right) t_{pjm} \\
&\quad + \sum_{j=1}^3 (1 + v_{mR} t_{\Pi m}) \mu_{jkm} s_{jm} - \sum_{j=1}^3 (\mu_{jkm} - \delta_{jk} v_{mR}) a_{jkm}
\end{aligned} \tag{C.38}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_m}{\partial t_{ckm}} &= \sum_{j=1}^3 \left[ (1 + v_{mR} t_{\Pi m}) \mu_{jkm} b_{jm} - \sum_{s=1}^3 (\mu_{skm} - \delta_{sk} v_{mR}) a_{sjm} \right] p_j \\
&\quad - \sum_{j=1}^3 \sigma_{jkm} q_j \lambda_m \\
&\quad + \sum_{j=1}^3 \left[ \sigma_{jkm} (1 + v_{mR}) - \sum_{s=1}^3 (\mu_{skm} - \delta_{sk} v_{mR}) a_{sjm} \right] t_{cjm} \\
&\quad + \sum_{j=1}^3 \left\{ (v_{mR} - v_{mR} t_{\Pi m}) \mu_{jkm} b_{jm} \right\} t_{pjm} \\
&\quad + \sum_{j=1}^3 (1 + v_{mR} t_{\Pi m}) \mu_{jkm} s_{jm} - \sum_{j=1}^3 (\mu_{jkm} - \delta_{jk} v_{mR}) a_{jkm}
\end{aligned} \tag{C.39}$$

The first order condition could then be written:

$$\begin{aligned}
& - \sum_{j=1}^3 \left[ (1 + v_{mR} t_{\Pi m}) \mu_{jkm} b_{jm} - \sum_{s=1}^3 (\mu_{skm} - \delta_{sk} v_{mR}) a_{sjm} \right] p_j \\
& \quad + \sum_{j=1}^3 \left[ \sum_{s=1}^3 (\mu_{skm} - \delta_{sk} v_{mR}) a_{sjm} - \sigma_{jkm} (1 + v_{mR}) \right] t_{cjm} + \sum_{j=1}^3 \sigma_{jkm} q_j \lambda_m \\
& = - \sum_{j=1}^3 (\mu_{jkm} - \delta_{jk} v_{mR}) a_{jkm} + \sum_{j=1}^3 \left\{ (v_{mR} - v_{mR} t_{\Pi m}) \mu_{jkm} b_{jm} \right\} t_{pjm} \\
& \quad + \sum_{j=1}^3 (1 + v_{mR} t_{\Pi m}) \mu_{jkm} s_{jm}
\end{aligned} \tag{C.40}$$

Then we define:

$$\begin{aligned}
\theta_{pksm} &= (1 + v_{mR} t_{\Pi m}) \mu_{skm} b_{sm} - \sum_{s=1}^3 (\mu_{jkm} - \delta_{jk} v_{mR}) a_{jkm} \\
\theta_{\lambda km} &= \sum_{j=1}^3 \sigma_{jkm} q_j \\
\theta_{tcksm} &= \sum_{j=1}^3 (\mu_{jkm} - \delta_{jk} v_{mR}) a_{jkm} - \sigma_{skm} (1 + v_{mR}) \\
\theta_{km} &= - \sum_{j=1}^3 (\mu_{jkm} - \delta_{jk} v_{mR}) a_{jkm} + \sum_{j=1}^3 \{ (v_{mR} - v_{mR} t_{\Pi m}) \mu_{jkm} b_{jm} \}_{pjm} \\
&\quad + \sum_{j=1}^3 (1 + v_{mR} t_{\Pi m}) \mu_{jkm} s_{jm}
\end{aligned} \tag{C.41}$$

The first order conditions could therefore be written as:

$$- \sum_{j=1}^3 \theta_{pkjm} p_j + \theta_{\lambda km} \lambda_m + \sum_{j=1}^3 \theta_{tckjm} t_{cjm} = \theta_{km} \tag{C.42}$$

Let us then define

$$\tilde{\mathcal{E}}_f = \sum_{n \in N} (s_{fn} - a_{fn} - b_{fn} t_{pfn}), f=1,2 \tag{C.43}$$

$$\tilde{\mathcal{E}}_{3r} = \sum_{n \in r} (s_{3n} - a_{3n} - b_{3n} t_{p3n}), r=A, E, P, \tag{C.44}$$

Then we could write the equilibrium conditions of the oil, coal and gas markets as follows:

$$\varphi_{11} p_1 + \varphi_{12} p_2 + \sum_{r \in R} \varphi_{13r} p_{3r} + \sum_{n \in N} \sum_{s=1,2,3} a_{1s} t_{csn} = \tilde{\mathcal{E}}_1 \tag{C.45}$$

$$\varphi_{21} p_1 + \varphi_{22} p_2 + \sum_{r \in R} \varphi_{23r} p_{3r} + \sum_{n \in N} \sum_{s=1,2,3} a_{2s} t_{csn} = \tilde{\mathcal{E}}_2 \tag{C.46}$$

$$\varphi_{3r1} p_1 + \varphi_{3r2} p_2 + \varphi_{3r3} p_{3r} + \sum_{n \in r} \sum_s a_{3s} t_{csn} = \tilde{\mathcal{E}}_{3r}, \quad \forall r \in R \tag{C.47}$$

Assuming that  $\rho_{nr} = \begin{cases} 1 & \text{if } n \in r \\ 0 & \text{if } n \notin r \end{cases}$ , we define the following set of matrixes and vectors:

$$\begin{aligned}
\Phi &= \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13A} & \varphi_{13E} & \varphi_{13P} \\ \varphi_{21} & \varphi_{22} & \varphi_{23A} & \varphi_{23E} & \varphi_{23P} \\ \varphi_{3A1} & \varphi_{3A2} & \varphi_{3A3} & 0 & 0 \\ \varphi_{3E1} & \varphi_{3E2} & 0 & \varphi_{3E3} & 0 \\ \varphi_{3P1} & \varphi_{3P2} & 0 & 0 & \varphi_{3P3} \end{bmatrix}, A_n = \begin{bmatrix} a_{11n} & a_{12n} & a_{13n} \\ a_{21n} & a_{22n} & a_{23n} \\ \delta_{rA} a_{31n} & \delta_{rA} a_{32n} & \delta_{rA} a_{33n} \\ \delta_{rE} a_{31n} & \delta_{rE} a_{32n} & \delta_{rE} a_{33n} \\ \delta_{rP} a_{31n} & \delta_{rP} a_{32n} & \delta_{rP} a_{33n} \end{bmatrix}, \\
\Theta_{pn} &= \begin{bmatrix} -\theta_{p11n} & -\theta_{p12n} & -\delta_{rA} \theta_{p13n} & -\delta_{rE} \theta_{p13n} & -\delta_{rP} \theta_{p13n} \\ -\theta_{p21n} & -\theta_{p22n} & -\delta_{rA} \theta_{p23n} & -\delta_{rE} \theta_{p23n} & -\delta_{rP} \theta_{p23n} \\ -\theta_{p31n} & -\theta_{p32n} & -\delta_{rA} \theta_{p33n} & -\delta_{rE} \theta_{p33n} & -\delta_{rP} \theta_{p33n} \end{bmatrix}, \\
\Theta_{in} &= \begin{bmatrix} \theta_{i11n} & \theta_{i12n} & \theta_{i13n} \\ \theta_{i21n} & \theta_{i22n} & \theta_{i23n} \\ \theta_{i31n} & \theta_{i32n} & \theta_{i33n} \end{bmatrix}, \Theta_{\lambda nM} = \begin{bmatrix} \delta_{1n} \theta_{\lambda 11} & \dots & \delta_{Mn} \theta_{\lambda 1M} \\ \delta_{1n} \theta_{\lambda 21} & \dots & \delta_{Mn} \theta_{\lambda 2M} \\ \delta_{1n} \theta_{\lambda 31} & \dots & \delta_{Mn} \theta_{\lambda 3M} \end{bmatrix} \\
\Xi_{pM} &= \begin{bmatrix} \xi_{11} & \xi_{21} & \rho_{1A} \xi_{31} & \rho_{1E} \xi_{31} & \rho_{1P} \xi_{31} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \xi_{1M} & \xi_{2M} & \rho_{MA} \xi_{3M} & \rho_{ME} \xi_{3M} & \rho_{MP} \xi_{3M} \end{bmatrix} \\
p &= \begin{bmatrix} p_1 \\ p_2 \\ p_{3A} \\ p_{3E} \\ p_{3P} \end{bmatrix}, t = \begin{bmatrix} t_{c11} \\ t_{c21} \\ t_{c31} \\ \vdots \\ t_{c1N} \\ t_{c2N} \\ t_{c3N} \end{bmatrix}, L_M = \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_M \end{bmatrix}, \tilde{\varepsilon} = \begin{bmatrix} \tilde{\varepsilon}_1 \\ \tilde{\varepsilon}_2 \\ \tilde{\varepsilon}_{3A} \\ \tilde{\varepsilon}_{3E} \\ \tilde{\varepsilon}^{3P} \end{bmatrix}, \theta = \begin{bmatrix} \theta_{11} \\ \theta_{21} \\ \theta_{31} \\ \vdots \\ \theta_{1N} \\ \theta_{2N} \\ \theta_{3N} \end{bmatrix}, X_M = \begin{bmatrix} \xi_{01} \\ \vdots \\ \xi_{0M} \end{bmatrix}
\end{aligned}$$

If all the emission constraints are binding in country 1,...,M, and non-binding in country M+1,...,N, then the tax rates in the Nash equilibrium are then determined in the following set of equations:

$$\begin{bmatrix} \Phi & A_1 & \dots & A_N & 0_{5 \times M} \\ \Theta_{p1} & \Theta_{i1} & \dots & 0_{3 \times 3} & \Theta_{\lambda 1M} \\ \vdots & \vdots & & \vdots & \vdots \\ \Theta_{pN} & 0_{3 \times 3} & \dots & \Theta_{iN} & \Theta_{\lambda NM} \\ \Xi_M & \Xi_{1M} & \dots & \Xi_{NM} & 0_{M \times M} \end{bmatrix} \begin{bmatrix} p \\ t \\ L_M \end{bmatrix} = \begin{bmatrix} \varepsilon \\ \theta \\ X_M \end{bmatrix} \quad (\text{C.48})$$

where  $0_{ij}$  is the null matrix of  $i$  rows and  $j$  columns. Assuming that country 1 is in North America and that country  $M$  and  $N$  are in the Pacific it is perhaps more transparent to write it in the following way:

$$\begin{bmatrix}
\varphi_{11} & \varphi_{12} & \varphi_{13A} & \varphi_{13E} & \varphi_{13P} & a_{111} & a_{121} & a_{131} & \dots & a_{11N} & a_{12N} & a_{13N} & 0 & \dots & 0 \\
\varphi_{21} & \varphi_{22} & \varphi_{13A} & \varphi_{13E} & \varphi_{13P} & a_{211} & a_{221} & a_{231} & \dots & a_{21N} & a_{22N} & a_{23N} & 0 & \dots & 0 \\
\varphi_{3A1} & \varphi_{3A1} & \varphi_{3A3} & 0 & 0 & a_{111} & a_{111} & a_{111} & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\
\varphi_{3E1} & \varphi_{3E2} & 0 & \varphi_{3E3} & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\
\varphi_{3P1} & \varphi_{11} & 0 & 0 & \varphi_{3P3} & 0 & 0 & 0 & \dots & a_{31N} & a_{32N} & a_{33N} & 0 & \dots & 0 \\
-\theta_{p111} & -\theta_{p121} & -\theta_{p131} & 0 & 0 & \theta_{i111} & \theta_{i121} & \theta_{i131} & \dots & 0 & 0 & 0 & \theta_{\lambda11} & \dots & 0 \\
-\theta_{p211} & -\theta_{p221} & -\theta_{p231} & 0 & 0 & \theta_{i211} & \theta_{i221} & \theta_{i231} & \dots & 0 & 0 & 0 & \theta_{\lambda21} & \dots & 0 \\
-\theta_{p311} & -\theta_{p321} & -\theta_{p331} & 0 & 0 & \theta_{i311} & \theta_{i321} & \theta_{i331} & \dots & 0 & 0 & 0 & \theta_{\lambda31} & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\theta_{p11N} & -\theta_{p12N} & 0 & 0 & -\theta_{p13N} & 0 & 0 & 0 & \dots & \theta_{i11N} & \theta_{i11N} & \theta_{i11N} & 0 & \dots & \theta_{\lambda1M} \\
-\theta_{p21N} & -\theta_{p22N} & 0 & 0 & -\theta_{p23N} & 0 & 0 & 0 & \dots & \theta_{i11N} & \theta_{i11N} & \theta_{i11N} & 0 & \dots & \theta_{\lambda2M} \\
-\theta_{p31N} & -\theta_{p32N} & 0 & 0 & -\theta_{p33N} & 0 & 0 & 0 & \dots & \theta_{i11N} & \theta_{i11N} & \theta_{i11N} & 0 & \dots & \theta_{\lambda3M} \\
\xi_{11} & \xi_{21} & \xi_{31} & 0 & 0 & \xi_{11} & \xi_{21} & \xi_{31} & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\xi_{1M} & \xi_{2M} & 0 & 0 & \xi_{3M} & 0 & 0 & 0 & \dots & \xi_{1M} & \xi_{2M} & \xi_{3M} & 0 & \dots & 0
\end{bmatrix}
\cdot
\begin{bmatrix}
p_1 \\
p_2 \\
p_{3A} \\
p_{3E} \\
p_{3P} \\
t_{c11} \\
t_{c21} \\
t_{c31} \\
\vdots \\
t_{c1N} \\
t_{c2N} \\
t_{c3N} \\
\lambda_1 \\
\vdots \\
\lambda_M
\end{bmatrix}
=
\begin{bmatrix}
\tilde{\varepsilon}_1 \\
\tilde{\varepsilon}_2 \\
\tilde{\varepsilon}_{3A} \\
\tilde{\varepsilon}_{3E} \\
\tilde{\varepsilon}_{3P} \\
\theta_{11} \\
\theta_{21} \\
\theta_{31} \\
\vdots \\
\theta_{1N} \\
\theta_{2N} \\
\theta_{3N} \\
\xi_{01} \\
\vdots \\
\xi_{0M}
\end{bmatrix}$$

## Appendix D Calibration of the parameters

We assume we know the price elasticities of demand and supply in a point in the price-tax space and calibrate the parameters from this information. In the case of linear demand functions this could be done in the following way:

$$a_{ifs} = e_{ifs} \frac{y_{if}}{p_s} \quad (\text{D.1})$$

$$a_{if} = y_{if} \left( 1 - \sum_{s=1}^3 e_{ifs} \left( 1 + \frac{t_{cis}}{p_s} \right) \right) \quad (\text{D.2})$$

$$b_{if} = \sigma_{if} \frac{x_{if}}{p_f} \quad (\text{D.3})$$

$$s_{if} = x_{if} \left( 1 - \sigma_{if} \left( 1 - \frac{t_{pif}}{p_f} \right) \right) \quad (\text{D.4})$$

The calibration is however more complicated because we need values also for the parameters in the utility function described in (C.1) and the chosen value must satisfy the restrictions in (C.3). The parameters of the utility functions are therefore found through iteration processes.

## Appendix E List of symbols

$M$	Set of regions, i.e. $M = \{A, E, P\}$ . $A$ is North America, $E$ is Europe, $P$ is the Pacific region.
$N$	Set of countries
$E_n$	Emission constraint of country $n$
$p_{3r}$	Price of natural gas in region $r = A, E, P$
$p_i$	World market price of fossil fuel $i=1,2$
$P_{in}$	$p_i + t_{cin}$ if $i=1,2$ . If $i = 3$ : $p_{3r} + t_{c3n}$ , $n \in r$
$R_{Fn}$	Public revenue collected from the taxation of consumption and production of fossil fuels in country $n$
$R_n$	Sum of public revenue in country $n$
$R_{On}$	Public revenue collected from other sources than taxation of other goods and services than fossil fuels in country $n$ .
$t_{cin}$	Tax on consumption of fossil fuel $i$ in country $n$ . Symbol used in the appendix to distinguish to producer taxes
$t_{pin}$	Tax on production of fossil fuel $i$ in country $n$
$t_{\pi in}$	Tax on profit in production of fossil fuel $i$ in country $n$
$W_n$	National welfare indicator
$x_{in}$	Indigenous production of fossil fuel $i=1,2,3$ in country $n$
$y_{in}$	Consumption of fossil fuel $i=1,2,3$ in country $n$
$z_n$	Production of the numeraire good in country $n$
$v_{Rn}$	Marginal Excess Burden of taxation in country $n$
$\lambda$	Shadow price of the emission constraint in country $n$



# ***This is CICERO***

CICERO was established by the Norwegian government in April 1990 as a non-profit organization associated with the University of Oslo.

The research concentrates on:

- International negotiations on climate agreements. The themes of the negotiations are distribution of costs and benefits, information and institutions.
- Global climate and regional environment effects in developing and industrialized countries. Integrated assessments include sustainable energy use and production, and optimal environmental and resource management.
- Indirect effects of emissions and feedback mechanisms in the climate system as a result of chemical processes in the atmosphere.

Contact details:

CICERO  
P.O. Box. 1129 Blindern  
N-0317 OSLO  
NORWAY

Telephone: +47 22 85 87 50  
Fax: +47 22 85 87 51  
Web: [www.cicero.uio.no](http://www.cicero.uio.no)  
E-mail: [admin@cicero.uio.no](mailto:admin@cicero.uio.no)

