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A Model for Global Responses to Anthropogenic Changes in the Environment (GRACE)

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Sammendrag:.
GRACE-modellen er en økonomisk likevektsmodell for hele verden med mange verdensregioner og mange økonomiske sektorer. Modellen er statisk, men med flere perioder. Den er utviklet for analyse av langsiktige økonomiske analyser av virkninger av klimaendringer og klimapolitikk. Denne rapporten beskriver modellstruktur og gir en detaljert presentasjon av relasjonene i modellen. Siktemålet er å gi en forståelse av hvordan modellen fungerer og danne en plattform for kritisk gjennomgang av resultater fra GRACE-modellen.

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Abstract:
The GRACE model is a multi-sector, multi-regional, recursive dynamic computable general equilibrium model. It was developed for long-term economic analysis of climate change impacts and greenhouse gas abatement policy. The purpose of this report is to provide a description of the model and make available the model equations and structure. This will help in understanding and critically examining the results obtained using GRACE.

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1 Introduction

The model for Global Responses to Anthropogenic Change in the Environment (GRACE) is a multi-sector, multi-regional, recursively dynamic global computable general equilibrium model (CGE) written in GAMS. It was developed for long-term economic analysis of climate change impacts and greenhouse gas abatement policy. The model allows for additional modules for analysis, including emissions permit trading and climate impacts on the forestry sector. Coupled with an atmospheric model, the model can also be used for integrated assessment modeling of the climate and economy. This note presents only the core model. A forestry module for GRACE is presented in Rive, Aaheim and Hauge (2005) and documentation of an atmospheric model that can be linked to GRACE is given in Fuglestvedt and Berntsen (1999).

Section 2 describes our data sources. Section 3 describes the general structure and flows within the model. Section 4 describes the final demand and production structures in the model, the treatment of income and trade, and the price structure. Section 5 outlines the greenhouse gas emissions inclusion, and Section 6 describes the dynamics of the model, and treatment of investments. The model equations are listed in Section 7.

2 Data

The GRACE employs the Global Trade Analysis Project (GTAP) database. The database contains “bilateral trade, transport, and protection data characterizing economic linkages among regions, together with individual-country input-output data bases that account for intersectoral linkages within each region” (Hertel, 1997: p.4). Version 5.4 of the data base (Dimaranan and McDougall, 2002) consists of data for 57 sectors and 78 regions. The original data base is converted into a GAMS readable format using the GTAPinGAMS conversion tool (Rutherford and Paltsev, 2000). These data are then aggregated in order to create a full social accounting matrix (SAM) for each region. A SAM is a general and consistent macroeconomic accounting framework, based on UN’s System for National Accounts. In GRACE, the global SAM consists of input-output matrices for each region (inputs of primary factors to sectors, and output to consumption and investment), and with trade between the regions. Adjustments are made to the GTAP database to ensure that the investments in the base year are made consistent with the growth assumptions applied in the model runs. These adjustments, to the initial capital stock and investments, are described in Section 6.

GRACE uses greenhouse gas emissions data from the GTAP/EPA project “Towards An Integrated Data Base for Assessing the Potential for Greenhouse Gas Mitigation” (Lee 2002). The database provides CO2 emissions from intermediate use of each of the six GTAP energy commodities, in each of the 57 GTAP regions and 66 sectors, with differentiation for either domestic or imported energy sources (a 6*57*66*2 matrix). The database also provides emissions for energy use in the household sector (a 6*57*2 matrix).

3 General Structure

The GRACE model is constructed and calibrated around the GTAP v5.4 database. Its general structure is based on a number of other models, with some adjustments. The production and final demand structure is based on the GTAP-E model by Rutherford and Paltsev (2000). The dynamics and treatment of investments were developed from the GTAP-Dyn (Ianchovichina and McDougall, 2000) model.

The GRACE model consists of five main elements: Production, an Armington aggregation of domestic goods and imports, a Regional Household, a Global Bank and a Global Trust. The flow of payments in the model is shown in Figure 1. Production is contingent on the availability of initial endowments, labor, capital and natural resources in each region. All endowments are fully employed at each point in time and allocated to the production sectors according to the global demand for cross deliveries from production sectors and from the final demand sectors. Each region demands a domestic and an imported share of each good (Armington good). The total demand for a good from one region is the domestic demand plus the sum of demand for imports from that region in all the other regions, and contributions to the international transport sector. All regions therefore produce according to their comparative advantage. Regions contribute savings to a global bank at a fixed savings rate to satisfy the exogenous assumptions of capital stock growth. The resulting amount of available capital is allocated to regions at the rate of return. Returns to capital are allocated to regions based on their historical investments.

In the basic GRACE model, there are nine regions (**r**, or **rr**), eleven production sectors (**i**, or **ii**), three final demand sectors (**j**), and three primary factors (**f**) (see Table 1). Production in each sector takes place using input from the primary factors; capital (**K(i,r)**), nature (**N(i,r)**), and labor (**L(i,r)**) and from other sectors (**IO(ii,i,r)**)[1]. Output from each sector (**XD(i,r)**) can be sold domestically (**XDD(i,r)**) or exported (**EX(i,r)**). The model allows some flexibility with regards to sector inclusion and aggregation. Due to the structure of production, however, certain sectors must be present on their own for the model to work. These are the final energy sectors: refined oil, electricity, gas, and coal. The GTAP database includes five factors of production, which are aggregated into the three primary factors in the GRACE model. In GRACE, there is full flexibility with regards to regional aggregation. In each region, domestically produced (**XDD(i,r)**) and imported goods (**IM(i,r)**) are differentiable, and aggregated into an Armington composite good (**X(i,r)**), which in turn is demanded by both the production and final demand sectors. A fixed factor markup for international transport is added to each traded commodity.

Each region includes a Regional Household, which is endowed with nature (**RTOT(r)**) and labor resources (**LTOT(r)**), and a capital stock (**VKB(r)**). Tax and factor income for each region, including returns to capital (**CAP(r)**), accrues to the Regional Household. This regional income (**REGINC(r)**) is distributed to the Public (**INC('PUB',r)**) and Private (**INC('PRI',r)**) final demand sectors and savings (**REGSAVE(r)**). The Final Demand sectors demand goods from the production sectors (**FD(i,j,r)**). All regional savings are sent to a Global Bank, which distributes funds to regional Investment sectors (**INC('INV',r)**) towards increasing the capital stock in each region. The end of period capital stock (**VKE(r)**), net of any depreciation (**CAPDEP(r)**) is carried forward to the next period. Given that capital ownership is independent of location (e.g. a region may have investments abroad), a Global Trust is used to track the investments of each region over time. This ensures that the returns to investments are sent to the originating Regional Household.

[1] As in the actual model code, abbreviations of each flow listed here may be indexed by region, sector, final demand, or factor. For example, production **XD(i,r)** represents domestic production of sector **i** in region **r**. Demand **IO(ii,i,r)** represents intermediate demand of good produced by **ii** in sector **i** in region **r**.

Table 1: GRACE regions and sectors aggregated from the GTAP database, and their abbreviations

Regions	Production Sectors	Final Demand Sectors	Factors of Production
China (CHN)	Agriculture (AGR)	Private (PRI)	Capital (CAP)
India (IND)	Forestry (FRS)	Public (PUB)	Nature (RES)
Rest of Asia (ROA)	Paper and Pulp Products (PPP)	Investment (INV)	Labor (LAB)
European Union 25 (EU)	Lumber (LUM)		
Rest of EFTA (NO)	Coal (COL)		
North American Free Trade Region (NAFTA)	Electricity (ELC)		
Central and South America (LAM)	Gas (GAS)		
Russian Federation (RUS)	Refined Oil (REF)		
Rest of World (RUS)	Crude Oil (CRU)		
	Services (SER)		
	Other products (PRO)		

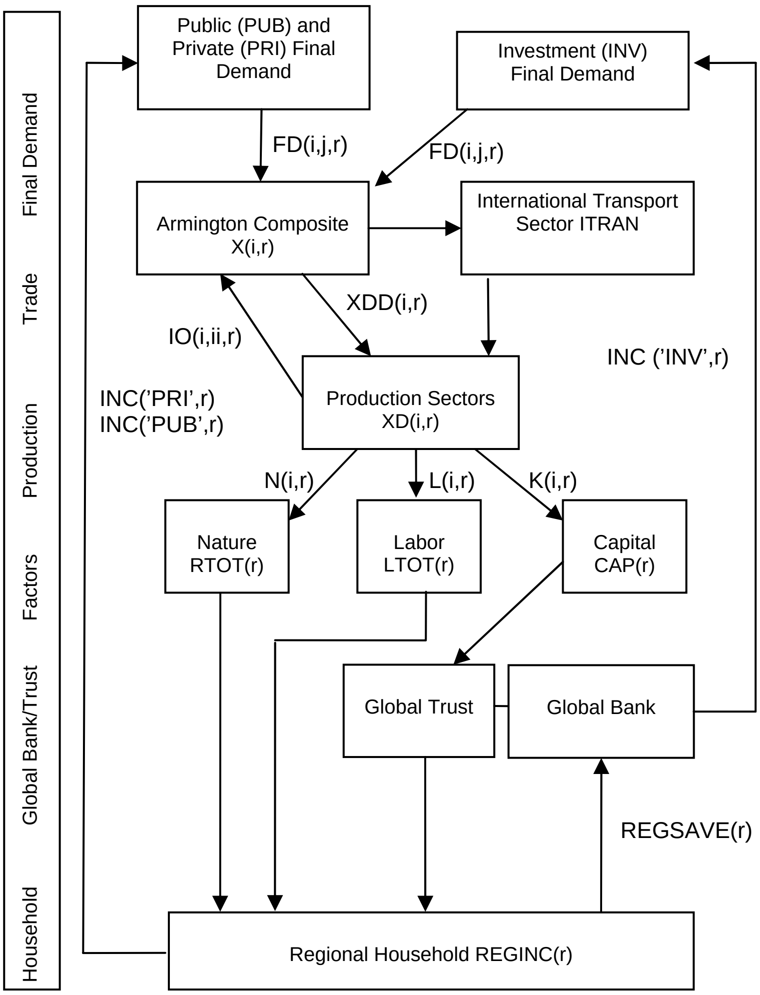


Figure 1: Flow of payments in the GRACE model

4 Production and Final Demand Structures

4.1 Production

Each production sector demands factor inputs (**K(i,r)**,**L(i,r)**, and **N(i,r)**) and intermediate goods (**IO(ii,i,r)**) from other sectors. The demand structure is a nested constant elasticity of substitution (CES) tree illustrated in Figure 2. The substitution elasticities at each nest are defined in the Figure, prefixed 'RX'. Sector abbreviations are found in Table 1. At the top level of the demand structure, substitution is made between Reserve (**XR(i,r)**) and Intermediate (**XI(i,r)**) aggregate goods. The Reserve aggregate is made up of demand for capital and nature, while the Intermediate aggregate is made up of labor and the Commodities aggregate (**XC(i,r)**). The Commodities aggregate nest requires fixed shares (i.e. Leontief technology) of the Goods (**XG(i,r)**) and Energy (**XU(i,r)**) aggregates. The Materials (**XM(i,r)**) aggregate nest requires fixed shares of the non-energy, non-service sectors. At the top level of the Energy aggregate, the substitution is made between refined oil (**IO('REF',i,r)**) (mobile energy source) and the Non-Oil (**XA(i,r)**) aggregate (stationary energy sources). Within the Non-Oil aggregate, substitution is made between electricity (**IO('ELC',i,r)**) and the Non-Electric aggregate (**XF(i,r)**).

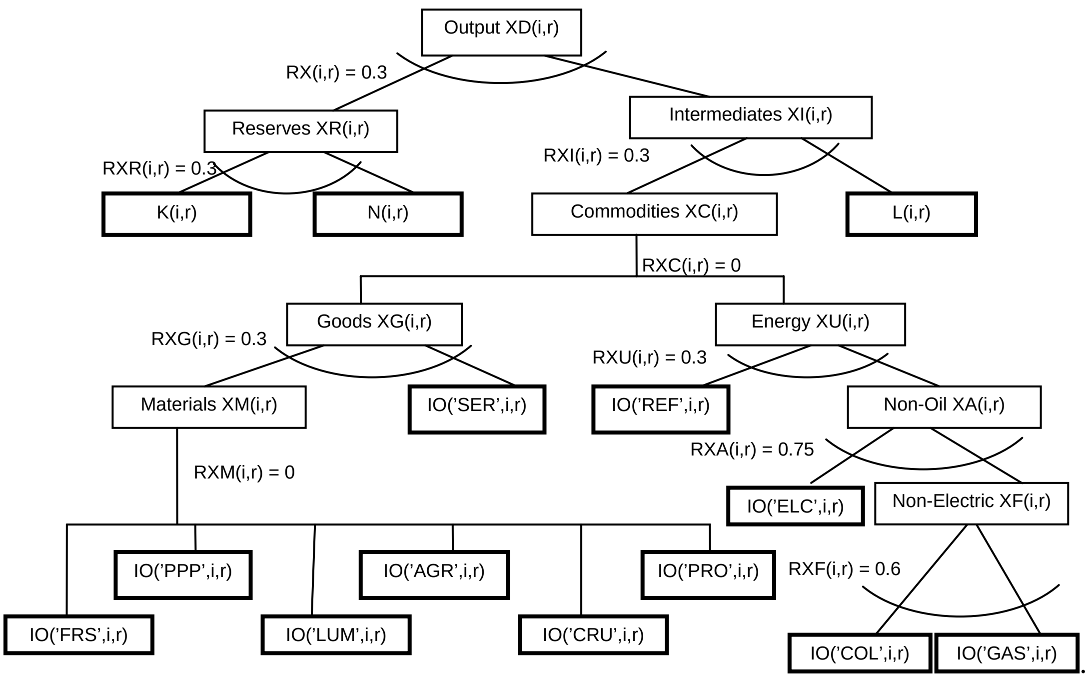


Figure 2: GRACE Production demand tree

4.2 The Regional Household and the Global Bank

As in GTAP, GRACE includes a Regional Household to which tax revenue and factor income accrues (**REGINC(r)**). In addition, there exist three final demand sectors: Private, Public, and Investment (see Figure 3). Income to the Private (**INC('PRI',r)**) and Public (**INC('PRI',r)**) sectors is provided by the Regional Household at fixed shares, calibrated in the base year. The Investment sector is funded through a Global Bank. Regional Households contribute to the Global Bank (**REGSAVE(r)**) at a fixed savings rate. The Global Bank distributes this income back to the individual Investment sectors (**INC('INV',r)**). Due to the fixed savings rate, we do not enforce a current account balance constraint.

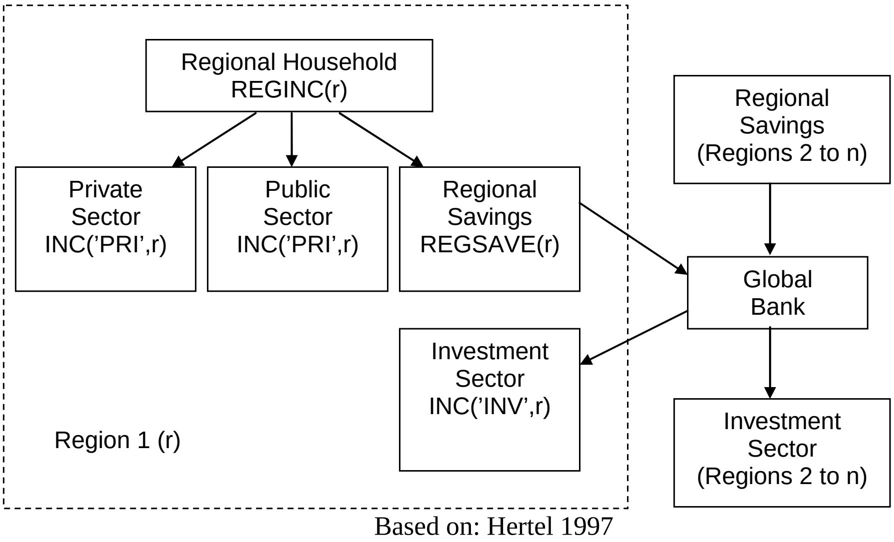


Figure 3: The Regional Household and the Global Bank

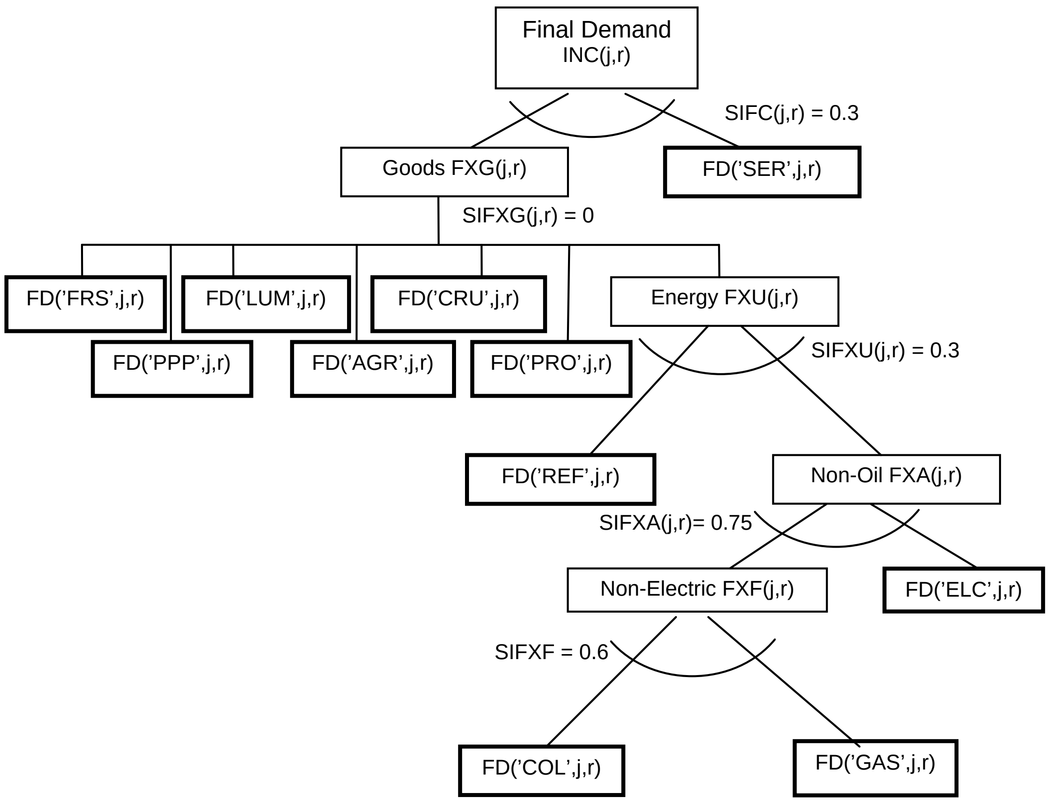


Figure 4: GRACE Final Demand tree

All three Final Demand sectors demand goods from the Production sectors (**FD(i,j,r)**). The structure of this demand is illustrated in Figure 4. The substitution elasticities at each level are denoted with the parameter prefixed 'SI'. At the top level, the Final Demand substitutes between the Goods aggregate (**FXG(j,r)**) and services. The Goods aggregate is made up of fixed share inputs (Leontief technology) of the Energy aggregate (**FXU(j,r)**) and the six remaining non-energy sectors. The Energy aggregate nest is constructed similarly to the Production demand structure in Figure 2. Final Demand agents must first substitute between mobile and stationary energy sources (refined oil vs. Non-Oil (**FXA(j,r)**), then electricity and Non-Electricity (**FXF(j,r)**) sources.

4.3 Trade and International Transport

In GRACE, the regional economies are linked through bilateral trade flows. All goods can be traded internationally, with the exception of the primary factors. Rather than assuming that goods are exported to a global pool, trade occurs bilaterally between countries. Bilateral imports of the same good from different regions (**EXR(i,rr,r)**) are combined into an import aggregate (**IM(i,r)**). This is aggregated with domestically produced goods into a single 'Armington good' (**X(i,r)**) which is then demanded by the Production (as **IO(i,ii,r)**) and Final Demand (as **FD(i,j,r)**) sectors. Substitution between bilateral imports of the same good and between domestic and aggregate imports is modeled through a CES function (Figure 5). The substitution elasticities are denoted in the figure with prefix 'RI'. Exceptions to the listed substitution elasticities are made for the following sectors: (a) REF (RIM = 6), (b) ELC (RIM = 0.5; RIMR = 0.3), and (c) GAS and COL (RIMR = 4).

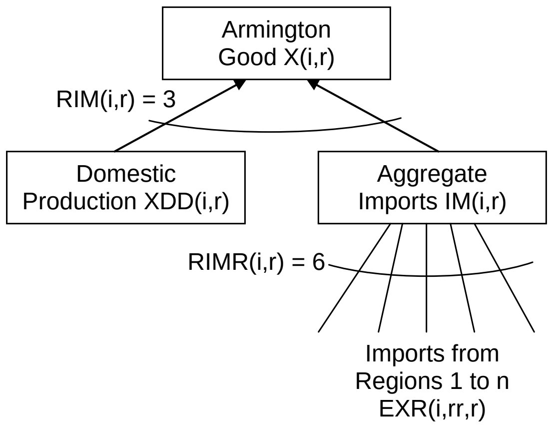
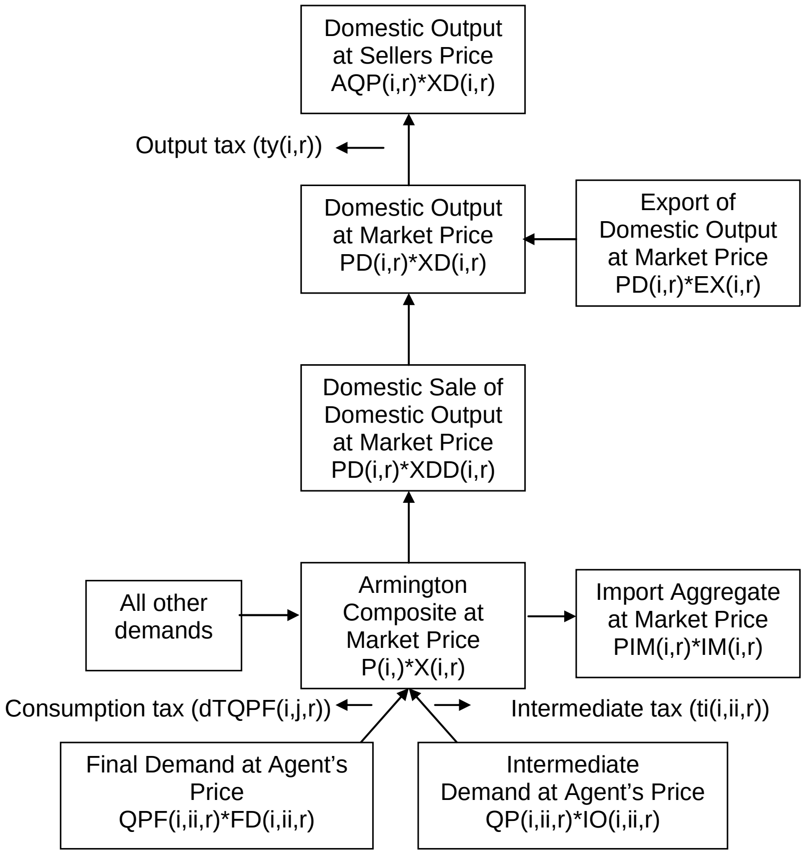


Figure 5: Bilateral imports and the Armington aggregate

When a good is traded, a price premium is paid by the importing country to the international transport sector. This price premium is determined by a fixed transport factor (**dTRANS(i,r,rr)**) derived from the base year data. The transport is provided by a Cobb-Douglas composite of the service good from the individual regions.

4.4 Price Structure

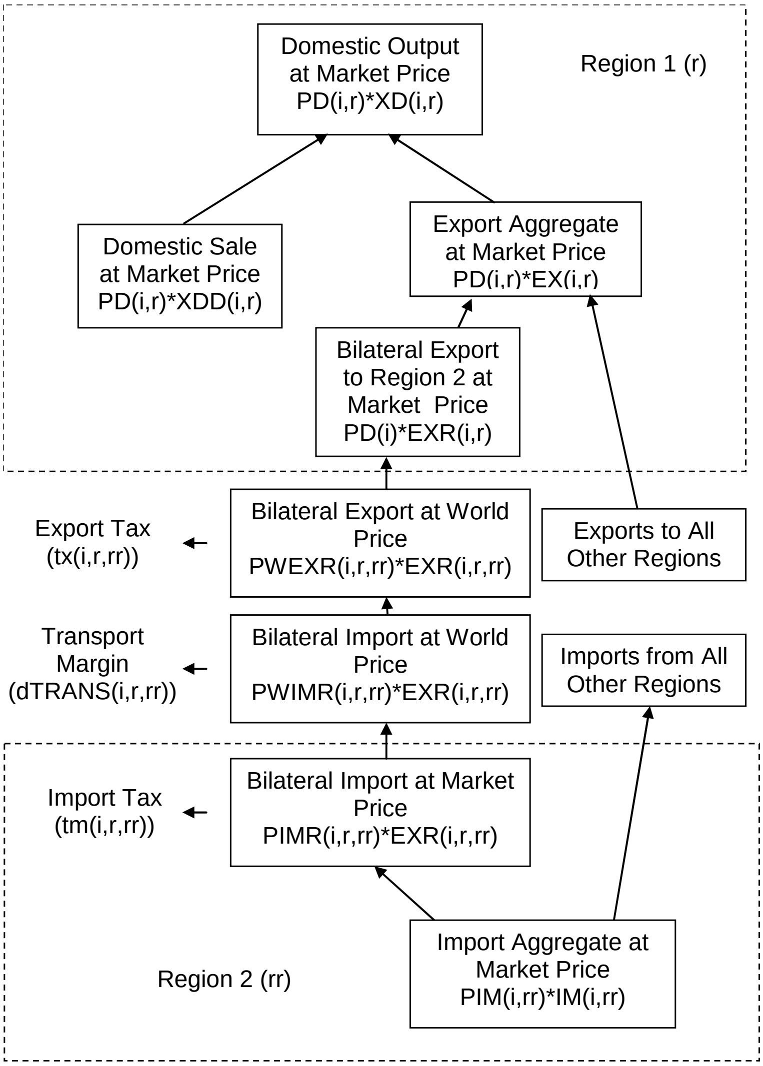
For simplicity, we have until now avoided discussion of prices in the economy. The pricing structure in GRACE is identical to that in the GTAP model (Hertel, 1997). In a CGE framework, there are no commodity unit "prices" as they are understood in the real world. The GTAP database provides only total values (price times quantity) of the good and payment flows in the economy. We are thus free to choose any representative price to use in the model, which is then associated with the quantity level found in the equilibrium database. We typically choose a market price of **P(i,r)** = 1 at equilibrium for all Armington composite goods (**X(i,r)**) in all regions. However, the actual price of each good faced by each agent in the economy will differ from this market price due to 'price wedges'. Within each region, these price wedges are brought about by taxes (or subsidies), which either raise (or lower) the price faced by each agent relative to the market price of 1. Figure 6 illustrates the price differentiation between agents. A price wedge occurs between the seller's price (**AQP(i,r)**) and the market price (**PD(i,r)**) of domestic outputs, brought about by an output tax. There is a second price wedge between the agent's price of intermediate (**QP(i,ii,r)**) and final (**QPF(i,j,r)**) demand, as a result of consumption and intermediate taxes.



Based on: Hertel (1997)

Figure 6: Price structure within each region. Arrows denote payment flows.

In trade, price wedges are brought about by tariffs and transport margins (see Figure 7). Domestic market (**PD(i,r)**) and region-specific world prices of bilateral export goods (**PWEXR(i,r,rr)**) are differentiated through an export tax. This price is further differentiated into the world price of import (**PWIMR(i,r,rr)**) through a transport margin, and the market price of import (**PIMR(i,r,rr)**) through an import tariff.



Based on: Hertel 1997

Figure 7: Pricing structure in trade. Arrows denote payment flows.

5 Greenhouse Gas Emissions

The GRACE model includes emissions of CO_2 from industrial and household sources. These emissions are relatively simple to model, as they are emitted at a roughly fixed rate of fossil fuel use. GRACE uses emissions data from the GTAP/EPA project “Towards An Integrated Data Base for Assessing the Potential for Greenhouse Gas Mitigation” (Lee 2002). The database provides CO_2 emissions from intermediate use of each of the six GTAP energy commodities, in each of the 57 GTAP regions and 66 sectors, with differentiation for either domestic or imported energy sources (a 6*57*66*2 matrix). The database also provides emissions for energy use in the household sector (a 6*57*2 matrix). In GRACE, the production sector emissions are aggregated to three energy sources (refined oil, gas, and coal), and the 9 regions and 11 sectors, with no differentiation for domestic or imported energy sources (a 3*9*11 matrix). For the final demand, we assume the same GTAP household emissions factors across Public, Private, and Investment sectors (a 3*9 matrix). CO_2 emissions are then calculated from the fixed emissions factors from intermediate and final fossil fuel use. Over time these factors fall, under assumptions of autonomous technological improvement, independent of abatement policy.

6 Investment and Dynamics

The GRACE model is a recursive dynamic model, wherein multiple static equilibria are solved to model the development of the economy over time. Each static equilibrium represents a single time period, and is connected to the next by the growth in the capital stock. Each time period is solved independently and thus unlike intertemporal dynamic models, computation complexity does not increase when time periods are added. The model is typically run for twenty 5-year periods from its base year.

In the model, economic growth is driven by growth in the capital stock over time. The value of the capital stock at the end of each time period is given by the value of the stock at the beginning of the period (net of depreciation) plus the value of investments made. This end of period capital stock is then available at the beginning of the next time period.

Investments in each region are made by the Global Bank (see Section 4). We follow the investment theory of the GTAP-Dyn model, where investors seek to equalize expected rates of return (**RORE(r)**) across all regions. If an imbalance arises, and one region develops a higher expected rate of return than the others, investments will be shifted toward that region until the expected rate of return is again equalized. The expected rate of return in each period becomes the actual rate of return (**RORC(r)**) in the next period. In a recursive dynamic setup, investors are myopic, and do not account for future prices when making their investment decisions.

As expected, we find that the GTAP base year data does not conform to our investment theory. In reality, investment choices are affected by a multitude of factors, including risk aversion, information availability, and preferences. To correct for the discrepancy, we make an adjustment to the database initial capital stocks so that the expected rate of return is equalized across all regions – and conforms to our theory from the start. While such an adjustment may not be preferable, it is simple and acceptable when we are less concerned with the short-term. Possible alternative treatments are discussed by Ianchovichina and McDougall (2000).

In the basic version of GRACE, we assume an annual depreciation (**dDEP(r)**) rate of 4%, and an interest/discount rate (**dINTEREST(r)**) of 5%. The economic growth in each region is moderated through the use of an assumed 'normal' growth rate (**KHAT(r)**) for the capital stock – at which the rate of return does not fall. This, in effect, guides the investments to each region so that the capital stock (**VKB(r)**) (and economy) grows at the rate assumed by the growth scenario. We make a second adjustment to the base data, related to this assumed growth rate. The base year model will exhibit growth rates that differ from our assumed rates. We thus adjust the investments (**INC('INV',r)**) in each region using an approach by Paltsev (1999), such that in the base year:

$$INC('INV',r) = VKB(r) \times \left(KHAT(r) + dDEP(r)\right) \tag{1}$$

Returns to capital (i.e. income from investments) are distributed to the Regional Households by the Global Trust based on ownership, rather than the location of capital. This accounts for any regional investments that are made abroad. Income from capital in each region (**CAP(r)**) first accrues to the Global Trust at the global rate of return. This income is then distributed back to the Regional Households based on the cumulative value of each region's savings over time. The distribution of income from the Global Trust to the Regional Households is based on each region's share of the total global cumulative savings. These savings, like capital, depreciate over time.

We must make an exception to this treatment, however, for the base year capital stock. In the GTAP database, the income from the capital stock in each region accrues only to its own Regional Household. Thus, the Households own the base year capital stock within their region, thereby violating our assumption that capital ownership is not location specific. As we wish for our first time period to replicate the base year data, we must account for this anomaly by assuming that (i) the base year capital stock is only owned by the region (i.e. Household) where it is located, and (ii) from only the second time period onwards is capital ownership not location-specific. This means that the Global Trust must track the depreciating value of the base year capital stock along with the value of the cumulative savings.

7 Model Equations

Variables

XD(i,r)	Output in sector i in region r
AQP(i,r)	Agents price of output in sector i
XR(i,r)	Demand for reserve aggregate in sector i
XI(i,r)	Demand for intermediates aggregate in sector i
QXR(i,r)	Price of reserve aggregate in sector i
QXI(i,r)	Price of intermediates aggregate in sector i
K(i,r)	Demand for real capital in sector i
N(i,r)	Demand for natural resources in sector i
PK(r)	Market price of capital
PN(r)	Market price of natural resources
QK(i,r)	User price of capital in sector i
QN(i,r)	User price of natural resources in sector i
L(i,r)	Demand for labour in sector i
XC(i,r)	Demand for commodities aggregate in sector i
PL(r)	Market price of labor
QL(i,r)	User price of labor in sector i
QXC(i,r)	Price of commodities aggregate in sector i
XU(i,r)	Demand for energy aggregate in sector i
XG(i,r)	Demand for goods aggregate in sector i
QXU(i,r)	Price of energy aggregate in sector i
QXG(i,r)	Price of goods aggregate in sector i

IO(i,ii,r)	Demand for intermediate good i in sector i
P(i,r)	Market price of Armington good i
QP(i,ii,r)	User price of good i in sector i
XM(i,r)	Demand for materials in sector i
QXM(i,r)	Price of materials in sector i
INC(j,r)	Income to final delivery j
REGINC(r)	Income to regional household
PUBIF(r)	Public tax income from production activities
PUBIH(r)	Public tax income from final deliveries
PUBIY(r)	Public tax income from output
PUBEX(r)	Public tax income from exports
PUBIM(r)	Public tax income from imports
REGSAVE(r)	Savings in regional household r
CAPDEP(r)	Capital depreciation in region r
REGINV(r)	Investment transfers to region r from global bank
GLOBINC	Income to global bank
VKE(r)	Value of capital stock (end of period)
RORE(r)	Expected rate of return on capital
RORG	Global expected rate of return on capital
PKTRUST	Price of global trust capital returns
PD(i,r)	Market price of domestic production of good i
X(i,r)	Armington good i
EX(i,r)	Exports of good i
XDD(i,r)	Domestic consumption of domestic good i
PEX(i,r)	Market price of export good i
ER(r)	Exchange rate
PIM(i,r)	Market price of import good i
IM(i,r)	Imports of good i
PDD(i,r)	Market price of domestic consumption of domestic good i
EXR(i,r,rr)	Exports of good i from r to rr
PEXR(i,r,rr)	Market price of good i export from r to rr

PWEXR(i,r,rr)	World price of good i export from r to rr
PWIMR(i,r,rr)	World price of good i import from r to rr
PIMR(i,r,rr)	Market price of good i import from r to rr
PITRAN	Price of international transport service
ITRANR(r)	Service provided to international transport sector by region r
ITRAN	International transport service
XF(y,r)	Demand for Non-Electric aggregate
QXF(y,r)	Price of Non-Electric aggregate
XA(y,r)	Demand for Non-Oil aggregate
QXA(y,r)	Price of Non-Oil aggregate
FD(i,j,r)	Demand for good i in final delivery j
QPF(i,j,r)	User price of good i in final delivery j
QFXG(j,r)	Price of goods aggregate to final delivery j
FXG(j,r)	Goods aggregate to final delivery j
FXU(j,r)	Energy aggregate to final delivery j
QFXU(j,r)	Price of energy aggregate to final delivery j
FXA(j,r)	Demand for non-oil aggregate
QFXA(j,r)	Price of non-oil aggregate
FXF(j,r)	Demand for non-electric aggregate
QFXF(j,r)	Price of non-electric aggregate
TOTCO2(r)	Regional CO2 emissions
WORLDCO2	World total CO2
GDP(r)	Price-indexed gross domestic product

Parameters

LTOT(r)	Total supply of labor in region r
CAP(r)	Returns to capital in region r
RTOT(r)	Total natural resource endowment

GX(i,r)	Scale parameter for output (Reserves – Intermediates nest) in sector i
AX(i,r)	Distribution parameter for output (Reserves – Intermediates nest) in sector i
RX(i,r)	Substitution parameter for output (Reserves – Intermediates nest) in sector i

Note: CES parameters for other nests are excluded for brevity, but follow the same prefix system

COMMU(i,r)	Leontief share of Energy (XU) in Commodities (XC)
COMMG(i,r)	Leontief share of Goods (XU) in Commodities (XC)
LEONM(i,ii,r)	Leontief share of intermediate i (IO) into sector ii Materials (XG) aggregate
FGEN(j,r)	Fixed factor share of energy aggregate in final demand goods aggregate
FGFD(i,j,r)	Fixed factor share of other inputs (FD) in final demand goods aggregate

ty(i,r)	Output tax on production sector i
ti(i,ii,r)	Intermediate input tax of good i demand in sector ii
tf(f,i,r)	Factor tax on demand of factor f in sector i
tx(i,r,rr)	Export tax rate on good i export from region r to rr
tm(i,r,rr)	Import tariff rate on good i import from region r to rr
dTQPF(i,j,r)	Tax rate of final demand of good i in j

INCSHARE(j,r)	Constant share of regional income to final demand j
ITRANSHARE(r)	Share of international transport service provided by region r
dTRANS(i,r,rr)	International transport premium for good i trade from region r to rr

dDEP(r)	Rate of depreciation of capital stock
RORC(r)	Current rate of return on capital

IOCINT(i,ii,r)	C intensity of intermediate use of energy i in sector ii
FDCINT(i,r)	C intensity of final demand use of energy

BY_VKB(r)	Base year capital stock value
SAVINGS(r)	Cumulative savings
t_size	Size of model time period
GLOBTRUST	Value of global trust of capital returns
dINTEREST(r)	Interest rate
GLOBTRUSTSHARE(r)	Regional share of global trust capital returns
KHAT(r)	Normal rate of growth in the capital stock in region r

Production Sector Demand

$$XD(i,r) = GX(i,r)\left[AX(i,r)\times XR(i,r)^{RX(i,r)} + (1-AX(i,r))\times XI(i,r)^{RX(i,r)}\right]^{\frac{1}{RX(i,r)}} \tag{2}$$

$$XR(i,r) = GXR(i,r)\left[AXR(i,r)\times K(i,r)^{RXR(i,r)} + (1-AXR(i,r))\times N(i,r)^{RXR(i,r)}\right]^{\frac{1}{RXR(i,r)}} \tag{3}$$

$$XI(i,r) = GXI(i,r)\left[AXI(i,r)\times XC(i,r)^{RXI(i,r)} + (1-AXI(i,r))\times L(i,r)^{RXI(i,r)}\right]^{\frac{1}{RXI(i,r)}} \tag{4}$$

$$XU(i,r) = XC(i,r)\times COMMU(i,r) \tag{5}$$

$$XG(i,r) = XC(i,r)\times COMMG(i,r) \tag{6}$$

$$XG(i,r) = GXG(i,r)\begin{bmatrix} AXG(i,r)\times XM(i,r)^{RXG(i,r)} \\ +(1-AXG(i,r))\times IO('SER',i,r)^{RXG(i,r)} \end{bmatrix}^{\frac{1}{RXG(i,r)}} \tag{7}$$

$$XU(i,r) = GXU(i,r)\begin{bmatrix} AXU(i,r)\times IO('REF',i,r)^{RXU(i,r)} \\ +(1-AXU(i,r))\times XA(i,r)^{RXU(i,r)} \end{bmatrix}^{\frac{1}{RXU(i,r)}} \tag{8}$$

$$XA(i,r) = GXA(i,r)\begin{bmatrix} AXA(i,r)\times IO('ELC',i,r)^{RXA(i,r)} \\ +(1-AXA(i,r))\times XF(i,r)^{RXA(i,r)} \end{bmatrix}^{\frac{1}{RXA(i,r)}} \tag{9}$$

$$XF(i,r) = GXF(i,r)\begin{bmatrix} AXF(i,r)\times IO('COL',i,r)^{RXF(i,r)} \\ +(1-AXF(i,r))\times IO('GAS',i,r)^{RXF(i,r)} \end{bmatrix}^{\frac{1}{RXF(i,r)}} \tag{10}$$

$$IO(ii,i,r) = XM(i,r)\times LEONM(ii,r)$$

s.t. $ii = FRS, PPP, LUM, AGR, CRU, PRO$ (11)

Production Sector Zero Profit

$$AQP(i,r)\times XD(i,r) = QXR(i,r)\times XR(i,r) + QXI(i,r)\times XI(i,r) \tag{12}$$

$$QXR(i,r)\times XR(i,r) = QK(i,r)\times K(i,r) + QN(i,r)\times N(i,r) \tag{13}$$

$$QXI(i,r)\times XI(i,r) = QXC(i,r)\times XC(i,r) + QL(i,r)\times L(i,r) \tag{14}$$

$$QXC(i,r)\times XC(i,r) = QXU(i,r)\times XU(i,r) + QXG(i,r)\times XG(i,r) \tag{15}$$

$$QXG(i,r)\times XG(i,r) = QXM(i,r)\times XM(i,r) + QP('SER',i,r)\times IO('SER',i,r) \tag{16}$$

$$QXU(i,r)\times XU(i,r) = QXA(i,r)\times XA(i,r) + QP('REF',i,r)\times IO('REF',i,r) \tag{17}$$

$$QXA(i,r)\times XA(i,r) = QXF(i,r)\times XF(i,r) + QP('ELC',i,r)\times IO('ELC',i,r) \tag{18}$$

$$\begin{aligned} &QXF(i,r)\times XF(i,r) = \\ &QP('GAS',i,r)\times IO('GAS',i,r) + QP('COL',i,r)\times IO('COL',i,r) \end{aligned} \tag{19}$$

$$QXM(i,r) \times XM(i,r) = \sum_{ii} QP(ii,i,r) \times IO(ii,i,r) \text{ s.t.}$$
$$ii = FRS, PPP, LUM, AGR, CRU, PRO \tag{20}$$

User Prices

$$QK(i,r) = PK(r) \times (1 + tf('CAP',i,r)) \tag{21}$$

$$QL(i,r) = PL(r) \times (1 + tf('LAB',i,r)) \tag{22}$$

$$QN(i,r) = PN(r) \times (1 + tf('RES',i,r)) \tag{23}$$

$$QP(ii,i,r) = P(ii,r) \times (1 + ti(ii,i,r)) \tag{24}$$

$$AQP(i,r) = PD(i,r) \times (1 - ty(i,r)) \tag{25}$$

$$QPF(i,j,r) = P(i,r) \times (1 - dTQPF(i,j,r)) \tag{26}$$

Regional Household Income

$$\begin{aligned} PUBIF(r) = {} & \sum_{i} \begin{bmatrix} PK(r) \times tf('CAP',i,r) \times L(i,r) + PL(r) \times tf('LAB',i,r) \\ \times L(i,r) + PN(r) \times tf('RES',i,r) \times N(i,r) \end{bmatrix} \\ & + \sum_{i,ii} \left[P(i,r) \times ti(i,ii,r) \times IO(i,ii,r) \right] \end{aligned} \tag{27}$$

$$PUBIH(r) = \sum_{i,j} \left[P(i,r) \times dTQPF(i,j,r) \times FD(i,j,r) \right] \tag{28}$$

$$PUBIY(r) = \sum_{i} \left[PD(i,r) \times ty(i,r) \times XD(i,r) \right] \tag{29}$$

$$PUBEX(r) = \sum_{i,rr} \left[PD(i,r) \times tx(i,r,rr) \times EXR(i,r,rr) \right] \tag{30}$$

$$PUBIM(r) = \sum_{i,rr} \left[PWIMR(i,rr,r) \times tm(i,rr,r) \times EXR(i,rr,r) \right] \tag{31}$$

$$\begin{aligned} REGINC(r) = {} & PL(r) \times LTOT(r) + PKTRUST \times GLOBTRUST \\ & \times GLOBTRUSTSHARE(r) + PN(r) \times RTOT(r) + PUBIF(r) + PUBIH(r) \\ & + PUBIY(r) + PUBEX(r) + PUBIM(r) \end{aligned} \tag{32}$$

Global Bank and Trust

$$PKTRUST = \frac{\sum_{r} \left(PK(r) \times CAP(r) \right)}{\sum_{r} CAP(r)} \tag{33}$$

$$INC('PRI',r) = INCSHARE('PRI',r) \times REGINC(r) \tag{34}$$

$$INC('PUB',r) = INCSHARE('PUB',r) \times REGINC(r) \tag{35}$$

$$REGSAVE(r) = INCSHARE('INV', r) \times REGINC(r) \quad (36)$$

$$\sum_r REGSAVE(r) = \sum_r INC('INV', r) \quad (37)$$

$$CAPDEP(r) = VKB(r) \times dDEP(r) \quad (38)$$

$$PK(r) \times VKE(r) = PK(r) \times VKB(r) + INC('INV', r) - PK(r) \times CAPDEP(r) \quad (39)$$

$$RORE(r) = RORC(r) \times \left[1 - RORFLEX(r) \times \left(\frac{VKE(r)}{VKB(r)} - 1 - KHAT(r)\right)\right] \quad (40)$$

$$RORE(r) = RORG \quad (41)$$

Final Demand

$$INC(j,r) = \begin{bmatrix} BFC(j,r) \times FD('SER', j,r)^{SIFC(j,r)} \\ + (1 - BFC(j,r)) \times FXG(j,r)^{SIFC(j,r)} \end{bmatrix}^{\frac{1}{SIFC(j,r)}} \quad (42)$$

$$FD(i,j,r) = FGFD(i,j,r) \times FXG(j,r)$$

s.t. $i = FRS, PPP, LUM, AGR, CRU, PRO$ (43)

$$FXU(i,j,r) = FGEN(i,j,r) \times FXG(j,r) \quad (44)$$

$$FXU(j,r) = GFXU(j,r) \times \begin{bmatrix} BFXU(j,r) \times FD('REF', j,r)^{SIFXU(j,r)} \\ + (1 - BFXU(j,r)) \times FXA(j,r)^{SIFXU(j,r)} \end{bmatrix}^{\frac{1}{SIFXU(j,r)}} \quad (45)$$

$$FXA(j,r) = GFXA(j,r) \times \begin{bmatrix} BFXA(j,r) \times FD('ELC', j,r)^{SIFXA(j,r)} \\ + (1 - BFXA(j,r)) \times FXF(j,r)^{SIFXA(j,r)} \end{bmatrix}^{\frac{1}{SIFXA(j,r)}} \quad (46)$$

$$FXF(j,r) = GFXF(j,r) \times \begin{bmatrix} BFXF(j,r) \times FD('COL', j,r)^{SIFXF(j,r)} \\ + (1 - BFXF(j,r)) \times FD('GAS', j,r)^{SIFXF(j,r)} \end{bmatrix}^{\frac{1}{SIFXF(j,r)}} \quad (47)$$

Final Demand Zero Profit

$$INC(j,r) = QFXG(j,r) \times FXG(j,r) + QPF(i,j,r) \times FD(i,j,r) \quad (48)$$

$$QFXG(j,r) \times FXG(j,r) = QFXU(j,r) \times FXU(j,r) + \sum_i \left[QPF(i,j,r) \times FD(i,j,r)\right]$$

s.t. $i = FRS, PPP, LUM, AGR, CRU, PRO$ (49)

$$\begin{aligned} &QFXU(j,r) \times FXU(j,r) = \\ &QFXA(j,r) \times FXA(j,r) + QPF('REF', j,r) \times FD('REF', j,r) \end{aligned} \quad (50)$$

$$QFXA(j,r)\times FXA(j,r)= \\ QFXF(j,r)\times FXF(j,r)+QPF('ELC',j,r)\times FD('ELC',j,r) \tag{51}$$

$$QFXF(j,r)\times FXF(j,r)= \\ QPF('GAS',j,r)\times FD('GAS',j,r)+QPF('COL',j,r)\times FD('COL',j,r) \tag{52}$$

Market Clearing

$$CAP(r)=\sum_i K(i,r) \tag{53}$$

$$RTOT(r)=\sum_i N(i,r) \tag{54}$$

$$LTOT(r)=\sum_i L(i,r) \tag{55}$$

$$X(i,r)=\sum_{ii} IO(i,ii,r)+\sum_j FD(i,j,r) \tag{56}$$

$$XD(i,r)=XDD(i,r)+EX(i,r)+ITRANR(r)\$(i=SER) \tag{57}$$

Armington Composite and Trade

$$X(i,r)=GIM(i,r)\begin{bmatrix} AIM(i,r)\times IM(i,r)^{RIM(i,r)} \\ +(1-AIM(i,r))\times XDD(i,r)^{RIM(i,r)}\end{bmatrix}^{\frac{1}{RIM(i,r)}} \tag{58}$$

$$EX(i,r)=\sum_{rr} EXR(i,r,rr) \tag{59}$$

$$PWEXR(i,r,rr)=PD(i,r)\times\left(1+tx(i,r,rr)\right) \tag{60}$$

$$PWIMR(i,r,rr)=PWEXR(i,r,rr)+PITRAN\times dTRANS(i,r,rr) \tag{61}$$

$$PIMR(i,r,rr)=PWIMR(i,r,rr)\times\left(1+tm(i,r,rr)\right) \tag{62}$$

$$PIM(i,rr)=\frac{\sum_r\left(PIMR(i,r,rr)\times EXR(i,r,rr)\right)}{IM(i,rr)} \tag{63}$$

$$P(i,r)\times X(i,r)=PIM(i,r)\times IM(i,r)+PD(i,r)\times XDD(i,r) \tag{64}$$

International Transport

$$PD('SER',r)\times ITRANR(r)=ITRANSHARE(r)\times PITRAN\times ITRAN \tag{65}$$

$$ITRAN=\sum_r ITRAN(r) \tag{66}$$

$$ITRAN=\sum_{i,r,rr}\left(dTRANS(i,r,rr)\times EXR(i,r,rr)\right) \tag{67}$$

Carbon Emissions

$$TOTCO2(r) = \sum_{i,j} \begin{matrix}[FD(i,j,r) \times FDCINT(i,j,r)] \\ + \sum_{i,ii} [IO(i,ii,r) \times IOCINT(i,ii,r)]\end{matrix} \tag{68}$$

$$WORLDCO2(r) = \sum_{r} TOTCO2(r) \tag{69}$$

Gross Domestic Product

$$GDP(r) = \left(PL(r) \times LTOT(r) + PN(r) \times RTOT(r) + PK(r) \times CAP(r)\right) \times \frac{\sum_i \left(PZ(i,r) \times X(i,r)\right)}{\sum_i \left(P(i,r) \times X(i,r)\right)} \tag{70}$$

Dynamics *(applied after each solution)*

$$BY\_VKB(r) = BYVKB(r) \times \left(1 - dDEP(r)\right)^{t\_size} \tag{71}$$

$$SAVINGS(r) = SAVINGS(r) \times \left(1 - dDEP(r)\right)^{t\_size} + REGSAVE(r) \times \frac{t\_size}{PK(r)} \tag{72}$$

$$LTOT(r) = LTOT(r) \times \left(\frac{VKE(r)}{VKB(r)}\right)^{t\_size} \tag{73}$$

$$RTOT(r) = RTOT(r) \times \left(\frac{VKE(r)}{VKB(r)}\right)^{t\_size} \tag{74}$$

$$VKB(r) = VKB(r) \times \left(\frac{VKE(r)}{VKB(r)}\right)^{t\_size} \tag{75}$$

$$RORC(r) = RORE(r) \tag{76}$$

$$CAP(r) = RORC(r) \times VKB(r) \tag{77}$$

$$GLOBTRUST = \sum_r CAP(r) \tag{78}$$

$$dINTEREST(r) = RORC(r) - dDEP(r) \tag{79}$$

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